

Preface

There are many excellent books and monographs dedicated to the fundamentals of quantum mechanics. The content of these books is mainly focused on what has to be done to describe a physical system in a quantum-mechanical way, namely to solve the Schrödinger equation for a wave function or statistical operator. This book is an attempt to discuss how to describe a real quantum mechanical system, and we use a variety of examples from the broad spectrum of physical problems in support of this attempt.

In contrast to equations of condensed matter physics or hydrodynamics, the direct numerical solution of the Schrödinger equation, using a finite-element approximation for differential operators, is practically ineffective due to the large number of variables. Therefore, the most widely used approach for the solution is to approximate the physical system by some models, which have an exact solution. Then construct the perturbation series for the perturbation operator, which is defined by the difference between the Hamiltonian of real system and the Hamiltonian of the model. The applicability of this approach depends on the existence of small parameters for the perturbation operator.

Modern theoretical physics, however, deals with physical systems, the majority of which do not permit us to define a small parameter bearing a physical sense and mathematically fitting to the formal method of perturbation theory. Even after being constructed, the perturbation series often have an asymptotic nature and zero convergence radius. Moreover, there are many practical physical problems where the parameters of Hamiltonian vary within a broad range of values, and the perturbation operator is not sorted out. These systems require a non-perturbative approach for the development of the quantum theory without a physical small parameter.

There are several methods capable of theoretically investigating quantum systems without introducing a small parameter. The direct numerical integration of the Schrödinger equation by using expansion over an artificial parameter, a step of finite-difference approximation derivatives, is a frequently used technique to avoid implementation of a small parameter. The use of lattice models of quantum field theory is another example of non-perturbative methods. The lattice constant is a non-physical parameter which determines the accuracy of the calculation for

functional integrals when the integration is replaced by summation. The drawbacks of numerical methods are the exponential growth of calculations and instability of the algorithm with increased degrees of freedom for the investigated physical system.

The physical non-perturbative methods include, for example, the variational principle, the Hartree–Fock method, a method of approximating Hamiltonians, the density functional theory, and others. In the framework of these methods, the estimate for the ground state energy can be obtained; however, the calculation of successive approximations with required accuracy and the calculus of the spectrum of the system provoke essential difficulties.

This monograph introduces and demonstrates the application of the original non-perturbative method for the description of quantum systems, which overcomes most of the above-mentioned obstacles typical for non-perturbative techniques. This method, named the operator method (OM), was first introduced and published in numerous works by authors in the early 1980s. For the construction of zeroth approximation of the operator method and further successive approximations there is no necessity to solve differential and integral equations because all calculations are reduced to algebraic calculus with matrix elements of the operators. A very prominent feature of the operator method is an ability to obtain zeroth approximation for the eigenfunctions and eigenvalues of the Hamiltonian, which are uniformly suitable in the entire range of physical parameters and quantum numbers of the system. The successive approximations of operator method deliver the converging sequence, which enables the calculation of physical characteristics with any required accuracy.

The book presents both the mathematical fundamentals of the operator method and its practical applications for real quantum systems, including ones with an infinite number of the degrees of freedom. A further generalization of the method is given for the quantum statistics, which implements the additional physical parameter, temperature. In comparison with the description of pure quantum states, the calculation of observables in statistics using the density matrix includes an additional complicated procedure: the summation over all states of physical system. For this purpose, the standard procedure of the operator method is supplemented by a non-perturbative algorithm for the summation over quantum states. This combined technique calculates the thermodynamic characteristics of physical systems in a wide diapason of temperatures and remains effective for the systems with multiple degrees of freedom.

The authors dedicate this book to the memory of Professor Lev Ivanovich Komarov, who was one of the pioneers of OM development and contributed essential research on operator methods. The major part of the studies described in this monograph was conducted by the scientists and alumnus of the Department of Theoretical Physics and Astrophysics of the Belarusian State University (Minsk, Belarus). We are thankful to all of our colleagues for their studies published in the recent decades together with the authors of this book: I.K. Dmitrieva, G.I. Plindov, L.I. Gursky, I.V. Nichipor, T.S. Romanova, A.L. Tolstik, S.I. Fisher,

I. Tsvetkov, P.A. Khomyakov, V.V. Triguk, A.V. Leonov, X.H. Ly, Z.A. Chan, N.T. Vu, A.T. Le, and N.T. Hoang-Do. We also appreciate C. Klein for the help with stylistic corrections.

The target audience for this monograph is researchers, post-graduate students, and students studying theoretical and mathematical physics.

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July 2014

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Non-perturbative Description of Quantum Systems

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2015, XV, 362 p. 63 illus., 43 illus. in color., Softcover

ISBN: 978-3-319-13005-7