

Exploring the Relations between Project Duration and Activity Duration

F. Acebes, J. Pajares, J. M. Galán and A. López-Paredes

1 Introduction

A project monitoring and control system is aimed at minimizing the deviations from the initial planning during the execution of the project. This process usually consists of four steps: identifying the status of the project, comparing it with the initial planning, analysing the deviations of the actual execution with regard to planning and proposing, and adopting corrective actions appropriate to redirect the project towards the planned goal.

In this work, uncertainty over the total project duration is referred to the probability that it ends in one or another date depending on the durations of the activities that make up the project.

Monte Carlo simulation is often used to create simulation models of projects [1, 2] which allow estimating uncertainty. If the durations of the activities are modelled using a probability distribution function, Monte Carlo simulation can give as result a probability distribution of the total project duration. This output is useful since it provides a probabilistic support to make predictions about the real duration of projects. However, project Monte Carlo models can be used to obtain additional

F. Acebes (✉) · J. Pajares · A. López-Paredes

Grupo INSISOC, Departamento de Organización de Empresas y CIM, Escuela de Ingenierías Industriales, Universidad de Valladolid, Pso del Cauce S/N, 47011 Valladolid, Spain
e-mail: facebes@yahoo.es

J. Pajares
e-mail: javier@insisoc.org

A. López-Paredes
e-mail: insisoc@insisoc.org

J. M. Galán
Grupo INSISOC, Área de Organización de Empresas, Departamento de Ingeniería Civil, Escuela Politécnica Superior, Universidad de Burgos, C. Villadiego S/N, 09001 Burgos, Spain
e-mail: jmgalan@ubu.es

interesting insights. For instance, we can try to figure out which activities are those with more influence in the duration or variability of the project.

In this article we analyse by means of a case study if shortening, lengthening or reducing the variability of certain activities can produce significant reductions in the expected duration of the entire project, or if the variability of that duration can be reduced. In order to do so, we base our analysis on previous research.

Williams [9] questions the consistency of the criticality index as a single indicator of the importance of each activity in project duration. Criticality is defined as the probability that an activity belong to the critical path of the project. It is usually calculated as a ratio between 0 and 1 of the number of times an activity is on the critical path to the total number of Monte Carlo simulation runs of the project. He proposes the cruciality index as an indicator to complement the analysis based on criticality. Cruciality is defined as the correlation between the duration of an activity and the project duration. In Elmaghraby [4], a short overview addresses the advantages and disadvantages of the sensitivity measures discussed by Williams [8]. He conjectures that the relative importance of project activities should be given by considering a combined version of these sensitivity measures.

Elmaghraby [4] also discusses the uncertainty of the whole project. This uncertainty is measured as the variability of the project duration (statistical variance of the duration), as a function of the expected duration of the activities (average duration). He reviews the research efforts focused on the sensitivity of the mean and variance of project's total duration consequence of changes in the mean and variance of individual activities. Cho and Yum [3] propose an uncertainty importance index to measure the effect of the variability in activity duration on the variability of the overall project duration. Elmaghraby et al. [5] investigate the impact of changing the mean duration of an activity on the variability of the project duration. Gutiérrez and Paul [6] present an analytical treatment of the effect of the activity variance on the expected project duration.

In this work, we explore the relation between the duration of each one of the activities of our case study (both the expected duration and the variability of the duration) with the criticality and cruciality indexes of the activities. We then analyse the influence of these variables on the duration of the entire project (both the expected duration and the variability of the duration). Additionally, we show which activities are the most influential in terms of the previous measures and their associated criticality and cruciality indexes.

We show that this type of analysis gives project managers two interesting additional insights to support decision making. They obtain relevant information to reduce the variability or uncertainty about the total project duration, and at the same time they can determine which are the most influential and decisive activities of the project under different perspectives.

The structure of the paper is as follows: in the next section we describe the project network used as case study in our research. Subsequently we give some brief background to criticality and cruciality concepts. Results and discussions are then provided for each of the variables analysed, and finally, conclusions are presented in the last section.

