

Contents

1	Introduction: A Historical Journey	1
	Part I Classical Results	7
2	Preliminaries	9
2.1	Derivatives	9
2.2	Families of Continuous Nowhere Differentiable Functions	12
2.3	The Denjoy–Young–Saks Theorem	12
2.4	Series of Continuous Functions	15
2.5	Hölder Continuity	16
3	Weierstrass-Type Functions I	19
3.1	Introduction	19
3.2	General Properties of $\mathbf{W}_{p,a,b,\theta}$	22
3.3	Differentiability of $\mathbf{W}_{p,a,b,\theta}$ (in the Infinite Sense)	24
3.4	An Open Problem	26
3.5	Weierstrass’s Method	27
3.5.1	Lerch’s Results	31
3.5.2	Porter’s Results	33
3.6	Cellérier’s Method	35
3.7	Dini’s Method	36
3.8	Bromwich’s Method	38
3.9	Behrend’s Method	39
3.10	Emde Boas’s Method	46
3.11	The Method of Baouche–Dubuc	48
3.12	Summary	49
4	Takagi–van der Waerden-Type Functions I	51
4.1	Introduction	51
4.2	Kairies’s Method	56
4.3	Cater’s Method	58
4.4	Differentiability of a Class of Takagi Functions	61

5	Bolzano-Type Functions I	65
5.1	The Bolzano-Type Function	65
5.2	Q -Representation of Numbers	69
5.2.1	Continuity of Functions Given via Q -Representation	70
5.2.2	Bolzano-Type Functions Defined via Q -Representation	71
5.3	Examples of Bolzano-Type Functions	73
5.3.1	The Hahn Function	73
5.3.2	The Kiesswetter Function	73
5.3.3	The Okamoto Function	79
5.4	Continuity of Functions Given by Arithmetic Formulas	84
5.5	Sierpiński Function	85
5.6	The Pratsiovytyi–Vasylenko Functions	86
5.7	Petr Function	87
5.8	Wunderlich–Bush–Wen Function	89
5.9	Wen Function	91
5.10	Singh Functions	93
6	Other Examples	99
6.1	Schoenberg Functions	99
6.2	Second Wen Function	102

Part II Topological Methods 105

7	Baire Category Approach	107
7.1	Metric Spaces and First Baire Category	107
7.2	The Banach–Jarnik–Mazurkiewicz Theorem	108
7.3	Typical Functions in the Disk Algebra	113
7.4	The Jarnik–Marcinkiewicz Theorems	115
7.5	The Saks Theorem	121
7.6	The Banach–Mazurkiewicz Theorem Revisited	124
7.7	The Structure of $\mathcal{ND}(\mathbb{I})$	127

Part III Modern Approach 131

8	Weierstrass-Type Functions II	133
8.1	Introduction	133
8.2	Hardy’s Method	134
8.3	Baouche–Dubuc Method	143
8.4	Kairies–Girgensohn Method	145
8.4.1	A System of Functional Equations	145
8.4.2	The Faber–Schauder Basis of $\mathcal{C}(\mathbb{I})$	146
8.4.3	Nowhere Differentiability and the Schauder Coefficients	149
8.4.4	Schauder Coefficients of Solutions of a System of Functional Equations	151
8.4.5	Nowhere Differentiability of $\mathbf{W}_{1,a,b,\theta}$ for $ab \geq 1$, $b \in \mathbb{N}_2$	154
8.5	Weierstrass-Type Functions from a General Point of View	157
8.6	Johnsen’s Method	161
8.7	Hata’s Method	171

8.7.1	Nowhere Differentiability of the Weierstrass-Type Functions: Finite One-Sided Derivatives	171
8.7.2	Knot Points of Weierstrass-Type Functions	174
8.7.3	Nowhere Differentiability of Weierstrass-Type Functions: Infinite Derivatives	178
8.8	Summary	186
9	Takagi–van der Waerden-Type Functions II	187
9.1	Introduction	187
9.2	The Case $ab > 1$	187
9.3	Infinite Unilateral Derivatives of $T_{1/2,2,0}$	191
9.4	Proof of Theorem 9.3.4	198
9.5	The Case of Normal Numbers	200
10	Bolzano-Type Functions II	203
10.1	Bolzano-Type Functions	203
11	Besicovitch Functions	209
11.1	Morse’s Besicovitch Function	209
11.1.1	Preparation	209
11.1.2	A Class of Continuous Functions and Its Properties	212
11.1.3	A New Function \bar{f} for Every $f \in \mathfrak{A}$	213
11.1.4	A Besicovitch–Morse Function	217
11.2	Singh’s Besicovitch Function	220
11.2.1	A Representation of Numbers	220
11.2.2	Definition of Singh’s Besicovitch Function	224
11.2.3	Continuity of S_4	226
11.2.4	Nowhere Differentiability of S_4	228
11.3	$\mathbf{BM}(\mathbb{I})$ Is Residual in a Certain Subspace of $\mathcal{C}(\mathbb{I})$	235
12	Linear Spaces of Nowhere Differentiable Functions	245
12.1	Introduction	245
12.2	\mathfrak{c} -Lineability of $\mathcal{ND}^\infty(\mathbb{R})$	246
12.3	Spaceability of $\mathcal{ND}_\pm(\mathbb{I})$	248
12.3.1	Two Matrices	248
12.3.2	Auxiliary Functions	249
12.3.3	The Closed Linear Subspace $E \subset \mathcal{ND}_\pm(\mathbb{I})$	250
Part IV	Riemann Function	255
13	Riemann Function	257
13.1	Introduction	257
13.2	Auxiliary Lemmas	258
13.3	Differentiability of the Riemann Function	261
Appendix A		265
A.1	Cantor Representation	265
A.2	Harmonic and Holomorphic Functions	266
A.3	Fourier Transform	268
A.4	Fresnel Function	270

A.5 Poisson Summation Formula	270
A.6 Legendre, Jacobi, and Kronecker Symbols	271
A.7 Gaussian Sums	273
A.8 Farey Fractions	274
A.9 Normal Numbers	275
Appendix B: List of Symbols	279
B.1 General Symbols	279
B.2 Symbols in Individual Chapters	280
Appendix C: List of Problems	283
List of Figures	285
References	287
Index	297

Continuous Nowhere Differentiable Functions

The Monsters of Analysis

Jarnicki, M.; Pflug, P.

2015, XII, 299 p. 15 illus., 14 illus. in color., Hardcover

ISBN: 978-3-319-12669-2