Some Recent Results on Monitoring the Rate of a Rare Event

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Abstract A growing number of applications involve monitoring with rare event data. The event of interest could be, for example, a nonconforming manufactured item, a congenital malformation, or an industrial accident. The most common approaches for monitoring such processes involve using an exponential distribution to model the time between the events or using a Bernoulli distribution to model whether or not each opportunity for the event results in its occurrence. The use of a sequence of independent Bernoulli random variables leads to a geometric distribution for the number of non-occurrences between the occurrences of the rare events. One surveillance method is to use a power transformation on the exponential or geometric observations to achieve approximate normality of the in-control distribution and then use a standard individuals control chart. We add to the argument that use of this approach is very counterproductive and cover some alternative approaches. We discuss the choice of appropriate performance metrics. The strong adverse effect of Phase I parameter estimation on Phase II performance of various charts is then summarized. In addition, the important practical issue of the effect of aggregation of counts over time, some generalizations of standard methods, and some promising research ideas are discussed.

Keywords Impact of data aggregation • Monitoring geometric distribution • Phase I parameter estimation • Power transformation

1 Introduction

In an increasing number of applications interest is in the monitoring of a relatively rare event. It is often assumed that the practitioner has the results of a sequence of independent Bernoulli random variables, where a value of one indicates the event occurred and a value of zero indicates nonoccurrence of the event. Thus the number of trials between events has a geometric distribution for a stable process. Under

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this scenario one wants to monitor the event probability. In other applications it is commonly assumed that the time between events has an exponential distribution and one monitors the average time between events. Most often the focus is on detecting process deterioration, i.e., an increase in the probability of the adverse event or a decrease in the average time between events.

Szarka and Woodall (2011) reviewed the extensive number of methods that have been proposed for monitoring processes using Bernoulli data. Generally, it is difficult to better the performance of the Bernoulli cumulative sum (CUSUM) chart of Reynolds and Stoumbos (1999). The Bernoulli and geometric CUSUM charts can be designed to be equivalent, as discussed by Szarka and Woodall (2012). With respect to monitoring the mean of an exponential distribution it is difficult to outperform the exponential CUSUM chart studied by Lucas (1985) and Gan (1994). Levinson (2011) argued that control charts should not be used with healthcare rare event data because in many situations there is an assignable cause for each error, e.g., each hospital-acquired infection or serious prescription error, and each incident should be investigated. We agree that serious adverse events should be investigated whether or not they result in a control chart signal. The investigation of rare adverse events, however, and the implementation of process improvements to prevent future such errors, does not preclude using a control chart to determine if the rate of such events has increased or decreased over time. In fact, a control chart can be used to evaluate the success of any process improvement initiative.

2 Performance Metrics

The choice of appropriate performance metrics for comparing surveillance schemes for monitoring Bernoulli and exponential data is quite important. The usual Average Run Length (ARL) metric refers to the average number of points plotted on the chart until a signal is given. This metric is most clearly appropriate when the time between the plotted points is constant. For exponential and geometric random variables each plotted point corresponds to the occurrence of the event of interest. Thus the ARL is the expected number of events until the chart signals. If the event is quite adverse such as a serious accident or medical error, then this interpretation of the ARL is very useful even though the time between events varies. If the process does deteriorate, then we would like to detect it with as few adverse events as possible.

In some cases, such as in monitoring the number of near-miss accidents, it may be informative to use a metric that reflects the actual time required to obtain an out-of-control signal. Thus one can consider the number of Bernoulli trials until an out-of-control signal is given for Bernoulli data, leading to its average, the ANOS. The ANOS will be proportional to the average time before a signal if the rate at which the Bernoulli trials are observed is constant over time. For exponentially distributed data one could consider the average time to signal, the ATS. If the process is stable, then ANOS = ARL/ p and ATS = ARL* θ, where p and θ are the Bernoulli probability and the exponential mean, respectively.

To assess out-of-control performance we believe it is most realistic to consider steady-state performance where the shift in the parameter occurs at some time after monitoring has begun. This approach was discussed by Zhang et al. (2007) and Szarka and Woodall (2011), among others. Under this scenario one cannot easily convert the ARL metric to the ANOS and ATS metrics. Consideration of steady-state performance of competing methods is important because some methods have an implicit headstart feature that results in good zero-state performance, but poor steady-state performance. An example is the sets method of Chen (1978) studied by Capizzi (1994) and Sego et al. (2008). The basic sets method based on Bernoulli data signals a rate increase if the most recent k geometric waiting times are all below a specified constant. The sets method and its variations have been proposed for monitoring the rate of congenital malformations.