Table 1 Sequence of project activities of the case study. Durations are modelled as normal distributions; μ represents the mean duration and σ the standard deviation

Previous activity	Activity	μ	σ
	Ai	0	0
A _i	A1	5.5	1.18
A _i	A2	13	3
A1	A3	7	1
A1	A4	16.5	2.5
A2, A3	A5	10	1.34
A4, A5	Af	0	0

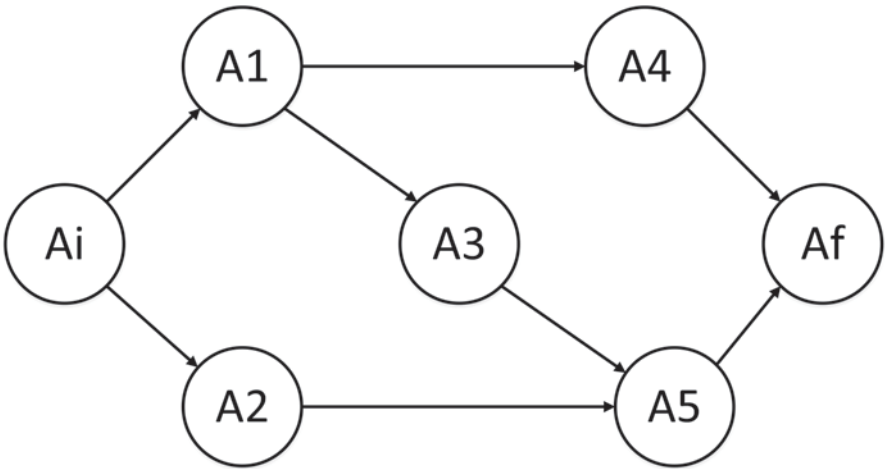


Fig. 1 Activity on node network diagram of the simulated project

2 Case Study

Our analysis is based on a stylized model of a project defined in Table 1 and Fig. 1. This model has been proposed and used by Elmaghraby et al. [5] in previous research. Each one of the activities is assumed stochastic and there are several finish-to-start precedence relationships.

Durations of activities are modelled as Normal probability distribution functions, whose parameters are shown in Table 1.

The used network combines series and parallel networks through the activity 3. Despite being a simple network, this feature increases the diversity of possible outcomes and combinations, which makes it interesting for academic and illustration purposes.

The start and end of the project are represented by two fictitious activities without duration: Initial activity (A_i) and End activity (A_f).

By means of Monte Carlo simulation we have analysed the following relations of the case study:

1. Expected project duration as function of the expected duration of activities.
2. Variability of the project duration as function of the expected duration of the activities.
3. Expected project duration as function of the variability of the duration of activities.
4. Variability of the project duration as function of the variability of the duration of activities.

We use the statistics obtained from the analysis of the total project duration as benchmark to compare the rest of results. Average (expected duration) and variance (measure to assess the variability or dispersion of the duration) are modified for each activity of the project. The results obtained are used to study every one of the cases A, B, C and D and to perform an analysis of criticality and cruciality.

3 Criticality and Cruciality

Criticality and cruciality of the project's activities can both be used as a measure of the influence of the duration of each activity on the duration of the entire project. As consequence, it is important to state clearly the difference between both indexes in order to correctly interpret the information provided for each measure.

Criticality or Criticality Index of an activity is the probability that such activity lies on the critical path or on one of the critical paths of the project. Therefore, using Monte Carlo simulation this measure is calculated as the times over the total number of simulations in which the activity belonged to the critical path.

According to Williams [10] the criticality index should be considered along with other indicators to determine more precisely the influence of the duration of each activity in the whole project. One option is the Cruciality Index.

Cruciality or Cruciality Index of an activity is the correlation between the duration of the activity and the duration of the entire project. Therefore, the influence of the activity duration on the duration of the entire project is measured as the correlation coefficient (Pearson correlation coefficient) between the duration of the activity and the duration of the entire project. In our case study, this information is extracted from the total of the simulations carried out.

This index represents the degree of linear association between the total duration of a project and the duration of a particular activity. In other words, if the total duration of a project is long when a particular activity is long, and the duration of the project is short when this activity is short, then we can say that this activity has a high Cruciality Index.

Both indexes provide complementary information to make decisions. Activities with high Cruciality Index must be carefully managed as consequence of the uncertainty that they create on the project, while activities with high criticality must be shortened, because they are likely to become the "bottlenecks" that hinder reducing the total duration.

4 Analysis of the Results

4.1 *Expected Project Duration as Function of the Expected Duration of the Activities*

Any increase in the duration of an activity will lead to an increase in the project duration, or, at least, the project duration will remain constant. Otherwise, any decrease in the duration of the activity will lead to a decrease in the project duration, or, at least, the project duration will remain constant.

