2 A Safety Performance Function for Real Populations

Abstract

An SPF provides estimates of the mean and standard deviation of the μ's for many populations of units. When the units of these populations are real, the estimation of their $E\{\mu\}$ and $\sigma\{\mu\}$ is straightforward and their meaning is clear. Attaining this clarity is the main aim of this chapter. A simple SPF for real units will be built using data for 2,228 Colorado road segments.

2.1 The Origin

Chapter 1 introduced the notion that units which share some safety-related traits form a population and that the safety of populations can be parsimoniously and usefully described by the mean of the μ's of its units ($E\{\mu\}$) and by their standard deviation ($\sigma\{\mu\}$). The SPF was said to be a tool which provides estimates of $E\{\mu\}$ and $\sigma\{\mu\}$ for a multitude of populations.

When the term "Safety Performance Function" (SPF) was first coined,[1] it referred to a relationship that gives the average number of accidents for various amounts of exposure.[2] Over the years, the term has been broadened in two directions. First, nowadays the SPF is a function not only of exposure but also of

[1] Hauer (1995).

[2] Exposure is a measure of opportunities for accidents to occur. The most commonly used measure of exposure is "vehicle miles of travel" (VMT). The concept of exposure is tied to that of risk. Risk is usually construed as the probability of a crash of a specified type and severity to occur per unit of exposure which, in probability theory, corresponds to a "trial" the outcome of which is either "accident" or "no accident." These definitions of exposure and risk are due to Hauer (1982). For examples of usage see, e.g., Keall and Frith (1999) and Hakkert et al. (2002).

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other traits.[3] Second, the SPF now provides estimates not only of the average number of accidents but also of the diversity of the μ's in a population.

The "F" in SPF stands for "function." While "function" evokes the image of an equation, "it ain't necessarily so." A function can also be an algorithm, a graph, or a table; any device which for specific values of "predictor variables" returns a value of the "dependent variable." To secure a clear understanding of the SPF and of the estimates which it provides, it is best to present this first SPF in the form of a table. Doing so will allow one to speak about real populations of units the μ's of which have a real mean $E\{\mu\}$ and a real standard deviation $\sigma\{\mu\}$.

2.2 The Estimate of $E\{\mu\}$

The data to be used throughout the book are described in Sect. 3.2. The subset used here is of 2,228 rural two-lane road segments in Colorado which are between 0.5 and 1.5 miles long.[4] These road segments were sorted into 20 bins by average AADT as shown in Table 2.1.

The ratio of the entries in columns 2 and 3 is in column 4. This is the estimate of $E\{\mu\}$, the primary element of a simple tabular SPF. The squares in Fig. 2.1 are the graphical representation of these $\hat{E}\{\mu\}$.

The ordinate of the squares – the average number of accidents per road segment – is the estimate of $E\{\mu\}$ for real populations. Thus, for example, for the population of road segments defined by the traits (1) State: Colorado, (2) Road Type: two-lane, (3) Setting: rural, (4) Segment Length: 0.5–1.5 miles, and by (5) AADT: 9,000–10,000 vehicles per day, the average number of accidents in 5 years was 5.37(=102/19). This is an estimate of the average of 19 μ's, the $\hat{E}\{\mu\}$ of this population. An SPF of this type makes it clear that what is listed in a row of the table and shown in the graph always pertains to a population of units. Here there are 20 such populations each defined by traits (1)–(4) and by an AADT bin.

The accuracy of this estimate is represented by the standard error of $\hat{E}\{\mu\}$ and shown in Fig. 2.1 by the horizontal bars that surround the squares. To illustrate, for segments with 9,000 < AADT < 10,000, the standard error of $\hat{E}\{\mu\}$ is $\sqrt{102}/19$ $= \pm 0.53$ accidents in 5 years.[5] Accordingly, the corresponding horizontal bars are placed at 5.37 + 0.53 and at 5.37–0.53.

[3] The Highway Safety Manual (AASHTO 2010, page G-13) defines SPF as "... an equation used to estimate or predict the expected average crash frequency per year at a location as a function of traffic volume and in some cases roadway or intersection characteristics (e.g., number of lanes, traffic control, or type of median)."

[4] To download the data, go to http://extras.springer.com/ and enter the ISBN of this book. The ISBN (International Standard Book Number) is found just after the title page. Look in the "Data" folder for "4 (a or b) Colorado condensed (xls or xlsx)." To make Table 2.1 out the data, the Pivot Table tool described in Sect. 3.3 was used.