In addition, we note, as an example, that Liu et al. (2006) used zero-state performance comparisons instead of steady-state comparisons. With zero-state performance comparisons the shift in the parameter is assumed to occur at the start of monitoring or when the chart statistic is at its initial value. They considered charts based on the times between r events, where $r > 1$, that will seem to be more effective than they would be for the delayed shifts assumed for steady-state analysis. It is important to note that with these methods one waits until r events have occurred to obtain one count, waits until r additional events occur to obtain the next count, and so forth. When methods are based on waiting times which are aggregated over time like this or when comparing methods based on counts taken at differing levels of aggregation over time, then the ARL metric is not meaningful and either the ATS or ANOS metric should be used.

3 The Use of a Power Transformation

We first consider the time-between-occurrence control chart as discussed by Montgomery (2013, pp. 332–333). We assume that when the process is in control that the successive times are independent exponential random variables with a mean of θ_0. Montgomery suggested the transformation method proposed by Nelson (1994) under which the exponential random variables, $X_1, X_2, X_3, \ldots, X_m$ in Phase I are transformed using a power transformation $Y = X^{0.2777}$ in order to achieve approximate normality. After the transformation is made then one uses the standard individuals control chart with 3-sigma control limits based on the average moving range. This chart is frequently referred to as a "t-chart" in the literature, where t refers to "time."

Even though this seems like a reasonable approach, the performance of the resulting t-chart is quite poor. McCool and Joyner-Motley (1998) studied Nelson's method and the use of a logarithmic transformation. More recently, Santiago and Smith (2013) showed that if the in-control parameter is assumed to be known then with 3-sigma control limits the out-of-control average run length (ARL) values when there are decreases in the mean are far higher than the in-control ARL. This

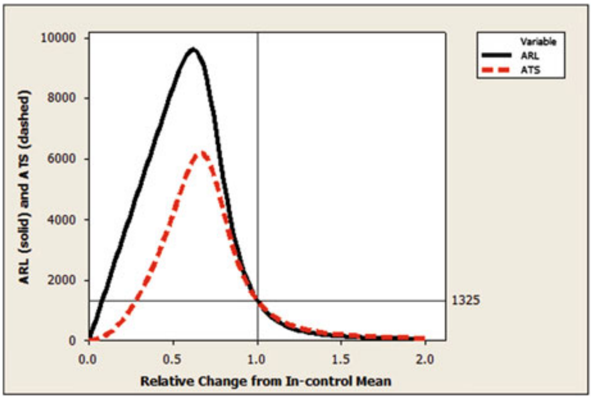

Fig. 1 Exact ARL and ATS values for the t-Chart based on Nelson (1994) Transformation, ($\theta_0 = 1$ and $ARL_0 = 1325$)

striking behavior is illustrated by the solid line in Fig. 1. The average time between events must decrease to a very small fraction of the in-control value θ_0 in order for a signal to be given on average with fewer plotted points.

Even though there are more points on average plotted on the chart before a signal is given when the mean decreases, one should note that there is a shorter average time period between the plotted points. Thus, in some cases it may be informative to consider the ATS metric, which is ARL*θ. This metric gives a better indication of how quickly a signal is given. One can see from the dashed line we added in Fig. 1 that the t-chart does not do well in detecting decreases in the exponential mean quickly, but the performance based on the ATS metric is not as poor as for the ARL metric. One could adjust the control limits of the chart to reduce the amount of ARL-bias, but this would take away the simplicity of the approach. Szarka and Woodall (2011) discussed this issue for similar charts.

