

Chapter 2

Tidal Disruption Rates from Two-Body Relaxation

2.1 Introduction

In this chapter, we review the standard TDE rate calculation in greater detail than was covered in the introduction. This is meant to provide context for the following two chapters of this thesis, which cover original work on ways in which tidal disruption rates are affected by SMBH recoil. These standard TDE rate estimates are also highly valuable calculations in their own right! Throughout this chapter, we will follow the relatively recent work of Wang and Merritt (2004), which represents a simplification of many other works on TDE rate calculations (Lightman and Shapiro 1977; Cohn and Kulsrud 1978; Magorrian and Tremaine 1999). This approach is therefore pedagogically appealing, but the interested reader is advised to thoroughly review the classical two-body relaxation literature for a fuller picture.

2.2 Two Body Relaxation

The loss cone of a SMBH can be defined as the region in angular momentum space for which

$$J^2 < J_{\text{LC}}^2(\epsilon) = 2r_t^2(\psi(r_t) - \epsilon) \approx 2GM_{\text{BH}}r_t, \quad (2.1)$$

where M_{BH} is the black hole mass, r_t is the stellar tidal radius, J is specific orbital angular momentum, ϵ is specific orbital energy, and $\psi(r)$ is the gravitational potential at a radius r from the SMBH, which is assumed to lie in the bottom of the stellar potential well¹. Throughout this chapter we employ the common stellar dynamical convention of negating the standard definitions of orbital energy and gravitational

¹ In this chapter we denote three dimensional radii with r rather than R to avoid confusion with an angular momentum-like variable.

potential; that is to say, $\psi > 0$ and, for bound stars, so is ϵ . The gravitational potential is defined as

$$\psi(r) = \psi_*(r) + \frac{GM_{\text{BH}}}{r}. \quad (2.2)$$

We will now make the assumption of a spherical star cluster surrounding the SMBH, with a stellar mass density profile $\rho(r)$. We will also employ the equivalent stellar number density profile, $n(r)$, throughout this section. This enables us to write the stellar potential as

$$\psi_*(r) = \frac{GM(r)}{r} + 4\pi G \int_{r'}^{\infty} \rho(r') r' dr', \quad (2.3)$$

where $M(r)$ is the total stellar mass enclosed at a radius r . With this potential in hand, we can calculate the stellar distribution function using the Eddington formula:

$$\begin{aligned} f(\epsilon) &= \frac{1}{\pi^2 \sqrt{8}} \frac{d}{d\epsilon} \int_0^\epsilon \frac{dn}{d\psi} \frac{d\psi}{\sqrt{\epsilon - \psi}} \\ &= \frac{1}{\pi^2 \sqrt{8}} \left(\int_0^\epsilon \frac{d^2 n}{d\psi^2} \frac{d\psi}{\sqrt{\epsilon - \psi}} + \frac{1}{\sqrt{\epsilon}} \left(\frac{dn}{d\psi} \right)_{\psi=0} \right). \end{aligned} \quad (2.4)$$

Using the distribution function $f(\epsilon)$, we can now calculate the flux of stars into the loss cone, $\mathcal{F}(\epsilon)$. Integrating this over all energies will give the total stellar disruption rate for a given galaxy,

$$\dot{N}_{\text{TDE}} = \int \mathcal{F}(\epsilon) d\epsilon. \quad (2.5)$$

To calculate $\mathcal{F}(\epsilon)$, we define the angular momentum proxy

$$R = \frac{J^2}{J_{\text{LC}}^2}, \quad (2.6)$$

which we shall use to define an orbit-averaged angular momentum diffusion coefficient,

$$\bar{\mu}(\epsilon) = \frac{2}{P(\epsilon)} \int_{r_p}^{r_a} \frac{dr}{v_r(r)} \lim_{R \rightarrow 0} \frac{\langle (\Delta R)^2 \rangle}{2R}. \quad (2.7)$$

Here $P(\epsilon)$ is the orbital period, $v_r(r)$ is the radial component of orbital velocity, and the locally-defined angular momentum diffusion coefficient is

$$\lim_{R \rightarrow 0} \frac{\langle (\Delta R)^2 \rangle}{2R} = \frac{32\pi^2 r^2 G^2 M_*^2 \ln \Lambda}{3J_c^2(\epsilon)} (3I_{1/2}(\epsilon) - I_{3/2}(\epsilon) + 2I_0). \quad (2.8)$$