[5] The standard error is the estimate of a standard deviation and is usually denoted by s The estimate of the mean of μ's $\equiv \hat{E}\{\mu\}$ = (Number of accidents)/(Number of segments). Assuming that the number of accidents is Poisson distributed, the standard error of the average crash rate is $s = \sqrt{\text{Number of accidents}}/(\text{Number of segments})$.

Table 2.1 A tabular SPF for 0.5–1.5 mile long two-lane rural road segments in Colorado

Data			Estimates		
1	2	3	4	5	6
Average AADT	Injury and fatal accidents 1994–1998	Number of segments	$\hat{E}\{\mu\}$, 1994–1998 I&F accidents/segment	$\hat{\sigma}\{\hat{E}\{\mu\}\}$, standard error of $\hat{E}\{\mu\}$	S^2, sample variance
500	376	975	0.39	0.02	0.53
1,500	445	466	0.95	0.05	1.36
2,500	382	260	1.47	0.08	2.80
3,500	381	178	2.14	0.11	4.47
…	…	…	…	…	…
9,500	102	19	5.37	0.53	35.18
10,500	81	18	4.50	0.50	9.81
…	…	…	…	…	…
18,500	14	1	14.00	3.74	0.00
19,500	13	1	13.00	3.61	0.00
Sum	3,011	2,228			

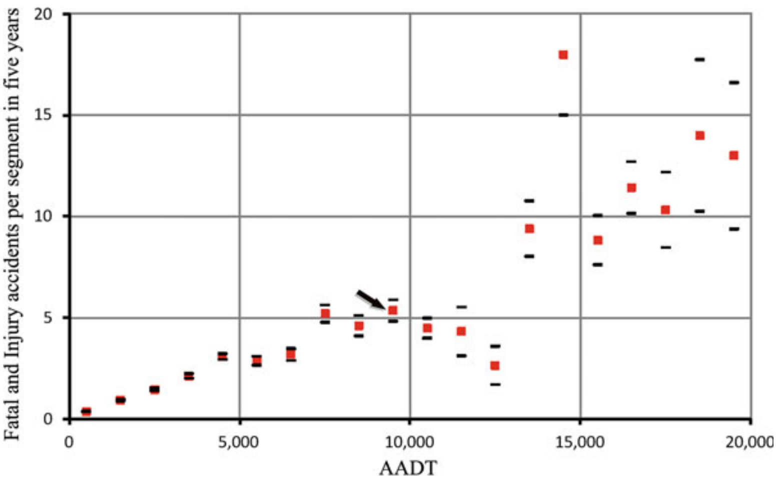

Fig. 2.1 An SPF which is a function but not an equation

The more segments there are in a bin, the more accurately can its $E\{\mu\}$ be estimated.[6] As is evident in Table 2.1, the larger the average AADT the fewer are

[6] The numerator in $s = \sqrt{\text{Number of accidents}}/(\text{Number of segments})$ can be written as $\sqrt{\text{Accidents per segment} \times \text{Number of road segments}}$. It follows that for any given crash rate ($\equiv$Accidents/segment) the standard error of $\hat{E}\{\mu\}$ is inversely proportional to the square root of the number of road segments which serve for estimation.

the segments in a bin. This is why, going from left to right in Fig. 2.1, the bars move further and further away from the squares.

What can be said on this basis? One can say that on Colorado rural two-lane road segments which are 0.5–1.5 miles long and have an AADT between 9,000 and 10,000 vehicles, the average of the μ's in the 1994–1998 period was about 5.37 I&F accidents and that the standard error of this estimate is ± 0.53.[7]

Because the $\hat{E}\{\mu\}$ of a population is a clue about the μ's of all units in that population one can say a bit more. Suppose that all we know about unit i is that it is a segment of a rural two-lane road in Colorado, that it is 1.0 miles long, and that it has an AADT of 9,500 vehicles per day. The estimate of $\hat{E}\{\mu\}$ for the population of road segments defined by these traits is 5.37 I&F accidents in 5 years. Because we know nothing about unit i to distinguish it from other segments in that population, 5.37 is also the best estimate of the μ of unit i, the μ_i. It is the "best estimate" because if $\hat{E}\{\mu\}$ was always used to estimate the μ of specific units with traits that match those of the population to which $\hat{E}\{\mu\}$ pertains, the variance of the estimates would be the smallest.