We next consider the case in which the value of the in-control parameter p_0 is unknown for a Bernoulli process. Table 1 shows the percentage of the time one will have a useful lower control limit (LCL) if one uses Nelson (1994) power transformation with geometrically distributed data and the standard individuals chart with limits based on the average moving range. Each geometric observation is the number of trials required for the event to occur. Because interest is most likely to be in detecting increases in the probability of the event probability, and thus decreases in the mean of the geometric waiting time, there is a very good chance that the control chart resulting from the use of Nelson (1994) transformation method will not have an LCL greater than one and not be able to detect such process deterioration. Here p_0 is the in-control probability of the event of interest occurring. We based each percentage in the table on 100,000 simulations of Phase I data containing m geometric observations. This table shows that as events become rarer, the probability of LCL > 1 increases, but still remains below 70 % even for $p_0 = 0.000001$ for $m = 100$. Note that the probability of a useful LCL decreases as m increases for all values of p_0 except $p_0 = 0.000001$. We do not know the reason for this phenomenon.

Table 1 Proportion of geometric Shewhart charts based on a Phase I sample of size m and Nelson's transformation which yield a useful LCL

p_0	$m = 25$	$m = 50$	$m = 100$
0.05	0.0095	0.0004	0.0000
0.01	0.0930	0.0277	0.0032
0.005	0.1545	0.0698	0.0182
0.001	0.3090	0.2314	0.1437
0.0001	0.4755	0.4535	0.4287
0.00005	0.5079	0.5018	0.4964
0.000001	0.6128	0.6466	0.6973

Table 2 Proportion of exponential Shewhart charts based on a Phase I sample size of m and Nelson's transformation which yield a useful LCL

$m = 25$	$m = 50$	$m = 100$
0.66029	0.71463	0.78248

Table 2 shows the corresponding percentages for exponentially distributed data, leading to the same conclusion. For the exponential distribution, we can assume without loss of generality that $\theta_0 = 1$ since for any other in-control value we can rescale the observations by dividing by θ_0.

Since the transformation method does not work well, other approaches must be used. A wide variety of methods have been proposed in the literature, including Shewhart, CUSUM, and EWMA charts based on sequences of exponential or Bernoulli data. Most papers in the literature are on Phase II methods with the in-control parameter value assumed to be known. In practice the practitioner must estimate the in-control parameters, so the effect of parameter estimation is discussed in the next section.

4 Performance Metrics

Steiner and MacKay (2004) pointed out that extremely large Phase I sample sizes are needed in order to establish control limits for high quality Bernoulli processes. If one uses Nelson's (1994) recommendation of basing the estimator of the in-control probability p_0 on 24 observed events in Phase I, then if $p_0 = 0.000005$ (i.e., 5 ppm) the expected number of items in Phase I would be 4.8 million units. Steiner and MacKay (2004) also pointed out that the out-of-control expected number of items to signal can also be impractically large.

In some respects the problem is worse than Steiner and MacKay (2004) portray it. Zhang et al. (2013) studied the effect of estimation error on the Shewhart-type geometric chart. For practitioners to have confidence in their control chart design in Phase II, they must have Phase II charts with the mean in-control ARL (or other metric) near the desired value with sufficiently small variation about that value. Figure 2 shows the average in-control ARL and Fig. 3 the standard deviation of the

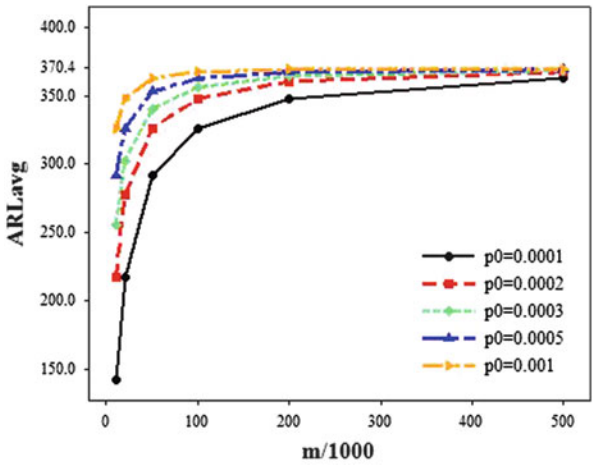

Fig. 2 The expected in-control ARL for a given in-control proportion p_0 and Phase I sample size m