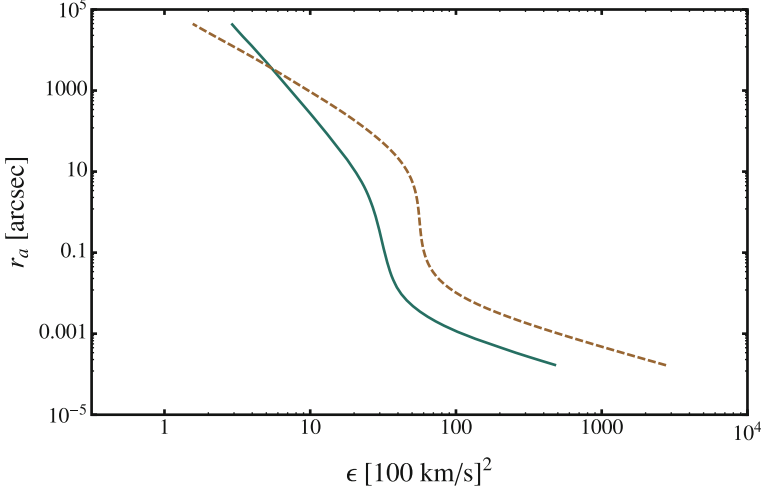


Fig. 2.1 The apocenter of a radial orbit, r_a , plotted against orbital energy ϵ , for the cusp galaxy NGC 4551 (*solid green line*) and the core galaxy NGC 4168 (*dashed brown line*). NGC 4551 has $M_{\text{BH}} = 10^{7.11} M_{\odot}$, while NGC 4168 has $M_{\text{BH}} = 10^{7.89} M_{\odot}$

For simplicity we assume that all stars in the cusp have the same mass², M_* . The angular momentum of a circular orbit with energy ϵ is $J_c(\epsilon)$, and the Coulomb logarithm is $\ln \Lambda$. We have also defined the auxiliary integrals

$$I_0(\epsilon) = \int_0^\epsilon f(\epsilon') d\epsilon' \quad (2.9)$$

and

$$I_{n/2} = (2\psi(r) - 2\epsilon)^{-n/2} \int_\epsilon^{\psi(r)} (2\psi(r) - 2\epsilon')^{n/2} f(\epsilon') d\epsilon'. \quad (2.10)$$

We can now specify the flux of stars per unit time per unit energy into the loss cone as

$$\mathcal{F}(\epsilon) d\epsilon = 4\pi^2 J_c^2(\epsilon) P(\epsilon) \bar{\mu}(\epsilon) \frac{f(\epsilon)}{\ln^{-1} R_0(\epsilon)} d\epsilon. \quad (2.11)$$

The quantity $R_0(\epsilon)$ accounts for the fact that our distribution function, which we have so far treated as isotropic, is not exactly so; very radial orbits have been depleted due to interactions with the loss cone. $R_0(\epsilon)$ is the value of R beneath which no more stars exist, and is given by

$$R_0(\epsilon) = R_{\text{LC}}(\epsilon) \times \begin{cases} \exp(-q(\epsilon)), & q(\epsilon) > 1 \\ \exp(-0.186q(\epsilon) - 0.824\sqrt{q(\epsilon)}), & q(\epsilon) < 1. \end{cases} \quad (2.12)$$

² Generalizing this calculation to a realistic present day mass spectrum increases TDE rates by a factor $\lesssim 2$ (Magorrian and Tremaine 1999).

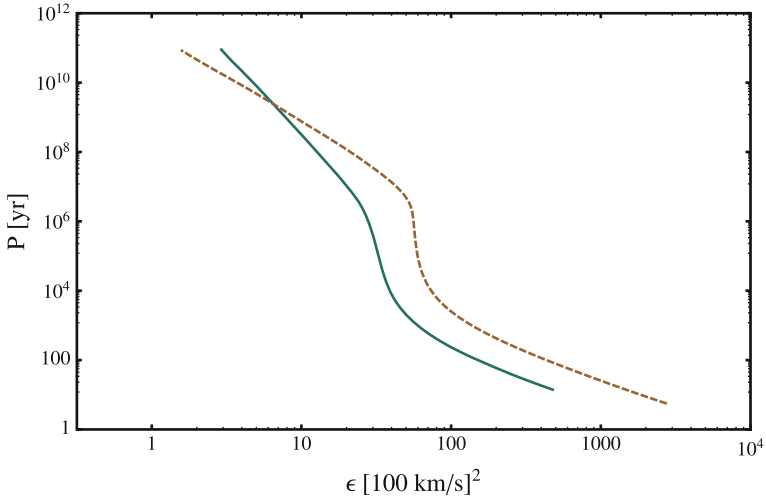


Fig. 2.2 The period of a radial orbit, $P(\epsilon)$, plotted against orbital energy ϵ . The line types are the same as in Fig. 2.1

