Chapter 2
Mathematical Background

Abstract This chapter provides a mathematical background for understanding of the mathematical tools used in the book: basic definitions of main terms, Markov model basics, and references for wavelets background.

Keywords Markov model · State transition · MTBF · MTTR · 2D DWT

In this chapter, we provide the basic definitions and the mathematical background necessary for understanding the context.

2.1 Definitions of Reliability and Availability

Availability is defined by [11]:

$$A_i = \frac{MTBF}{MTBF + MTTR} \tag{2.1}$$

where *MTTR* is Mean Time To Repair and Mean Time Before Failure (*MTBF*). Availability is often expressed as:

$$A_i = \frac{\mu}{\mu + \lambda} \tag{2.2}$$

where λ is the intensity of failures and μ the intensity of repairs. Intensity of failures can be determined by [11, 33]:

$$\lambda = \frac{1}{MTBF} \tag{2.3}$$

Intensity of repairs is defined with [11]:

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$$\mu = \frac{1}{MTTR} \tag{2.4}$$

2.2 Markov Models

The reliability and availability of a system can be established after an in-depth evaluation of the system. Some aspects meriting attention are:

- configuration of elements in the system,
- modes of operation of the system,
- component failure processes,
- conditions indicating that the system failed, and
- reparability of the system.

If the system is in one of the finite number of states in the observer time instant and if components fail in stochastic manner, the reliability and availability of the system can be established with the Markov theory.

Markov's models are functions or two variables:

- states of the system, $X(t)$, and
- observation time, t.

Both can be discrete or continuous in time. Based on the type of variables, Markov models can have four different forms:

- both variables are of discrete type,
- both variables are of continuous type,
- $X(t)$ is continuous and t discrete, and
- $X(t)$ is discrete and t continuous.

Markov models are called Markov chains if t is discrete. Markov models are called Markov processes if t is continuous. They depend on a set of probabilities, p_{ij}, indicating the transition of the system from state i to state j. A special case in the Markov process, interesting from the reliability and availability point of view, is the Poisson's process, which is, in fact, a model with discrete system's states and continuous time.

Equations, such as those presented below can be obtained from the table of transitions:

$$P_0(t + \Delta t) = (1 - \lambda_{01}\Delta t)P_0(t) \tag{2.5}$$

$$P_1(t + \Delta t) = \lambda_{01}\Delta t P_0(t)(1 - \lambda_{12}\Delta t)P_1(t) \tag{2.6}$$

$$P_2(t+\Delta t) = \lambda_{12}\Delta t P_1(t) + P_2(t) \tag{2.7}$$

If we put limit for $\Delta t \to 0$, we get differential equations, i.e.:

$$\frac{dP_0(t)}{dt} + \lambda P_0(t) = 0 \tag{2.8}$$

$$\frac{dP_1(t)}{dt} + \lambda_{12}P_1(t) - \lambda_{01}P_0(t) = 0 \tag{2.9}$$

$$\frac{dP_2(t)}{dt} - \lambda_{12}P_1(t) = 0 \tag{2.10}$$

By applying Laplace transform, the set of differential equations is transformed into a set of algebraic equations in the s-domain:

$$(s+\lambda_{01})L[P_0(t)] = P_0(0) \tag{2.11}$$

$$-\lambda_{01}L[P_0(t)] + (s+\lambda_{12})L[P_1(t)] = P_1(0) \tag{2.12}$$

$$-\lambda_{12}L[P_1(t)] + sL[P_2(t)] = P_2(0) \tag{2.13}$$

By solving equations and taking the inverse Laplace transform, we reach the solution for the desired probabilities:

$$P_0(t) = e^{-\lambda_{01}t} \tag{2.14}$$

$$P_1(t) = \frac{\lambda_{01}}{\lambda_{12}-\lambda_{01}}\left(e^{-\lambda_{01}t} - e^{-\lambda_{12}t}\right) \tag{2.15}$$

$$P_2(t) = 1 - \frac{\lambda_{12}}{\lambda_{12}-\lambda_{01}}e^{-\lambda_{01}t} + \frac{\lambda_{01}}{\lambda_{12}-\lambda_{01}}e^{-\lambda_{12}t} \tag{2.16}$$

In simplified cases, we can define three states for a two-component system:

- state S_0, in which both components are operating correctly,
- state S_1, in which one of the components is not operating correctly, and
- state S_2, when both components are malfunctioning.

The relationship between these three states can be graphically represented as in Fig. 2.1. Figure 2.1 shows that the intensity of failures leads to the next state and the intensity of repairs to the previous state and values are probabilities for the considered time period.