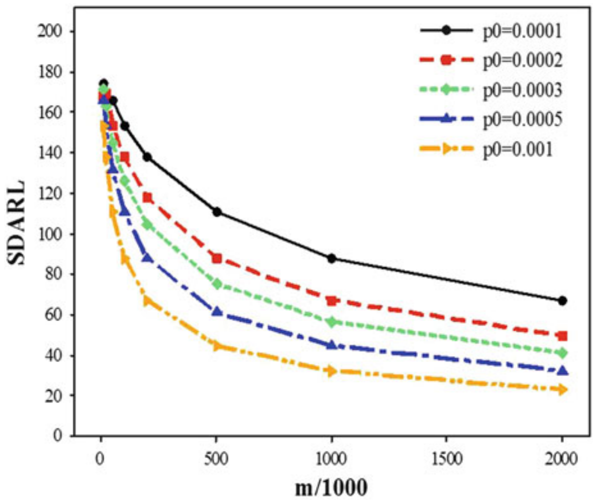

Fig. 3 The standard deviation of the in-control ARL for a given in-control proportion p_0 and Phase I sample size m

in-control ARL when the desired in-control ARL is 370.4. Note that the horizontal scales of Figs. 2 and 3 are not the same. Also, if needed, the in-control ANOS = in-control ARL/ p_0. The number of Phase I Bernoulli observations is denoted by m.

Even though the average in-control ARL converges relatively quickly to the desired value as m increases in Fig. 2, the variation in the in-control ARL converges very slowly to zero in Fig. 3. This means that it is very difficult to design the geometric control chart to have a specified in-control performance. The situation becomes worse for lower values of p_0. Using Nelson's (1994) recommendation of using a sample size required to observe 24 events in Phase I is inadequate. Lee et al. (2013) showed an even larger adverse effect of estimation error in designing the

Table 3 The average and standard deviation (in parentheses) of the in-control ARL for one-sided exponential CUSUM charts based on a Phase I sample size m and a shift size of interest δ

m	$\delta = 0.8$	$\delta = 0.5$	$\delta = 0.2$
50	10740.7	1157.3	642.6
	(213171.2)	(2561.7)	(474.4)
500	611.0	538.8	512.3
	(402.5)	(197.4)	(101.9)
10,000	504.8	501.8	500.6
	(60.8)	(38.9)	(21.9)
50,000	500.9	500.4	500.1
	(26.8)	(17.3)	(9.8)
100,000	500.5	500.2	500.1
	(18.9)	(12.2)	(6.9)

Bernoulli CUSUM chart. The effect of estimation error on the Bernoulli CUSUM chart increases as the targeted shift size in the underlying proportion decreases. The overall conclusion here is that practitioners should not expect charts based on estimated in-control parameters to perform like they would if these parameters were known, even if the Phase I sample is large.

Zhang et al. (2014) studied the effect of estimation error on the one-sided exponential CUSUM chart designed to detect a change in the mean from θ_0 to $\delta\theta_0$, where $0 < \delta < 1$. Some of their results are shown in Table 3, where one can see that the effect of estimation error is greater if one wishes to detect a smaller shift in the mean time between events. A large number of in-control observations are needed to have the expected in-control ARL near the targeted value of 500. In order for the variation of the in-control ARL to be reasonably low, however, an inordinate number of Phase I observations is required. Note that, if needed, the in-control ATS = in-control ARL* θ.

5 The Effect of Aggregating Data

Data aggregation is frequently done when monitoring rare events and for count data generally. For example, one might monitor the number of accidents per month in a plant or the number of patient falls per week in a hospital. Montgomery (2013, pp. 332–333) indicated that when there are many samples (i.e., time periods) with no events, then a c-chart is not useful and one should use a time-between-occurrence control chart. Schuh et al. (2013) showed, however, that there can be significantly long expected delays in detecting process deterioration when data are aggregated over time even when there are few samples with zero events. One can always aggregate data over long enough time periods to avoid zero counts, but the consequence is slower detection of increases in the rate of the adverse event.

We assume a homogeneous Poisson process with an average time between events of θ time units. Under this model the interarrival times are independently distributed exponential random variables with mean θ. For an aggregation period of length d time units, counts are independent and Poisson distributed with a mean of $\mu = d/\theta$. The time unit considered can vary depending on the application. We assume that the in-control value of the parameter is $\theta = \theta_0 = 1$ and that we wish to detect only decreases in the average time between events. Our results, however, are generalizable. For example, if events occur at an average rate of 4 per 28-day period, then this is equivalent to one event per week. If we consider an aggregation period d of seven-time units, then this would correspond to aggregating the data over a 7 week (or 49 day) period. Thus, it is only necessary to consider the $\theta_0 = 1$ case. Basically, the expected count is always one for some time period.