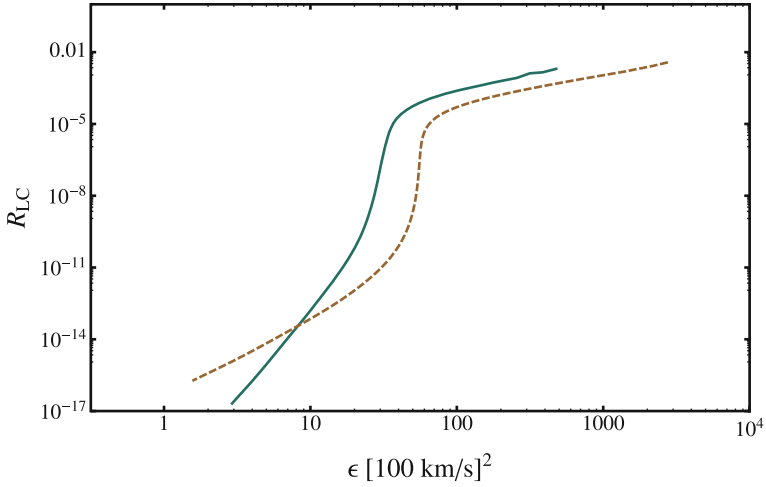


Fig. 2.3 The geometric size of the loss cone, $R_{LC}(\epsilon) = J^2/J_c(\epsilon)$, plotted against orbital energy ϵ . The line types are the same as in Fig. 2.1

The dimensionless quantity $q(\epsilon)$ can be thought of as the ratio of per-orbit changes in R to the R value of a loss cone orbit (all at fixed energy). It is defined as

$$q(\epsilon) = \frac{P(\epsilon)\bar{\mu}(\epsilon)}{R_{LC}(\epsilon)}, \quad (2.13)$$

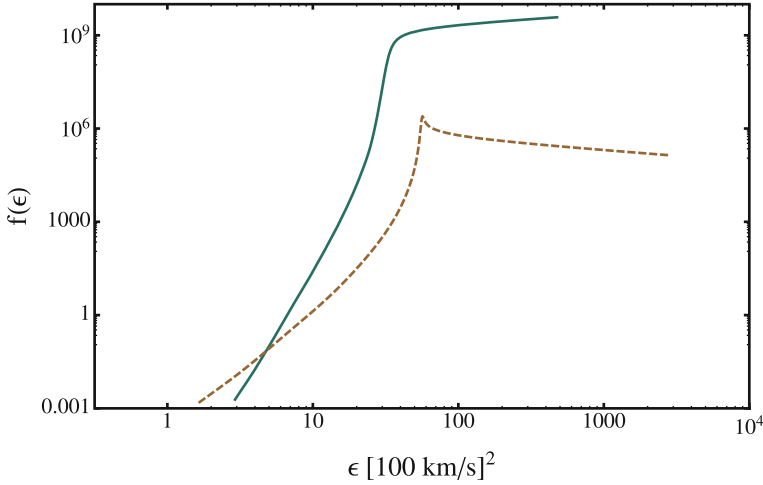


Fig. 2.4 The distribution function of stars, $f(\epsilon)$, plotted against orbital energy ϵ . Qualitatively different behavior is seen for the core galaxy (whose distribution function peaks at a finite ϵ) and the cusp galaxy (whose distribution function monotonically increases toward small radii). The line types are the same as in Fig. 2.1

and it demarcates the boundary between two different regimes of loss cone dynamics. When $q \ll 1$, stars diffuse slowly through angular momentum space, taking small steps that eventually result in grazing disruptions ($r_p \approx r_t$; $\beta \approx 1$). This “diffusive” regime is the one that dominates for stars near the SMBH. Beyond a critical radius r_{crit} , $q(\epsilon)$ grows quickly in value until $q \gg 1$. In this “pinhole” regime, the loss cone is close to full because stars can jump in and out of it multiple times per orbit. Notably, this produces TDEs that sample a full distribution of β , with $\dot{N}_{\text{TDE}}(\beta) \propto 1/\beta$.