This section was about the estimates of $E\{\mu\}$ for 20 populations that comprise real units. Taken together, these estimates together with their standard error make up the principal element of the SPF. However, for most applications estimates of $E\{\mu\}$ are insufficient. As noted earlier,[8] to determine whether a unit is a "blackspot," to predict what might be the safety benefit of a treatment or of a design change, and to estimate what the safety effect of an intervention was, one needs to know how diverse are the μ's in the population. For these applications, estimates of $\sigma\{\mu\}$ are also needed.[9] This element of the SPF is discussed next.

2.3 The Estimate of $\sigma\{\mu\}$

The $\sigma\{\mu\}$ characterizes the diversity of μ's in a population of units. To explain how an estimate of $\sigma\{\mu\}$ can be extracted from accident counts consider Fig. 2.2.[10]

The circles of the top tier represent the μ's of five units and the squares of the bottom tier represent their accident counts. Obviously there is a relationship

[7] An approximate rule of thumb is that the "true value" is within ± 2 standard deviations of the estimated value 19 times out of 20. This rule is based on the assumption that the estimate is unbiased and normally distributed.

[8] See Sects. 1.3 and 1.4.

[9] The $\hat{\sigma}\{\hat{E}\{\mu\}\}$ in column five of Table 2.1 and the $\sigma\{\mu\}$ to be discussed next are two entirely different constructs. One measures the accuracy with which $E\{\mu\}$ is estimated, the other measures the diversity of μ's in a population. How they combine to determine the accuracy of an estimate of μ for a specific unit is discussed in Sect. 2.4.

[10] Figure 2.2 is the same as Fig. 1.3 and is reproduced here for convenience.

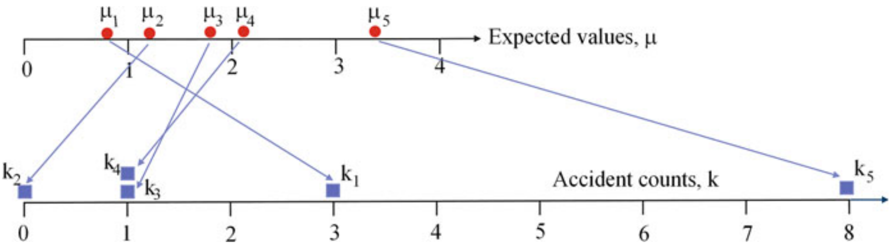

Fig. 2.2 The relationship between μ's and k's

between the diversity of the μ's and that of the k's. It can be shown[11] that when the accident counts for each unit are Poisson distributed then

$$\begin{aligned}&V\{\mu\} = V\{k\} - E\{\mu\}\\ &\text{and therefore}\\ &\hat{\sigma}^2\{\mu\} = \big(\text{Sample variance of accident counts in population}\\ &\qquad\qquad - \text{Sample mean of accident counts in population}\big)\\ &\qquad\qquad \text{if positive, and 0 otherwise.}\end{aligned} \tag{2.1}$$

To illustrate, for the 19 segments with $9{,}000 < \text{AADT} < 10{,}000$ that are 0.5–1.5 miles long the sample mean is 5.37 ± 0.53 accidents in 5 years and the sample variance (Column 6 in Table 2.1) is 35.18 accidents^2. Therefore, $\hat{\sigma}\{\mu\} = \pm\sqrt{35.18 - 5.37} = \pm 5.46$ accidents in 5 years.[12] Proceeding in the same manner for all bins in Table 2.1 for which there is sufficient information[13] Fig. 2.3 shows how in our data $\hat{\sigma}\{\mu\}$ depends on AADT. With this, the second element of this simple SPF, the estimate of $\sigma\{\mu\}$, is also in hand.

2.4 The Two σ's; Homogeneity Versus Accuracy

Two different σ's were discussed in this chapter: $\sigma\{\mu\}$ and $\sigma\{\hat{E}\{\mu\}\}$. The first describes the diversity of μ's amongst the units of a population; the second characterizes the accuracy with which the mean of these μ's is estimated. The nature of both, as well as the differences between them, can be clarified with the help Fig. 2.4. In this figure what is computed from data is in black and what is unknown and estimated is in grey.

[11] For proof, see Appendix C or Hauer (1997), pages 204–205.

