

Preface

The aim of this book is an introduction to the mathematical theory of optimal control for a relatively new class of equations: stochastic difference Volterra equations of neutral type. Equations of such type arise as independent objects of research, as mathematical models of systems with discrete time, or as difference analogues of stochastic differential and integral Volterra equations of neutral type [9, 109] by their numerical simulation.

These equations belong to the more general class of hereditary systems (also called systems with aftereffect, or systems with memory, or equations with deviating arguments, or equations with delays, or equations with time lag, and so on) and describe the processes whose behavior depends not only on their present state but also on their past history [9, 40, 42, 65–69, 72, 96, 103–106, 109, 138, 153, 154, 178, 179, 187, 199]. Systems of such type are very popular in researches and are widely used to model processes in physics, mechanics, automatic regulation, economy, finance, biology, ecology, sociology, medicine, etc. (see, e.g., [11, 15, 19, 21–23, 26, 27, 33, 34, 53, 62, 75, 86, 115, 126, 132, 144, 148, 152, 162, 166–169, 177–181, 188, 192, 197, 198]).

The general theory of difference equations, including difference equations with delay is represented in [2, 3, 19, 25, 38, 39, 56, 59, 95, 98, 121, 123, 135, 151, 156, 178, 183]. A lot of works are devoted, in particular, to investigation of integral and difference Volterra equations [4, 5, 10, 17, 18, 32, 41, 49–52, 54, 55, 57, 58, 60, 64, 76, 81, 93, 94, 101, 102, 108, 117, 118, 133, 143, 145, 157, 189, 190, 193, 213], to numerical analysis of systems with continuous time by virtue of appropriate difference analogues [1, 35–39, 73, 83, 123, 129, 136, 137, 194, 195, 199], to description of different mathematical models of systems with discrete time [6, 7, 44–46, 78, 79, 82, 91, 92, 127, 128, 131, 139, 149, 150, 155, 206–209].

A very important and popular (in particular, in the last years) direction in the theory and different applications of systems of such type is the optimal control theory. To problems of existence and construction of optimal control in the sense of the given performance criterion and stabilization problems for deterministic and stochastic systems with continuous or discrete time, both without and with delays,

plenty of works are devoted [8, 9, 12–14, 16, 20, 23, 24, 28–30, 32, 43, 47, 48, 61, 63, 68, 74, 77–80, 82, 84, 85, 87–91, 99, 109, 112–114, 116, 119, 122, 124, 125, 130, 134, 140–142, 146, 147, 149, 150, 155, 158–161, 163–165, 170–176, 185, 186, 191, 196, 200, 201, 203–212]. There are also many works devoted to problems of optimal estimation (see, for instance, [17, 18, 31, 97, 100, 107, 111, 120, 182, 202]).

In this book, consisting of six chapters, all the above-mentioned basic problems of the mathematical theory of optimal control and optimal estimation are extended on stochastic difference Volterra equations of neutral type. It is shown, in particular, that the difference analogues of the solutions of optimal control problems and optimal estimation problems obtained for stochastic integral Volterra equations cannot be optimal solutions of corresponding problems for stochastic difference Volterra equations.

The introductory Chap. 1 presents an origin of stochastic difference Volterra equations of neutral type, a necessary condition for the optimality of a control for abstract optimal control problem and some auxiliary definitions and lemmas.

In Chap. 2 a necessary condition for the control optimality of nonlinear stochastic difference Volterra equation is obtained and via this condition a synthesis of the optimal control for a linear-quadratic problem is constructed. A synthesis of the optimal control means the control constructed as a feedback control. Some demonstrative examples of calculating the optimal control in a final form are shown.

In Chap. 3 the problem of construction of successive approximations to the optimal control of the stochastic quasilinear difference Volterra equations with quadratic performance functional is considered. An algorithm for constructing such approximations is described. It is shown that successive approximations can be considered both as a program control and as a feedback control.

In Chap. 4 the problem of the optimal stabilization for a linear stochastic difference Volterra equation and quadratic performance functional is considered. The optimal control in the sense of a given quadratic performance functional that stabilizes the solution of the considered equation to mean square stable and mean square summable is constructed. For the quasilinear stochastic difference Volterra equation with quadratic performance functional a zeroth approximation to the optimal control is constructed that stabilizes the solution of the considered difference equation to mean square stable and mean square summable.

In Chap. 5 the filtering problem is formulated. More exactly, the problem of constructing the optimal (in the mean square sense) estimate of an arbitrary partially observable Gaussian stochastic process from its observations with delay is considered. It is proved that the desired estimate is defined by a unique solution of the fundamental filtering equation of the Wiener–Hopf type. Qualitative properties of this equation are discussed and several cases where it can be solved analytically are considered. The relationship between the observation error and the magnitude of delay in observations is investigated. It is shown that the fundamental filtering equation describes also the solutions of the forecasting and interpolation problems. In the case where the unobservable process is given by a stochastic difference Volterra equation an analogue of the Kalman–Bucy filter is constructed: the system

of four stochastic difference equations defined the optimal in the mean square sense estimate.

In Chap. 6 two different methods for solution of the optimal control problem for partly observable linear stochastic process with a quadratic performance functional are proposed: the separation method and the method of integral representations. An ε -optimal control of the optimal control problem for a quasilinear stochastic difference Volterra equation and a quadratic performance functional is constructed. A special method is proposed for solution of the optimal control problem for stochastic linear difference equation with unknown parameter and quadratic performance functional. The optimal control in final form is obtained. Numerical calculations illustrate and continue the theoretical investigations.

The bibliography at the end of the book does not pretend to be complete and includes some of the author's publications and publications of his coauthors [31–34, 99, 100, 107–109, 116–120, 171–176, 182] as well as the literature used by the author during his preparation of this book.

The book is mostly based on the author's results and ideas, closely related with the results of other researchers in the theory of difference equations, the theory of hereditary systems, and the optimal control theory [2, 14, 19, 47, 55, 59, 68, 79, 103–106, 112, 130, 183]. It is addressed to experts in the mathematical optimal control theory as well as to a wider audience of professionals and students in pure and applied mathematics.

Taking into account that the possibilities for further improvement and development are endless the author will appreciate receiving useful remarks, comments, and suggestions.

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Optimal Control of Stochastic Difference Volterra
Equations

An Introduction

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