2.3 Tidal Disruption Rates in Realistic Galaxies

We now have defined the theoretical formalism to calculate TDE rates in a galactic nucleus with known SMBH mass M_{BH} and stellar mass density profile $\rho(r)$ (assuming a mean stellar mass M_* to calculate $n(r) = \rho(r)/M_*$). However, $\rho(r)$ can only be inferred indirectly from observations of distant galactic nuclei. What is actually measured is the surface brightness profile $I(s)$, where s is a projected radial coordinate. In order to retrieve $\rho(r)$ from $I(s)$, we must assume (or measure) the mass to light ratio Υ , and also make the assumption of spherical symmetry.

Although many parametrizations for $I(s)$ exist in the literature, we will employ the “Nuker” surface brightness density profile

$$I(s) = I_b 2^{\frac{B-\Gamma}{\alpha}} \left(\frac{s}{s_b} \right)^{-\Gamma} \left(1 + \left(\frac{s}{s_b} \right)^\alpha \right)^{\frac{\Gamma-B}{\alpha}}. \quad (2.14)$$

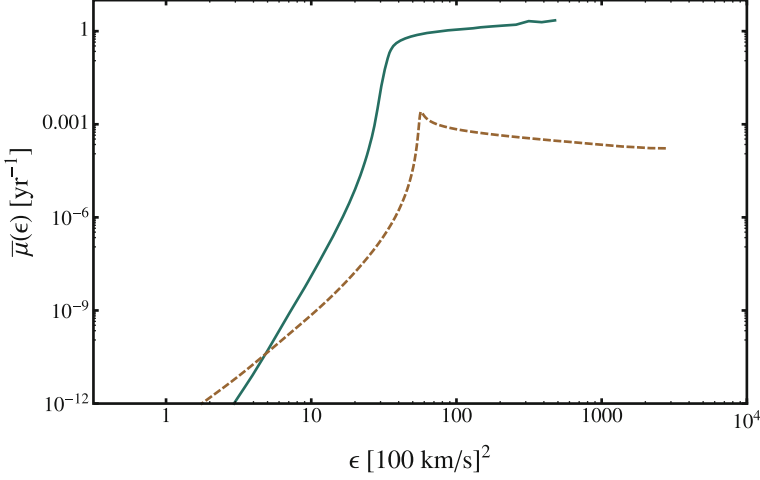


Fig. 2.5 The orbit-averaged angular momentum diffusion coefficient, $\bar{\mu}(\epsilon)$, plotted against orbital energy ϵ . The line types are the same as in Fig. 2.1

This five-parameter model is essentially a broken power law for the surface density profile; B is the inner power law slope, Γ is the outer power law slope, and α is a parameter governing the behavior at the break radius s_b . The normalization I_b is the magnitude of the surface brightness at $s = s_b$. Using an assumed or measured Υ , we can deproject a measured $I(s)$ into $\rho(r)$ using the Abel inversion

$$\rho(r) = -\frac{\Upsilon}{\pi} \int_r^\infty \frac{dI}{ds} \frac{ds}{\sqrt{s^2 - r^2}}. \quad (2.15)$$

Once we know $\rho(r)$, it is straightforward to calculate $M(r)$ and $\psi(r)$, and from there, to calculate TDE rates using the formalism of the previous section.

As a demonstration of this method, we now follow the example of Wang and Merritt (2004), and plot several quantities of interest for the galaxies NGC 4551 and NGC 4168. The SMBH masses for these galaxies, as inferred from the $M_{\text{BH}} - \sigma$ relation, are $M_{\text{BH}} = 10^{7.11} M_\odot$ and $M_{\text{BH}} = 10^{7.89} M_\odot$, respectively. In Figs. 2.1 and 2.2 we plot the apocenters and periods of radial orbits against orbital energy. Figure 2.3 shows the small geometric size of the loss cone, $R_{\text{LC}}(\epsilon)$, against orbital energy, while Fig. 2.4 shows the stellar distribution functions for both galaxies. Interestingly, $f(\epsilon)$ shows qualitatively different behavior for core and cusp galaxies: in core galaxies, the distribution function peaks near the SMBH influence radius, but in cusp galaxies it increases monotonically toward small radii. Figure 2.5 plots the orbit-averaged angular momentum diffusion coefficient, $\bar{\mu}(\epsilon)$, which like $f(\epsilon)$ shows different behavior for core and cusp galaxies. The related quantity $q(\epsilon)$ is plotted in Fig. 2.6, and, finally, we show flux into the loss cone, $\mathcal{F}(\epsilon)$, in Fig. 2.7. The colored text in this figure shows the integrated loss cone flux, i. e. the TDE rate for each galaxy.