[12] If a yearly crash rate is of interest, these results have to be divided by 5.

[13] For AADTs > 11,000, there are too few segments per bin to compute useful estimates of the sample variance of accident counts.

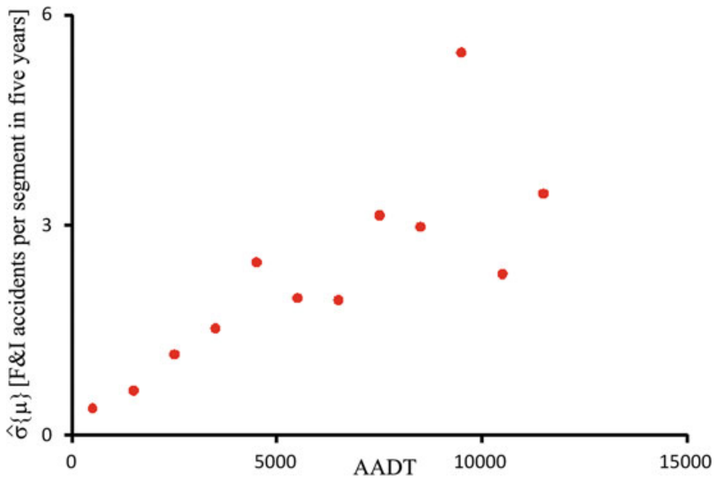

Fig. 2.3 $\hat{\sigma}\{\mu\}$, the second element of an SPF

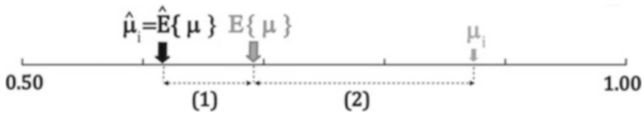

Fig. 2.4 Two variance components

The expected value of the square of distance (1) measures the accuracy[14] with which $E\{\mu\}$ is estimated; it is the $\sigma^2\{\hat{E}\{\mu\}\}$. The expected value of the square of distance (2) measures how diverse are the μ's of the units in population; it is the $\sigma^2\{\mu\}$. For the real populations and data in Table 2.1, estimates of both σ's are shown in Fig. 2.5.

In important applications both σ's are in play. Thus, for example, suppose that we are interested in the accuracy with which μ_i (the μ of unit i) is estimated when $\hat{E}\{\mu\}$ serves as its estimator[15]. That is, we need the variance of the sum of distances (1) and (2) in Fig. 2.4. In principle, the accuracy with which $E\{\mu\}$ is estimated and the diversity of the μ's in a population are causally independent.[16] If so,

[14] According to the Joint Committee for Guides in Metrology (JCGM 2008), the term "accuracy" refers to the degree of closeness of measurements of a quantity to that quantity's true value. In contrast, the word "precision" refers to the degree to which repeated measurements under unchanged conditions show the same results. To illustrate, if an experiment contains a systematic error then repeating the same flawed experiment would yield a string of possibly precise but still inaccurate (biased) results. Eliminating the systematic error would improve accuracy but may not change precision of the results.

[15] An estimator is a rule for calculating an estimate from data. That $\hat{E}\{\mu\}$ is the estimator of $\hat{\mu}_i$ is indicated in Fig. 2.4 by setting $\hat{\mu}_i$ to equal $\hat{E}\{\mu\}$.

[16] In estimation, however, the two summands must be correlated inasmuch as the statistic $\sqrt{\text{Number of accidents}}/(\text{Number of segments})$ features in both.

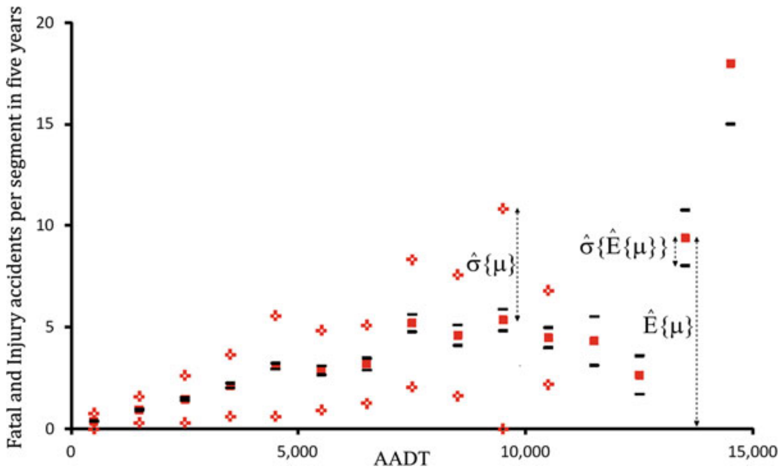

Fig. 2.5 Estimates of $\sigma\{\mu\}$ versus $\sigma\left\{\hat{E}\{\mu\}\right\}$

