Preface

Mathematics is the queen of the sciences
Carl Friedrich Gauss

This text is directed at undergraduate students in mathematical sciences who wish to have solid foundations for modern analysis, a meeting point of classical analysis with other parts of mathematics, like functional analysis, operator theory, nonlinear analysis, etc. These foundations are necessary for applications of mathematics in sciences or engineering. Moreover, students planning to pursue graduate work in mathematics will find this text useful, especially those who did not have a chance to go through the honors programs at their respective universities or colleges.

It is assumed the reader has a good understanding of elementary linear algebra and arithmetics, as well as some training in simple logic. We shall try to fill foreseeable gaps to help the reader in this direction.

The text consists of a rigorous yet gentle self-contained introduction to real analysis with various visual supplements. Moreover, we have enriched the material with several excursions to mathematical areas such as functional analysis, descriptive statistics, or Fourier analysis (some chapters that are rather self-contained can be used as a material for independent optional course in some undergraduate programs). Aside from the theoretical part, the text contains an ample amount of exercises of various difficulties with hints for their solutions. We have prepared a number of figures (by using the free-distribution programs Veusz and IPE, and in a few opportunities also the registered package Mathematica) that are intended to help with understanding of the material covered. We tried to touch on quite a few "folklore" things that are frequently used in real analysis. We hope that instructors in service calculus courses may find the text to be a source for more advanced problems.

In the first chapter we introduce the real number system, discuss the principle of the supremum, and first meet the important principle of compactness and the Baire Category theorem.

In the second chapter we encounter the notion of convergent and Cauchy sequences of real numbers and the approximation by rational numbers.

Chapter 3 contains an introduction to Lebesgue measure on the real line and its applications.

Chapter 4 contains basic notions and results in the theory of real-valued functions and their differentiability, together with an introduction to sequences of real-valued functions and their convergence.

Chapter 5 upgrades the discussion on function convergence. We discuss pointwise, uniform, measure, and almost everywhere convergences. The focus is on approximation and the properties preserved through it. In particular, global and local approximations are considered. Applications of those concepts include a discussion on real analytic functions and rigorous definitions of the basic functions in analysis.

Chapter 6 deals with metric spaces. This is a wide setting in which most of the former discussions find their place. The reader may find here Tietze's extension theorem, a discussion on separable spaces, with an emphasis on Polish spaces, a deeper analysis of compactness, including the Arzelà– Ascoli theorem, more on the Baire category theorem, and applications to metric fixed point theory.

Chapter 7 deals with integration in the Riemann and Lebesgue senses. Lebesgue's approach is intertwined with the measure theory already developed, and allows for a finer analysis of functions and convergence.

Chapter 8 introduces the reader to the basic theory of convex functions.

Chapter 9 is a basic introduction to the theory of Fourier series and integrals, including applications. An extension to the more general setting of periodic distributions will be done in Chap. 11.

Chapter 10 presents a basic introduction to descriptive statistics. The emphasis is on discrete probability, which may help to understand the subsequent, more general approach.

In Chap. 11, named "Excursion to Functional Analysis," we present an introduction to basic concepts and results in a few selected topics in functional analysis, like Banach spaces, operator theory, and nonlinear functional analysis, with applications to real analysis. In fact, we shall try to illustrate to some extent how "abstract" functional analysis emerges from the waters of real analysis as a lighthouse to orientate and overlook the whole sea. We believe that this chapter may be used as a basic introduction to these subjects, and may foster the interest of the reader to enlarge his/her knowledge of modern techniques used in many fields. Together with Chap. 6, this chapter may constitute a basic material for an introductory graduate course in linear and nonlinear functional analysis.

We include an Appendix (Chap. 12), mainly on number systems, and on three fundamental principles in set theory—the axiom of choice, the well-ordering principle, and Zorn's lemma.

The last chapter (Chap. 13) is formed by exercises that are organized according to the chapters in the text. They are of various levels of difficulty. Some of them just briefly review the basic techniques of rigorous elementary calculus, and some of them upgrade the material in the chapters of the text. All of them are accompanied by hints for their solutions.

Optional sections are denoted by the symbol ♣.

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