Instead of monitoring the exponentially distributed time-between-event data, it is very common to monitor instead the Poisson distributed data obtained by aggregating the event data over time intervals of a specified length. Schuh et al. (2013) showed that there is a price to be paid, however, for the data aggregation. As an example, Table 4 shows some of their results when monitoring a Poisson process with an average of one time unit between events of interest. Table 4 shows the additional number of adverse events that would be expected to occur before a signal is given for a decrease in the average time between events to given values θ, where $\theta < \theta_0 = 1$. The comparison is to the chart with the lowest steady-state

Table 4 Number of additional adverse events expected for each less-effective chart when detecting various decreases in the average time between events.

θ	POIS30	POIS4	POIS7	POIS1	EXP	Performance for best chart
0.95	-	105.8	178.2	57.7	54.0	2239.37
0.9	-	127.2	201.3	95.9	103.3	1084.78
0.85	-	87.4	130.8	72.9	70.1	551.06
0.8	-	44.4	68.6	33.1	36.9	298.87
0.75	-	19.9	28.3	8.9	10.7	178.13
0.7	8.0	9.1	10.9	2.6	-	113.00
0.65	7.8	8.9	8.0	0.5	-	73.54
0.6	18.8	9.5	6.7	0.5	-	51.67
0.55	23.6	10.5	6.4	0.5	-	38.73
0.5	28.8	11.8	6.6	0.6	-	30.60
0.45	34.0	13.6	6.9	0.9	-	25.11
0.4	39.8	16.0	7.8	1.3	-	21.25
0.35	46.0	18.9	8.9	1.1	-	18.57
0.3	53.7	23.0	10.7	1.3	-	16.33
0.25	63.6	28.0	12.8	1.2	-	14.80
0.2	79.5	35.5	17.0	2.0	-	13.00

(The — symbol appears when a chart has the lowest steady-state ATS). For reference the average number of adverse events until detection is given in the far right column for the best performing chart

ATS for the given shift. The exponential CUSUM chart is compared here to Poisson CUSUM charts based on four different levels of aggregation. In the table POIS14, for example, refers to the Poisson CUSUM chart based on counts aggregated over 14 time periods. All charts are designed to optimally detect a sustained shift to $\theta = 0.5$ under zero-state conditions. The in-control Average Time to Signal (ATS) values are near 4,500 for all five charts. The out-of-control performance is based on the steady-state ATS values with a shift to the out-of-control parameter value at some point in time after monitoring has begun. For more details, the reader is referred to Schuh et al. (2013).

Some other work has been done on the effect of aggregating count data. Reynolds and Stoumbos (2000) compared the performance of Bernoulli CUSUM charts to that of binomial CUSUM charts for aggregated samples of a specified size, finding that Bernoulli CUSUM charts showed better overall performance, especially for detecting large increases in the rates of nonconforming items. Szarka and Woodall (2011) further discussed this topic in their review of Bernoulli-based charts. Another type of aggregation is to wait until one has observed a given number of events before updating a control chart based on a proportion or waiting time. See, for example, Zhang et al. (2007) and Dzik et al. (2008). This type of aggregation, however, does not appear to delay the detection of process changes nearly as much as aggregating data over fixed time periods.

6 Some Generalizations

There have been some generalizations of the Bernoulli and exponential distribution-based methods we have discussed. Ryan et al. (2011), for example, extended the monitoring of Bernoulli data to monitoring with more than two categories through use of the multinomial distribution. Having more than two categories provides more information about the process.

Steiner et al. (2000) proposed a widely used method for monitoring Bernoulli data when the in-control probability varies over time. Their risk-adjusted CUSUM method is used to monitor surgical and other health-related outcomes while adjusting for patient risk factors. In these applications one typically works with serious events, such as death within 30 days of surgery, but the overall rate is frequently too high, e.g., around 0.01, for the adverse event to be considered rare. See Woodall et al. (2015) for a review of risk-adjusted monitoring.

Mousavi and Reynolds (2009) considered the monitoring of autocorrelated Bernoulli data where adverse events are more likely to follow other adverse events than to follow trials where the event does not occur. Finally, as generalizations of methods designed for exponentially distributed time-between-event data, methods have been proposed for Weibull and gamma distributed time-between-event data. See, for example, Xie et al. (2002) and Zhang et al. (2007).