$$\sigma\{\hat{\mu}_i\} = \sqrt{\sigma^2\left\{\hat{E}\{\mu\}\right\} + \sigma^2\{\mu\}} \tag{2.2}$$

To illustrate, consider again that segment of a rural two-lane road in Colorado that is 1.0 mile long and has an AADT of 9,500 vehicles. If to estimate its μ we use the $\hat{E}\{\mu\} = 5.37$ then, by Eq. 2.2, $\sigma\{\hat{\mu}_i\} = \sqrt{0.53^2 + 5.46^2} = \pm 5.47$ accidents in 5 years.[17]

Equation 2.2 leads to a key insight. When dealing with real populations (such as that of the Colorado road segments), the $\hat{E}\{\mu\}$ is obtained by dividing the number of accidents in a bin by the number of segments in it. The wider the bin the more segments will be in it and therefore the smaller will be the $\sigma\left\{\hat{E}\{\mu\}\right\}$. However, the wider the bin the more diverse will tend to be the μ's of the units in it. If so, making the bin wider will usually cause $\sigma\{\mu\}$ to increase. The compromise between these opposing tendencies defines a valley in the sum of the radical in Eq. 2.2 and thereby an optimal bin width.

The same key insight and same conflicting tendencies will apply when, instead of tabulating bin averages, the SPF will be obtained by fitting a function to data. In this case it will not be the bin width that matters but the number of variables in the function. The larger the number of variables the larger will tend to be the uncertainty surrounding the estimates of $E\{\mu\}$ but the less diverse will be the μ's of units belonging to the imagined populations.

[17] In this case, the accuracy with which the μ can be estimated is governed by the diversity of μ's in the population of units with the same traits, and not by the accuracy by which the mean of the μ's is estimated.

2.5 Summary

An SPF consists of two main elements: (1) Estimates of $E\{\mu\}$, the mean of the μ's in each population and the standard deviation of this estimate, the $\sigma\{\hat{E}\{\mu\}\}$; (2) estimates of $\sigma\{\mu\}$, the standard deviation of the μ's in each population. Both are needed for practical applications. In this chapter a simple table-and-graph SPF was built out of data for real units. The aim was to make the SPF tangible and the meaning of $E\{\mu\}$ and $\sigma\{\mu\}$ unambiguous. It is now clear that whatever is measured on the vertical axes of Figs. 2.1, 2.3, or 2.5 pertain to populations of units.

When dealing with real populations consisting of many units, the estimation of $E\{\mu\}$ and $\sigma\{\hat{E}\{\mu\}\}$ is straightforward. To estimate $\sigma\{\mu\}$ Eq. 2.1 can be used. Equation 2.2 can be used to estimate the standard error of $\hat{\mu}_i$ when $\hat{E}\{\mu\}$ is its estimator.

The simple SPF used in this section was sufficient to illustrate its essential nature and to highlight its two main elements. However, to be of practical use, the simple tabular SPF has to be enriched along three lines. First, the bins are too broad. Segment Length is usually known to the next hundredth of a mile and it does not make sense to use bins in which the length of segments may vary by as much as a mile. Second, while the simple SPF in this chapter accounts for Segment Length and AADT, many safety-related traits (variables) are missing. Thus, example, one cannot know what is normal for a road segment without taking into account its lane width. Neither can one meaningfully compare the mean of μ's for two populations without considering possible differences in, say, terrain. Nor does it make sense to use the mean of the μ's to estimate the μ of unit i if it is on a sharp curve while the curvature of the segments in the population is unknown. For the SPF of practical use, one has to add to the SPF some important population-defining traits. Third, one may expect that underneath the squares in Fig. 2.1 or the circles in Fig. 2.3 there is some presently unknown continuous curve. After all, AADT and Segment Length are nearly continuous variables. Which traits to add, what tools to use for this purpose, and how to fit mathematical functions to data are the question discussed in detail in later chapters.

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