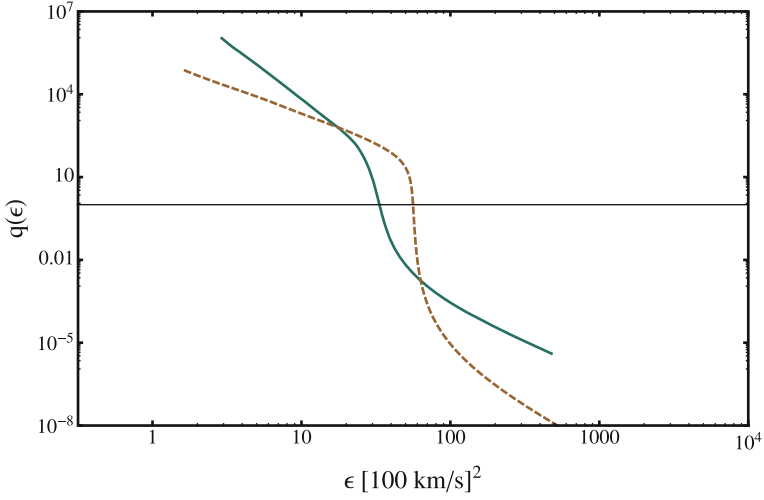


Fig. 2.6 The normalized per-orbit diffusion in R , $q(\epsilon) \approx (\Delta J(\epsilon)/J_{\text{LC}}(\epsilon))^2$, plotted against orbital energy ϵ . The thin black line indicates where $q(\epsilon) = 1$; this defines a critical energy (and therefore a critical radius) where flux into the loss cone transitions from diffusive to pinhole. The line types are the same as in Fig. 2.1

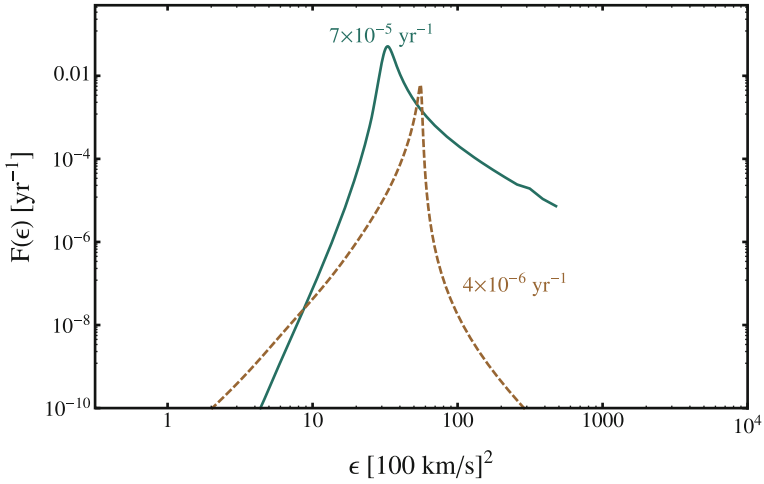


Fig. 2.7 The flux of stars into the loss cone, per unit energy per unit time, $\mathcal{F}(\epsilon)$, plotted against orbital energy ϵ , for the galaxies NGC 4551 and 4168. Integrals of these curves $d\epsilon$ are shown as colored text (i.e. the total number of TDEs per galaxy per year). The line types are the same as in Fig. 2.1. Note that this function is very sharply peaked in energy space, with the vast majority of tidally disrupted stars coming from a relatively narrow range in orbital energies