7 Research Ideas

Given the disappointing performance of many of the methods for monitoring the rate of a rare event, it is important to identify improved methods if at all possible. We recommend the alternative methods proposed by Saleh et al. (2015) and others. The focus of much of the research on monitoring rare events has been on detecting sustained step shifts corresponding to process deterioration. Additional research on other forms of process deterioration, such as drifts, and on detecting process improvement is needed.

We believe that the adverse effect of aggregating data over time has not been fully appreciated in practice and more research work is needed on this topic. Only a couple of the most basic scenarios for count data have been studied. Some interesting and important topics include the effect of data aggregation on the performance of charts based on seasonal or autocorrelated count data, risk-adjusted data, multinomial and multivariate data, and Weibull-distributed time-between-event data.

Virtually all of the work on monitoring the rate of rare events is based on the assumption that there is a sustained shift in the rate. In some applications the rate change may be transient. In this scenario other performance metrics would be needed, such as the probability of detecting the process shift during the transient period. The effect of data aggregation over time might be larger if shifts in the parameter are not sustained.

As reviewed by Szarka and Woodall (2011), there are several dozen papers on Phase II methods for monitoring a Bernoulli probability. Even though recent research has shown that very large Phase I samples are needed, relatively little work has been done on Phase I analysis to check the adequacy of the Bernoulli model and the stability of the process. Some related references are the papers by Pettitt (1980), Worsley (1983), Wallenstein et al. (1994), Bell et al. (1994), Borror and Champ (2001), Balakrishnan et al. (2001), Krauth (2003), and Tikhomirova and Christyakov (2010). It is not clear which approach, or combinations of approaches, is the best. Similarly, work on the Phase I analysis of exponential data is also needed. The Phase I methods of Jones and Champ (2002) and Dovoedo and Chakraborti (2012) have quite low power to detect shifts in the exponential mean.

In many applications the event of interest may vary in severity. The event of interest may be an industrial accident, for example, but the impact and consequences of accidents vary. How should both rate and severity be monitored? A related paper is that of Wu et al. (2010).

Although we and Xie et al. (2010) have reviewed some of the work on monitoring continuous time-between event data, we believe a more thorough review is needed of the rather extensive literature on this subject. This review would be similar to that done by Szarka and Woodall (2011) for monitoring with Bernoulli data.

Toubia-Stucky et al. (2012) proposed a Bayesian approach to monitoring the proportion associated with Bernoulli-trials. Although a Bayesian approach may seem appealing, their method does not seem to be a viable alternative to the frequentist methods due to the required assumptions and its ad hoc construction.

Finally,Wheeler (2011) was very critical of using either the Bernoulli or exponential models and preferred the use of an individuals control chart with empirically determined 3-sigma control limits based on the median moving range. He advocated plotting either the time-between-event data or converting the time-between event data to an "instantaneous rate." For example, if an adverse event occurred after 110 days, this would be equivalent to an instantaneous rate of $(1/110)*365 = 3.32$ events per year. Study of these methods is needed since Wheeler (2011) gave only a case study illustration of his proposed methods. Our preliminary investigation shows, however, that his methods have an unacceptably large false alarm rate. His recommendation of using a minimum of five events to determine the control limits will likely prove to lead to highly unpredictable chart performance.

8 Conclusions

We have provided a review of some recent results on the monitoring of rare events along with our perspective. This recent work has demonstrated that the effect of estimation error in Phase I is more severe than for other charts studied in the literature (Jensen et al. 2006). There are compelling arguments for the use of steady-state performance metrics. In addition, recent work has demonstrated that aggregating event data over fixed time intervals, as frequently done in practice, can result in significant delays in detecting increases in the rate of adverse events. We believe that the monitoring of the rate of rare events is an important and challenging area and have offered some research ideas.

We agree with Steiner and MacKay (2004) that the monitoring of the rate of rare events is indeed a very difficult problem. Steiner and MacKay (2004) proposed solution to many of these difficulties is to use logistic regression or some other approach to try to identify a continuous underlying variable, if possible, to monitor instead of tracking simply the occurrence or non-occurrence of events. There could be considerably more information in such an underlying continuous variable, which could lead to more effective monitoring.

