
Contents

1	Numerical series	1
1.1	Round-up on sequences	1
1.2	Numerical series	4
1.3	Series with positive terms	9
1.4	Alternating series	16
1.5	The algebra of series	19
1.6	Exercises	21
	1.6.1 Solutions	24
2	Series of functions and power series	33
2.1	Sequences of functions	34
2.2	Properties of uniformly convergent sequences	37
	2.2.1 Interchanging limits and integrals	38
	2.2.2 Interchanging limits and derivatives	39
2.3	Series of functions	41
2.4	Power series	44
	2.4.1 Algebraic operations	52
	2.4.2 Differentiation and integration	53
2.5	Analytic functions	56
2.6	Power series in \mathbb{C}	60
2.7	Exercises	60
	2.7.1 Solutions	64
3	Fourier series	75
3.1	Trigonometric polynomials	76
3.2	Fourier Coefficients and Fourier series	79
3.3	Exponential form	88
3.4	Differentiation	89
3.5	Convergence of Fourier series	90
	3.5.1 Quadratic convergence	90

3.5.2	Pointwise convergence.....	93
3.5.3	Uniform convergence.....	95
3.5.4	Decay of Fourier coefficients	96
3.6	Periodic functions with period $T > 0$	96
3.7	Exercises	98
3.7.1	Solutions	100
4	Functions between Euclidean spaces	111
4.1	Vectors in \mathbb{R}^n	111
4.2	Matrices	114
4.3	Sets in \mathbb{R}^n and their properties	120
4.4	Functions: definitions and first examples.....	126
4.5	Continuity and limits	130
4.5.1	Properties of limits and continuity.....	137
4.6	Curves in \mathbb{R}^m	138
4.7	Surfaces in \mathbb{R}^3	142
4.8	Exercises	145
4.8.1	Solutions	147
5	Differential calculus for scalar functions.....	155
5.1	First partial derivatives and gradient.....	155
5.2	Differentiability and differentials.....	160
5.2.1	Mean Value Theorem and Lipschitz functions	165
5.3	Second partial derivatives and Hessian matrix	168
5.4	Higher-order partial derivatives.....	170
5.5	Taylor expansions; convexity	171
5.5.1	Convexity	173
5.6	Extremal points of a function; stationary points	174
5.6.1	Saddle points	178
5.7	Exercises	183
5.7.1	Solutions	186
6	Differential calculus for vector-valued functions	201
6.1	Partial derivatives and Jacobian matrix	201
6.2	Differentiability and Lipschitz functions	202
6.3	Basic differential operators	204
6.3.1	First-order operators.....	204
6.3.2	Second-order operators.....	211
6.4	Differentiating composite functions	212
6.4.1	Functions defined by integrals.....	214
6.5	Regular curves	217
6.5.1	Congruence of curves; orientation.....	220
6.5.2	Length and arc length	222
6.5.3	Elements of differential geometry for curves.....	225
6.6	Variable changes	227

6.6.1	Special frame systems	230
6.7	Regular surfaces	236
6.7.1	Changing parametrisation	240
6.7.2	Orientable surfaces	241
6.7.3	Boundary of a surface; closed surfaces	243
6.7.4	Piecewise-regular surfaces	247
6.8	Exercises	248
6.8.1	Solutions	251
7	Applying differential calculus	261
7.1	Implicit Function Theorem	261
7.1.1	Local invertibility of a function	267
7.2	Level curves and level surfaces	268
7.2.1	Level curves	269
7.2.2	Level surfaces	273
7.3	Constrained extrema	274
7.3.1	The method of parameters	277
7.3.2	Lagrange multipliers	278
7.4	Exercises	285
7.4.1	Solutions	288
8	Integral calculus in several variables	297
8.1	Double integral over rectangles	298
8.2	Double integrals over measurable sets	304
8.2.1	Properties of double integrals	313
8.3	Changing variables in double integrals	317
8.4	Multiple integrals	322
8.4.1	Changing variables in triple integrals	328
8.5	Applications and generalisations	330
8.5.1	Mass, centre of mass and moments of a solid body	330
8.5.2	Volume of solids of revolution	332
8.5.3	Integrals of vector-valued functions	335
8.5.4	Improper multiple integrals	335
8.6	Exercises	337
8.6.1	Solutions	343
9	Integral calculus on curves and surfaces	367
9.1	Integrating along curves	368
9.1.1	Centre of mass and moments of a curve	374
9.2	Path integrals	375
9.3	Integrals over surfaces	377
9.3.1	Area of a surface	381
9.3.2	Centre of mass and moments of a surface	383
9.4	Flux integrals	383
9.5	The Theorems of Gauss, Green, and Stokes	385

9.5.1	Open sets, admissible surfaces and boundaries	386
9.5.2	Divergence Theorem	391
9.5.3	Green's Theorem	393
9.5.4	Stokes' Theorem	395
9.6	Conservative fields and potentials	397
9.6.1	Computing potentials explicitly	404
9.7	Exercises	406
9.7.1	Solutions	410
10	Ordinary differential equations	421
10.1	Introductory examples	421
10.2	General definitions	424
10.3	Equations of first order	430
10.3.1	Equations with separable variables	430
10.3.2	Homogeneous equations	432
10.3.3	Linear equations	433
10.3.4	Bernoulli equations	437
10.3.5	Riccati equations	437
10.3.6	Second-order equations reducible to first order	438
10.4	The Cauchy problem	440
10.4.1	Local existence and uniqueness	440
10.4.2	Maximal solutions	444
10.4.3	Global existence	446
10.4.4	Global existence in the future	448
10.4.5	First integrals	451
10.5	Linear systems of first order	454
10.5.1	Homogeneous systems	456
10.5.2	Non-homogeneous systems	459
10.6	Linear systems with constant matrix \mathbf{A}	461
10.6.1	Homogeneous systems with diagonalisable \mathbf{A}	462
10.6.2	Homogeneous systems with non-diagonalisable \mathbf{A}	466
10.6.3	Non-homogeneous systems	470
10.7	Linear scalar equations of order n	473
10.8	Stability	478
10.8.1	Autonomous linear systems	480
10.8.2	Two-dimensional systems	481
10.8.3	Non-linear stability: an overview	487
10.9	Exercises	489
10.9.1	Solutions	494

Appendices	509
A.1 Complements on differential calculus	511
A.1.1 Differentiability and Schwarz's Theorem	511
A.1.2 Taylor's expansions	513
A.1.3 Differentiating functions defined by integrals	515
A.1.4 The Implicit Function Theorem	518
A.2 Complements on integral calculus	521
A.2.1 Norms of functions	521
A.2.2 The Theorems of Gauss, Green, and Stokes	524
A.2.3 Differential forms	529
Basic definitions and formulas	533
Index	545



<http://www.springer.com/978-3-319-12756-9>

Mathematical Analysis II

Canuto, C.; Tabacco, A.

2015, XIII, 559 p., Softcover

ISBN: 978-3-319-12756-9