As expected, we can appreciate that increasing the duration of any given activity produce an increase on its criticality and in those which belong to its same critical path (see Fig. 2). Once an activity exceeds a given threshold in its criticality index (95 % in this case), the expected project duration increases almost exactly the amount that we have increased in that activity, or in other words, this activity has become practically the only one with influence on the expected project duration.

Given the topology of the network, activity A1 becomes the most critical activity when we increase his duration. We obtain a similar result if we simulate with activity A5. The rest of activities will have minor criticality index.

In the same way, the Cruciality Index of the activities increases if we increase the duration of each one of them. We highlight the results of the activity A1 and activity A4. Increasing the duration of activity A1, increases its cruciality; however due to the long duration of activity A4, which belongs to the same path as activity A1, the cruciality index of both is similar.

4.2 *Variability of Project Duration as Function of the Expected Duration of Activities*

In all cases, the variability of the project duration is stabilized and remains constant from a certain threshold. This happens when the criticality of the activity that we have increased in each case reaches approximately 1.

In this case, if we increase the expected activity duration, we can find three types of activities: (1) those in which an increase in the duration of the activity involves a reduction in the variability of the project duration (see Fig. 3), (2) those in which an increase in the duration of the activity produces an increase in the variability of the project duration and (3) a final set of activities in which, after some oscillations, variability stabilizes at a constant value.

These results are insightful since they illustrate that, if certain activities extend their duration, they may contribute to reduce the uncertainty about the date of completion of the project without delaying it too much.

For networks with paths of more diverse criticality, increasing the more critical activities may reduce even more the uncertainty about project duration. On the other

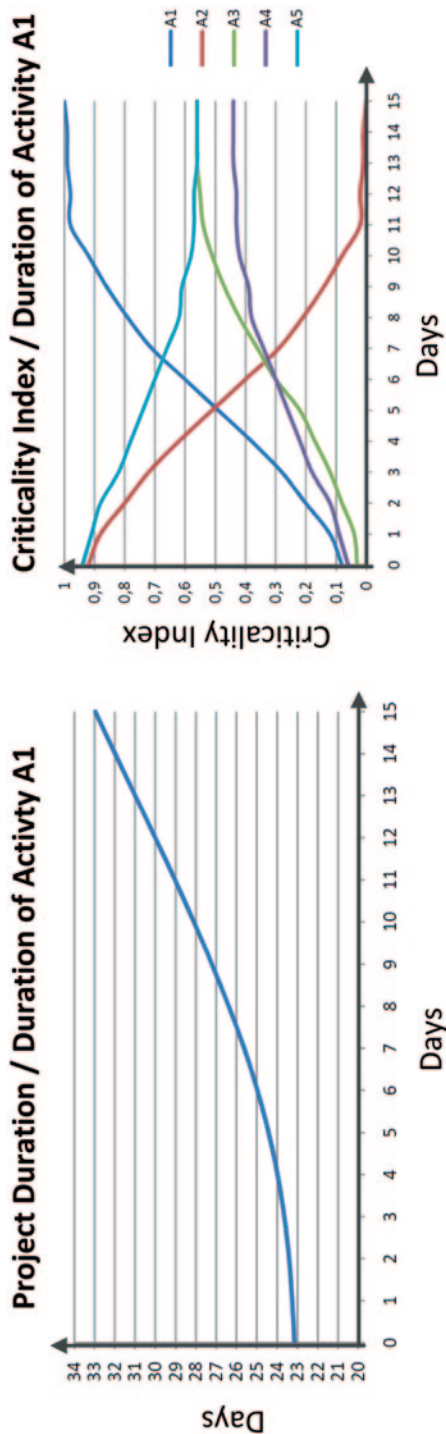


Fig. 2 Average project duration and criticality based on the duration of the activity A1