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References

Balakrishnan, N., Brito, M. R., & Quiroz, A. J. (2001). Using large order statistics of runs for tracking a changing Bernoulli probability. *Communications in Statistics – Theory and Methods, 31*(5), 719–732.

Bell, C., Gordon, C., & Pollak, M. (1994). An efficient nonparametric detection scheme and its application to the surveillance of a Bernoulli process with unknown baseline. In E. Carlstein, H. -G. Müller, & D. Siegmund (Eds.), *Change-point problems*. IMS Lecture Notes – Monograph Series (Vol. 23, pp. 7–27). Hayward, CA: Institute of Mathematical Statics.

Borror, C. M., & Champ, C. W. (2001). Phase I control charts for independent Bernoulli data. *Quality and Reliability Engineering International, 17*(5), 391–396.

Capizzi, G. (1994). On stopping rules for the sets scheme: An application of the success runs theory. In E. Carlstein, H. -G. Müller, & D. Siegmund (Eds.), *Change-point problems*. IMS Lecture Notes – Monograph Series (Vol. 23, pp. 66–77). Hayward, CA: Institute of Mathematical Statics.

Chen, R. A. (1978). Surveillance system for congenital malformations. *Journal of the American Statistical Association, 73*, 323–327.

Dovoedo, Y. H., & Chakraborti, S. (2012). Boxplot-based Phase I control charts for time between events. *Quality and Reliability Engineering International, 28*, 123–130.

Dzik, W. S., Beckman, N., Selleng, K., Heddle, N., Szczepiorkowski, Z., Wendel, S., & Murphy, M. (2008). Errors in patient specimen collection: Application of statistical process control. *Transfusion, 48*, 2143–2151.

Gan, F. F. (1994). Design of optimal exponential CUSUM control charts. *Journal of Quality Technology, 26*(2), 109–124.

Jensen, W. A., Jones-Farmer, L. A., Champ, C. W., & Woodall, W. H. (2006). Effects of parameter estimation on control chart properties: A literature review. *Journal of Quality Technology, 38*(4), 349–364.

Jones, L. A., & Champ, C. W. (2002). Phase I control charts for times between events. *Quality and Reliability Engineering International, 18*, 479–488.

Krauth, J. (2003). Change-points in Bernoulli trials with dependence. In W. Gaul, & M. Schader (Eds.), *Between data science and everyday web practice*. Heidelberg, Germany: Springer.

Lee, J., Wang, N., Xu, L., Schuh, A., & Woodall, W. H. (2013). The effect of parameter estimation on upper-sided Bernoulli CUSUM charts. *Quality and Reliability Engineering International, 29*, 639–651.

Levinson, W. A. (2011). Are control charts suitable for health care applications? Quality Digest. Available at http://www.qualitydigest.com/inside/health-care-article/are-control-charts-suitable-health-care-applications.html.

Liu, J. Y., Xie, M., Goh, T. N., & Sharma, P. R. (2006). A comparative study of exponential time between events charts. *Quality Technology & Quantitative Management, 3*(3), 347–359.

Lucas, J. M. (1985). Counted data CUSUM's. *Technometrics, 27*(2), 129–144.

McCool, J.I., & Joyner-Motley, T. (1998). Control charts applicable when the fraction nonconforming is small. *Journal of Quality Technology, 30*(3), 240–247.

Montgomery, D. C. (2013). *Introduction to statistical quality control* (7th ed.). Hoboken, NJ: John Wiley.

Mousavi, S., & Reynolds, M. R., Jr. (2009). A CUSUM chart for monitoring a proportion with autocorrelated binary observations. *Journal of Quality Technology, 41*(4), 401–414.

Nelson, L. S. (1994). A control chart for parts-per-million nonconforming items. *Journal of Quality Technology, 26*, 239–240.

Pettitt, A. N. (1980). A simple cumulative sum type statistic for the change-point problem with zero-one observations. *Biometrika, 67*(1), 79–84.

Reynolds, M. R., Jr., & Stoumbos, Z. G. (1999). A CUSUM chart for monitoring a proportion when inspecting continuously. *Journal of Quality Technology, 31*(1), 87–108.

Reynolds, M. R., Jr., & Stoumbos, Z. G. (2000). A general approach to modeling CUSUM charts for a proportion. *IIE Transactions, 32*, 515–535.

