

Preface

The intriguing figure from the previous page presents a visualization of the type of manifolds this monograph is concerned with. First, obtained as a graph over some chosen linearized spatial modes from a nonlinear stochastic partial differential equation (SPDE), such a manifold—and its geometric properties—depends naturally on the spatial variable. Second, the shape of this manifold is nonlinear as a fingerprint of the nonlinear effects conveyed by the nonlinear SPDE. Third, this manifold is stochastic, in other words, it depends on time for a given realization of the noise, meaning that “mutations” of the shape—such as changes in the (Gaussian) curvature—can occur as time flows. Manifolds that share these three features and more importantly for which the dynamics of the underlying nonlinear SPDE is either attracted to or meanders around, are the manifolds of interest in this two-volume series.

In this respect, a *pathwise* approach from the theory of random dynamical systems (RDS) is adopted in both volumes. Volume I deals with *approximation* of stochastic manifolds that are invariant for the SPDE dynamics, while Volume II [37] deals with stochastic manifolds that may not be invariant nor attractive and that can still capture essential features of the dynamics through appropriate parameterizations of the small spatial scales by the large ones. The small-scale parameterizations proposed in Volume II are articulated around a new concept of stochastic manifolds, namely the stochastic *parameterizing manifolds* (PMs).

An important common feature is shared by the (pathwise) approximation formulas derived in Volume I and the (pathwise) parameterization techniques introduced in Volume II: both are characterized as pullback limits from backward–forward systems which are only partially coupled, facilitating the calculation of such limits, either analytically or numerically.

The aforementioned pullback limits arise under the form of Lyapunov–Perron integrals which are useful for the rigorous treatment of the problem of approximation to the leading order,¹ of important stochastic manifolds such as—but not

¹ With respect to the nonlinearity involved in the SPDE at hand.

limited to—stochastic critical manifolds² built as random graphs over a fixed number of critical modes which lose their stability as a control parameter varies.

In this respect, Chaps. 6 and 7 contain explicit formulas for the leading-order Taylor approximation of stochastic (local) critical manifolds or more general stochastic hyperbolic manifolds. The pullback characterization of these formulas provides a useful interpretation of the corresponding approximating manifolds which gives rise to a simple framework that allows us, furthermore, to unify the previous approximation approaches of stochastic invariant manifolds, as discussed in Volume II; see [37, Sect. 4.1].

To help the reader appreciate this unification as well as the more prospective material presented in Volume II, we took the freedom to include a self-contained (short) survey on the theory of existence and attraction of one-parameter families of stochastic invariant manifolds in Chaps. 4 and 5.

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² Including stochastic manifolds such as the center or center-unstable manifolds.



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