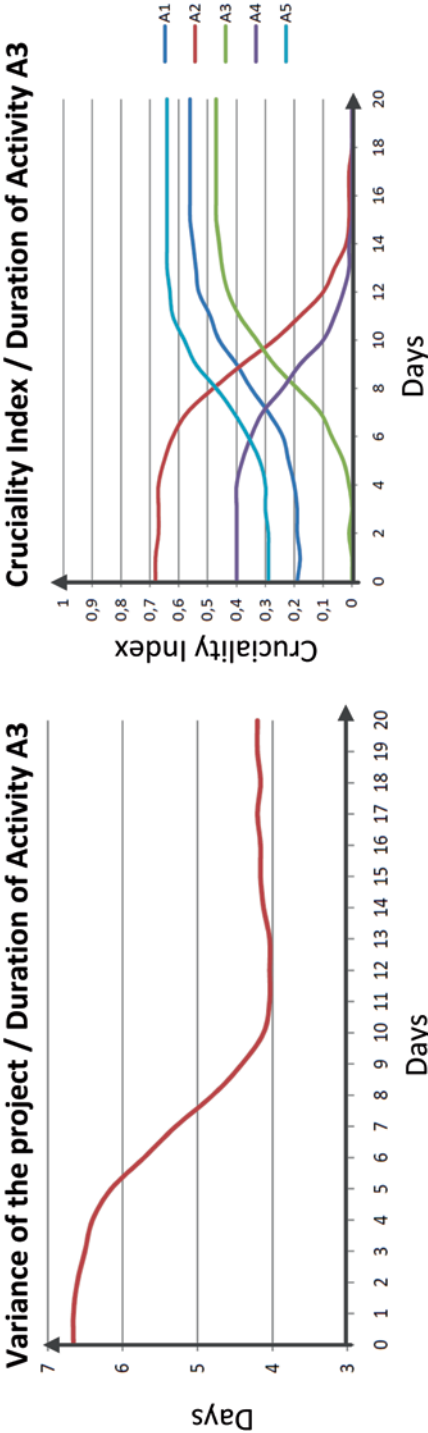


Fig. 3 Variance of the project and cruciality based on the duration of the activity A3

hand, increasing the less critical activities may contribute to raise considerably the uncertainty about the project duration, as the criticalities between the different paths become similar.

In relation to activities that cause oscillations in the variability of the project duration, they are usually the final activities of the project. Initial activities are the cause of an important part of the uncertainty of the project since they are performed in the periods with less information. However, as the project advances, there are more available information and uncertainty about the date of completion is lower, or in other words, final activities generate less uncertainty about the project because they depend on the development of activities that precede them.

Thus, activities which constitute the initial phases of the project are those that generate more uncertainty about the date of completion of the project and consequently they are the activities that project managers should study more carefully, because if they are managed in an appropriate way, uncertainty about the end of the project might be reduced.

Our results suggest that if the goal is to reduce the variability in total project duration, two decisions can be taken: Increase the criticality of the most critical path significantly reduce the variability of the project, i.e., increasing the more critical initial activities or shorten the less critical to reduce uncertainty about the project. A second option is to try to reduce the uncertainty generated by the most crucial activities, particularly the initial tasks.

4.3 Expected Project Duration as Function of the Variability of the Activities Duration

Most of the projects in reality include parallel paths. Our analysis for these projects shows that an increase of the variability in the duration of an activity leads to increase the expected duration of the entire project. This result confirms the conjecture proposed by Schonberger [7] who hypothesized that an increase in the variability of task durations will result in an increase of the expected total project duration.

The criticality is an indicator that does not provide complete information about the expected project duration. Observing Fig. 4, if we increase the variability of the activity, the length of the critical path remains more or less constant, whereas the project expected duration increases lightly. The criticality index remains constant, whereas the cruciality index increases. Therefore, in this case the cruciality index measures better the influence of the uncertainty of the activities on the project.

4.4 Variability of Project Duration as Function of the Variability of the Activities Duration

Instead of focusing our study just on the analysis of cruciality, we have used multiple linear regression to estimate the importance of the uncertainty of the activity.