Ryan, A. G., Wells, L. J., & Woodall, W. H. (2011). Methods for monitoring multiple proportions when inspecting continuously. *Journal of Quality Technology, 43*(3), 237–248.

Saleh, N. A., Mahmoud, M. A., Jones-Farmer, L. A., Zwetsloot, I., & Woodall, W. H. (2015). Another look at the EWMA control chart with estimated parameters. To appear in the Journal of Quality Technology.

Santiago, E., & Smith, J. (2013). Control charts based on the exponential distribution: Adapting runs rules for the *t* chart. *Quality Engineering, 25*(2), 85–96.

Schuh, A., Woodall, W. H., & Camelio, J. A. (2013). The effect of aggregating data when monitoring a Poisson process. *Journal of Quality Technology, 45*(3), 260–272.

Sego, L. H., Woodall, W. H., & Reynolds, M. R., Jr. (2008). A comparison of surveillance methods for small incidence rates. *Statistics in Medicine, 27*(8), 1225–1247.

Steiner, S. H., Cook, R. J., Farewell, V. T., & Treasure, T. (2000). Monitoring surgical performance using risk-adjusted cumulative sum charts. *Biostatistics, 1*, 441–452.

Steiner, S. H., & MacKay, R. J. (2004). Effective monitoring of processes with parts per million defective – A hard problem! In H. J. Lenz, & P. -Th. Wilrich (Eds.), *Frontiers in statistical quality control 7*. Heidelberg, Germany: Springer.

Szarka, J. L., III., & Woodall, W. H. (2011). A review and perspective on surveillance of Bernoulli processes. *Quality and Reliability Engineering International, 27*, 735–752.

Szarka, J. L., III, & Woodall, W. H. (2012). On the equivalence of the Bernoulli and geometric CUSUM charts. *Journal of Quality Technology, 44*(1), 54–62.

Tikhomirova, M. I., & Christyakov, V. P. (2010). On properties of the likelihood function in the problem of estimation of the instants of changes of the success probability in nonhomogeneous Bernoulli trials. *Discrete Mathematics and Applications, 20*(2), 213–230.

Toubia-Stucky, G., Liao, H., & Twomey, J. (2012). A sequential Bayesian cumulative conformance count approach to deterioration in high yield processes. *Quality and Reliability Engineering International, 28*, 203–214.

Wallenstein, S., Naus, J., & Glaz, J. (1994). Power of the scan statistic in detecting a changed segment in a Bernoulli sequence. *Biometrika, 81*(3), 595–601.

Wheeler, D. J. (2011). Working with rare events – What happens when the average count gets very small? Quality Digest. Available at http://www.qualitydigest.com/inside/quality-insider-article/\working-rare-events.html.

Woodall, W. H., Fogel, S. L., & Steiner, S. H. (2015). The monitoring and impovement of surgical outcome quality. To appear in Journal of Quality Technology.

Worsley, K. J. (1983). The power of likelihood ratio and cumulative sum tests for a change in a binomial probability. *Biometrika, 70*(2), 455–464.

Wu, Z., Liu, Y., He, Z., & Khoo, M. (2010). A cumulative sum scheme for monitoring frequency and size of an event. *Quality and Reliability Engineering International, 26*, 541–554.

Xie, M., Goh, T. N., & Ranjan, P. (2002). Some effective control chart procedures for reliability monitoring. *Reliability Engineering & System Safety, 77*, 143–150.

Xie, Y. J., Tsui, K. -L., Xie, M., & Goh, T. N. (2010). Monitoring time-between-events for health management. In *Prognostics & System Health Management Conference*, Macau.

Zhang, C.W., Xie, M., Liu, J.Y., & Goh, T.N. (2007). A control chart for the gamma distribution as a model of time between events. *International Journal of Production Research, 45*(23), 5649–5666.

Zhang, M., Peng, Y., Schuh, A., Megahed, F. M., & Woodall, W. H. (2013). Geometric charts with estimated parameters. *Quality and Reliability Engineering International, 29*, 209–223.

Zhang, M., Megahed, F. M., & Woodall, W. H. (2014). Exponential CUSUM charts with estimated control limits. *Quality and Reliability Engineering International, 30*, 275–286.

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