

# Preface

Optical collapse is a fascinating research topic. The propagation of intense laser beams in a transparent medium is usually modeled by the two-dimensional nonlinear Schrödinger equation (NLS), and beam collapse corresponds to NLS solutions becoming singular. Since collapse occurs only when nonlinearity is stronger than diffraction, the analysis of singular NLS solutions requires a genuinely nonlinear approach.

This research field was started by experimental and theoretical physicists in the 1960s when high-power lasers became available. In the late 1970s, just as physicists started to lose interest, mathematicians joined in and began to develop the rigorous and asymptotic mathematical theory for NLS collapse. The availability of ultrashort lasers and new applications such as filamentation in air “brought back” the physicists, and nowadays this field is studied by both communities.

Because physicists and mathematicians tend to speak “different languages”, all too often the flow of information between them has been limited. One of the goals of this book is to lower the communication barrier between these communities, so that mathematicians would know the physical context of the mathematical theory and become familiar with the experimental research, while physicists know the mathematical theory and how it relates to the physics. To achieve this goal, the book adopts a “multi-lingual approach” and combines rigorous analysis, asymptotic analysis, informal arguments, numerical simulations, and physical experiments, repeatedly emphasizing the relations between these approaches and the intuition behind results.

The book covers material from the early 1960s and up to the present. Chapter 1 provides an informal derivation of the two-dimensional cubic NLS from Maxwell’s equations. Chapter 2 covers the relevant linear theory. Chapter 3 presents the pioneering early studies from the 1960s, and offers a historical perspective. NLS models in nonlinear optics and in Bose-Einstein condensates (BEC) are discussed in Chap. 4. The rigorous NLS theory begins in Chap. 5 with the topic of global existence. Properties of solitary waves are discussed in Chap. 6. Chapter 7

introduces the variance identity and its consequences. NLS symmetries are presented in Chap. 8, with special emphasis on the consequences of lens transformation. Stability of solitary waves is briefly discussed in Chap. 9. Chapters 10–12 present the explicit blowup solutions  $\psi_R^{\text{explicit}}$ ,  $\psi_G^{\text{explicit}}$ , and  $\psi_Q^{\text{explicit}}$ . Properties shared by all blowup solutions of the critical NLS are presented in Chap. 13, and those that are unique to solutions that collapse with the  $\psi_{R(0)}$  profile are presented in Chap. 14. Chapter 15 concerns singular vortex solutions. Chapter 16 studies the effect of reflecting boundaries, by considering the NLS on bounded domains.

The blowup rate of peak-type solutions of the critical NLS is called the *loglog law*. The asymptotic analysis that leads to the loglog law, its “failure” in the regime of physical interest, and how to correct it with an adiabatic approach are presented in Chaps. 17 and 18. Other kinds of collapsing solutions of the critical NLS, namely ring-type and vortex solutions, are discussed in Chaps. 19 and 20. Collapsing peak-type and ring-type solutions of the supercritical NLS are covered in Chaps. 21–23. Going back to the nonlinear optics context, Chaps. 24 and 25 discuss the critical power for collapse and the breakup of high-power laser beams into multiple filaments. An asymptotic theory for strongly nonlinear solutions, the nonlinear geometrical optics (NGO) method, is presented in Chap. 26. Theoretical and experimental results on the location of singularity and on how to control it are given in Chap. 27. Numerical methods are discussed in Chaps. 28–30.

One of the advantages of this research field is that theoretical predictions that are based on the NLS can be observed in nonlinear optics experiments. This is because the NLS model is valid before and during the initial stages of the collapse. To model beam propagation beyond the NLS singularity, however, the mathematical model should include some of the terms that were neglected in the derivation of the NLS from Maxwell’s equations. A systematic asymptotic method for approximating perturbations of the critical NLS by simpler equations, called modulation theory, is derived in Chap. 31. The effects of various small terms neglected in the NLS model (high-order nonlinearities, linear and nonlinear damping, non-paraxiality, backscattering, and dispersion) on collapsing beams are discussed in Chaps. 32–37. Chapters 38 and 39 conclude with recent results on continuations of singular NLS solutions beyond the singularity.

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