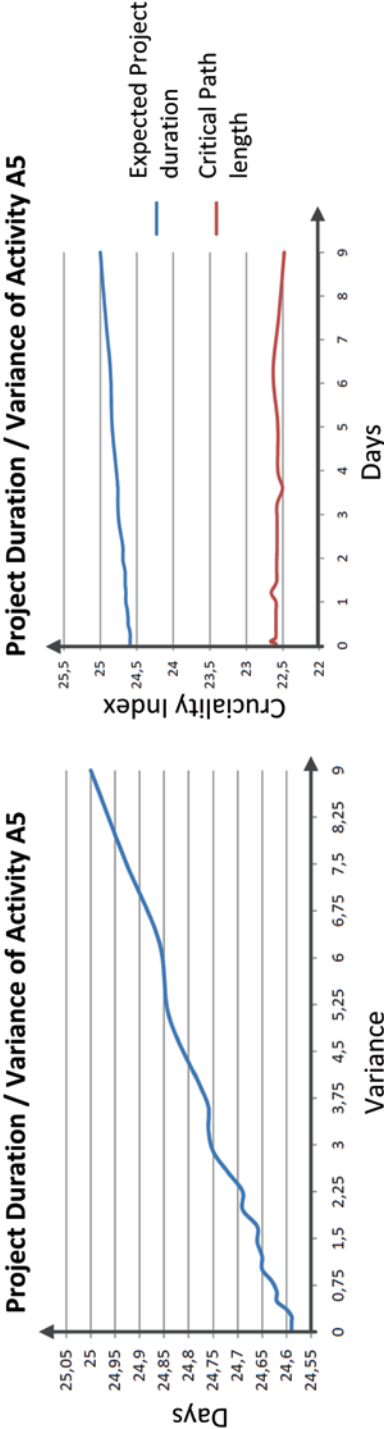


Fig. 4 Average project duration and length of the critical path based on the duration of the activity A5

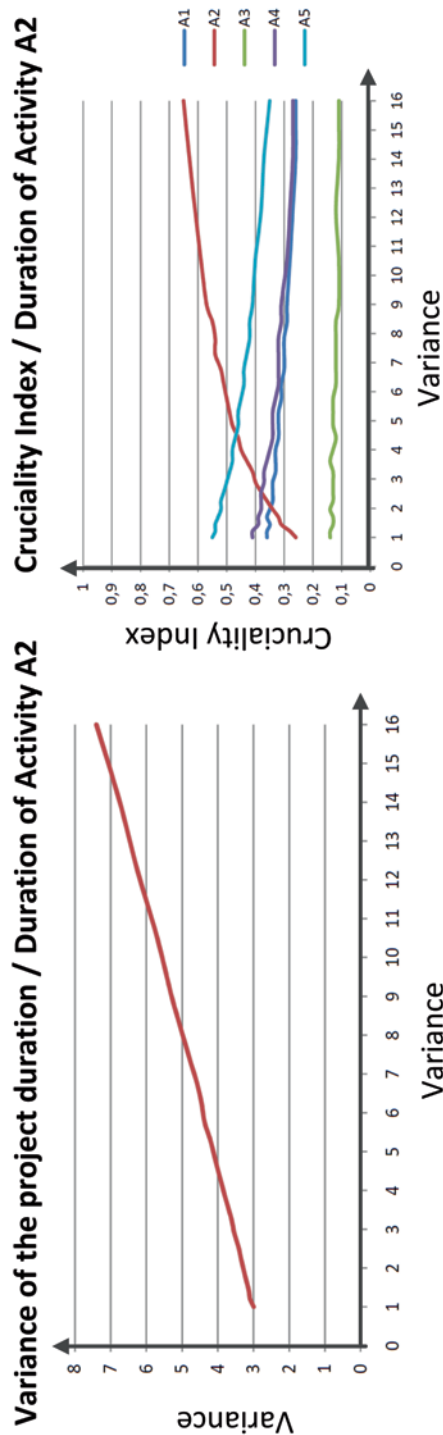


Fig. 5 Variance of the project and cruciality based on the duration of the activity A2

In Fig. 5 we show the evolution of the variability of the project duration as function of the variance of the activity A2. We show also how the evolution of cruciality index is when the variance of that activity is modified.

Our results indicate that all the variables are significant, however, given the parameters of the analysed network there are relevant differences between activities.

Activity A2 is notably the most influential because it creates almost 50% of the variability of the project duration, and the activity A5 origins 35% of this variability. With 85% of variability created over the project duration, the variability in the duration of activities A2 and A5 are considerably the most influential. So these activities are those that project managers should manage more carefully to reduce the uncertainty of the project.

5 Conclusions

We have illustrated with a case study that an analysis of different possible scenarios for the development of the project by means of simulation Monte Carlo may be very insightful to support decision making for project managers.

Once a simulation model of the project is properly formalized, specific characteristics of activities in terms of criticality and cruciality can be identified. This information may allow detecting the key activities which are decisive in the development of the project. These tasks are consequently those in which project managers should pay special attention. We have shown that modifying their expected duration or reducing their duration variability, the uncertainty about the total project duration can be reduced.

In addition, we have confirmed that criticality is not a complete meaningful indicator to assess how a modification in the duration of an activity influences the duration of the total project, and hence cruciality index provides complementary useful information.

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