

Preface

As its title indicates, this book aims to be a comprehensive, self-contained compendium of results on continuous nowhere differentiable functions, collecting many results hitherto accessible only in the scattered literature.

Motivation for Writing This Book

Why did the authors, both specialists in several complex variables, decide to write a book on continuous nowhere differentiable functions? Let us try to answer this question:

- (a) Whenever we would give a lecture on real analysis, we felt unsatisfied, since there was almost no time to discuss continuous nowhere differentiable functions in detail. Therefore, we could only mention the existence of such functions in most of our lectures. Moreover, whenever we wanted to deal with such functions in a proseminar, it was difficult to find a source book. Some information could be found in a master's thesis by J. Thim (see [Thi03]), which presented a more detailed description of these functions. Later, during the writing of this book, we found another survey article by A.N. Singh (see [Sin35]). With few sources available, we thought that a modern and complete description of how these functions appeared would be of great use, both for students and for colleagues creating their lectures and preparing proseminars.
- (b) Looking back to the middle of the nineteenth century, we see that that was an important time in the history of mathematics, when many arguments turned from being based more or less on heuristics into being grounded in precise definitions and proofs. We are still experiencing the consequences of this birth of mathematical precision. It is interesting to see how the methods used to discuss continuous nowhere differentiable functions has changed over time and to observe that there are still problems that have not been solved.

We hope that the reader will accept our motivation and that our book can be used for learning some very nice mathematics or for preparing proseminars or lectures on analysis.

Remarks for the Reader

To make a big part of the material accessible even to high-school graduates, we ordered the content into four main parts:

- Part I: Classical results.
In this part are collected all results from the middle of the nineteenth century up to about 1950. The proofs are based on complicated arguments, but to understand them requires only some basic facts from analysis.
- Part II: Topological methods.
This part is based on standard techniques from functional analysis that are certainly taught in any beginning course.
- Part III: Modern approach.
This part requires some more highly developed ideas from analysis, such as measure theory and Fourier transforms.
- Part IV: The Riemann function.
This part is in some sense unusual. On the one hand, it does not directly follow the theme of the book, since the Riemann function discussed here does not belong to the class of nowhere differentiable functions. On the other hand, it is more difficult and requires knowledge from several different fields of mathematics. To help the reader, we have placed such information in an appendix.

Nevertheless, we are convinced that at least 10 % of the book may be understood by high-school graduates, 40 % by students of mathematics who have completed a first analysis course, and the remainder by master's-level students.

We did not include any exercises, as they can be found in many textbooks. But the reader will find the word EXERCISE at different places in the text. It is at such points that the reader is asked to stop reading and to extend our arguments into greater detail.

Moreover, whenever some function is discussed in the book, the reader is asked to continue its study. For example, if f is claimed to be nowhere differentiable on the interval $[0, 1]$ and nothing, even later in the text, is said about infinite derivatives, then the reader should try to discuss this question on his own. In any case, any additional information in such directions that we have found in the literature has been added to the text.

Each chapter begins with a brief summary of its content. Moreover, the reader will find open problems in some chapters. They are indicated by the sign $\boxed{?} \dots \boxed{?}$. All these problems are collected at the end of the book, see List of Problems section in Appendix C. The reader is asked to work on these questions, although they do not seem to be simple to solve. For notation that may appear in the text without explanation, the reader is asked to consult Sect. B.1.

We wish to thank all our colleagues who told us about gaps in this book during its writing. In particular, we thank Dr. P. Zapałowski for all the corrections he made. It would not have been possible to reach the current level of presentation without his precise and detailed observations. Nevertheless, according to our experiences with our former books, we are sure that many errors have remained, and we are responsible for not detecting them.

We will be pleased if readers inform us about any observations they may have while studying the text. Please use the following e-mail addresses:

- Marek.Jarnicki@im.uj.edu.pl
- Peter.Pflug@uni-oldenburg.de

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Continuous Nowhere Differentiable Functions

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