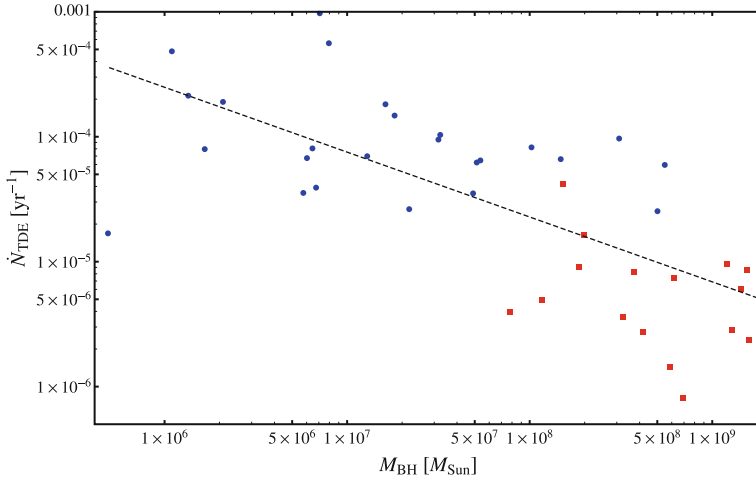


Fig. 2.8 The per galaxy rate of tidal disruption events, $\dot{N}_{\text{TDE}}$, in units of yr^{-1} , plotted against M_{BH} . The blue circles show cusp galaxies, and the red squares show core galaxies; both are taken from the sample in Wang and Merritt (2004). The dashed *black line* shows a simple best fit power law to the entire sample ($\dot{N}_{\text{TDE}} \propto M_{\text{BH}}^{-0.52}$). The large scatter above and below the best fit power law is due to intrinsic scatter in Nuker parameters, particularly Υ and Γ

Generally speaking, the highest TDE rates occur in smaller galaxies, for two reasons. The first reason is that high-resolution *HST* imaging indicates a bimodality in nuclear surface brightness profiles: smaller (“cusp”) galaxies tend to have higher values of the Nuker parameter $\gamma \approx 1$, indicating steeper central density profiles and denser star clusters with shorter relaxation times. Larger galaxies preferentially have low-density “cores,” with $\gamma \approx 0$ and correspondingly longer relaxation times. These cores are thought to be scoured by the inspiral of a binary SMBH system following a galaxy merger; smaller galaxies generally lack them because the larger gas fractions in smaller galaxies allow the rebuilding of stellar cusps after the SMBH merger. The second reason is that in dense cusp galaxies, $\dot{N}_{\text{TDE}}$ actually increases with decreasing SMBH mass, while the opposite is true for core galaxies. Specifically, Wang and Merritt (2004) find that for constant σ ,

$$\begin{aligned} \dot{N}_{\text{TDE}} &\propto M_{\text{BH}}^{\delta}, \\ \delta &= \frac{27 - 19\gamma}{6(4 - \gamma)}, \end{aligned} \quad (2.16)$$

where $\gamma \approx 1 + \Gamma$ is defined as the 3D power law slope of stellar density, i.e. $\rho(r) \propto r^{-\gamma}$. When $\gamma < 27/19$, $\delta > 0$ and the TDE rate increases with increasing SMBH mass.

Because the specific TDE rate is highest for low-mass galaxies (as can be seen in Fig. 2.8), and because low-mass galaxies are more numerous, it has long been thought that large samples of TDEs could serve as excellent probes for the uncertain bottom end of the SMBH mass function. In particular, while almost all galaxies

with stellar bulge masses $M > 10^9 M_\odot$ are believed to host a central SMBH, the occupation fraction below this mass is unclear. Estimating this occupation fraction represents an important scientific goal for future TDE observations.

Ultimately, tidal disruption events are quite rare. Past work has estimated typical values of $\dot{N}_{\text{TDE}} \sim 10^{-4} - 10^{-5} \text{ yr}^{-1}$ (Magorrian and Tremaine 1999; Wang and Merritt 2004), although this may be increased by a factor of a few due to the effects of nonspherical, axisymmetric stellar cusps. Interestingly, this theoretical estimation is somewhat above the rates inferred from observational samples of TDEs (Donley et al. 2002), which is puzzling, as the TDE rate due to two-body relaxation is a lower limit that can be increased by, e.g., massive perturbers or strongly triaxial cusps. However, inferred rates are convolved with complicated selection effects, currently suffer from small number statistics, and in any event are being compared to theoretical estimates which themselves carry large uncertainties, so this discrepancy is not yet severe.

The following two chapters of this thesis describe original work by the author on TDE rates in a specific and unusual type of galaxy: one where the central SMBH is recoiling out of the center due to the anisotropic emission of gravitational radiation following the merger of two SMBHs.

The Tidal Disruption of Stars by Supermassive Black
Holes

An Analytic Approach

Stone, N.C.

2015, XV, 154 p. 45 illus., 39 illus. in color., Hardcover

ISBN: 978-3-319-12675-3