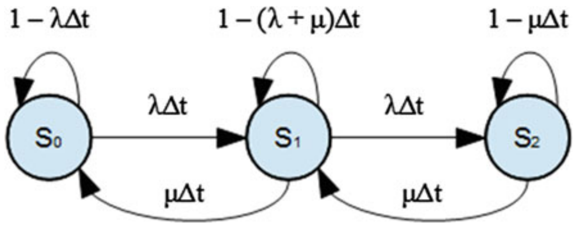

Fig. 2.1 Graphical representation of states' transitions [11]

2.3 Wavelet Role in Image Processing

Wavelets derive their strength from the Heisenberg uncertainty principle [34–36], but there are misunderstandings. Wavelets are, basically, used for non-stationary signal processing in the time-frequency domain [37–40]. The change of frequency and time resolution is possible due to the trade-off within the constraints of the Heisenberg principle. However, there are several ambiguities when dealing with images [41–43]. First, which value to assign to frequency and which to time. Furthermore, since the images are multidimensional we have to know the position of a pixel and one or more color values. When dealing with video, the time when the frame is taken is also of importance [44]. First, the wavelet transform (WT) is defined by mathematical expressions. Since the meaning of a particular parameter in the physical world is irrelevant, any physical parameter can be scaled or translated through WT. The only thing that matters is which physical parameter can be called frequency or time in a particular application.

Two-Dimensional Discrete WT (2D-DWT) takes care of pixel position and color values or intensity (in case of gray image), where the first parameter is location and the second color [34, 39, 41]. So, we localize frequency of color occurrence in an image. This property can be used in different applications, from histogram, edge detection or shadow detection to advanced computer vision algorithms.

When dealing with image sequences, videos, we must bear discrete time in mind. Every time instant corresponds to a single frame or the entire matrix with 2 coordinates of the pixel and values of the pixel (colors or intensity). Such data are commonly saved in multidimensional structures.

Wavelet can be used in preprocessing of images of visual quality control, such as denoising of the input data. Figure 2.2b, c shows an example of denoising of the source image Fig.2.2a. Figure 2.2b shows noised image and Fig. 2.2c denoised.

Another application is in the core of the algorithm, such as in feature extraction. Figure 2.2d, e shows an example of feature extraction—edge detection with Canny edge detector and gradient wavelet method. Figure 2.2d shows results of Canny edge detection for the sourced image.

The third possible application of wavelets is in post-processing, including archiving or image compression. Figure 2.2f, g, h shows compressed image at different levels in multiresolution compression.

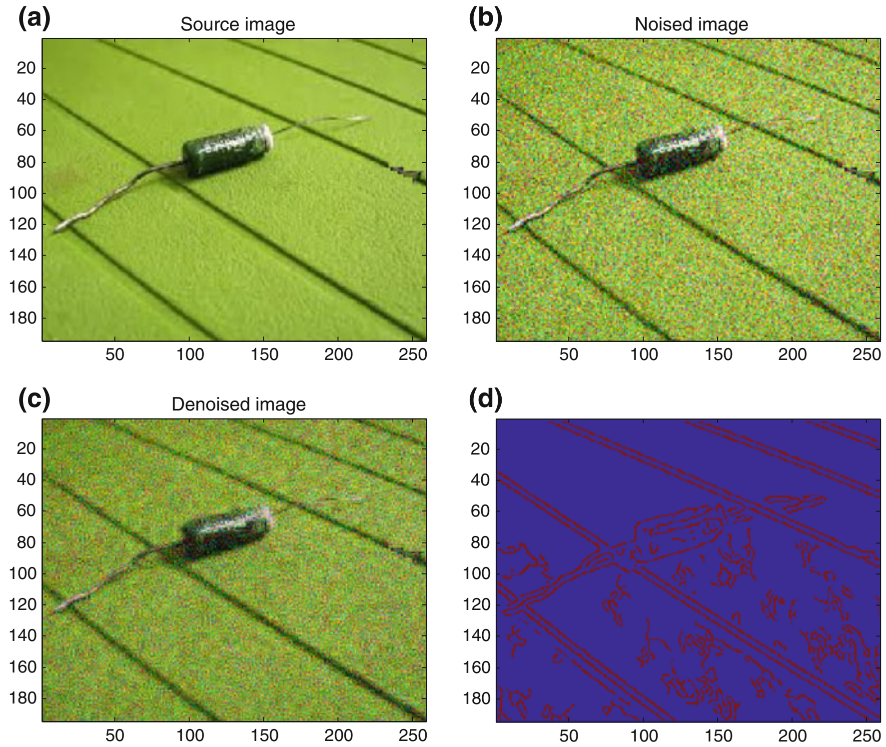

Fig. 2.2 Usage of wavelets: **a** source image, **b** noised source image, **c** denoised image, **d** Canny edge detection, **e** wavelet edge detection, **f** 1st level of the wavelet compression, **g** 2nd level of the wavelet compression, **h** 3rd level of the wavelet compression

Several novel transforms were presented in a past few decades, which poses enhanced characteristics for image processing purposes. Some are better for curves or edges and some for sharpness, compression, noise removal, etc. Some of these transforms are:

- curvelets [45],
- wedgelets [46],
- shapelets [47],
- bandelets [48],
- edgelets [49], etc.

Furthermore, WT is improved by introducing complex numbers [50, 51]. One should always keep in mind what are the benefits of advanced transforms usage, or the ration of gains and losses. For example, some transform can exhibit better characteristics, but the execution can take more time than it can be spared in some application. Therefore, such transform is useless in practice.

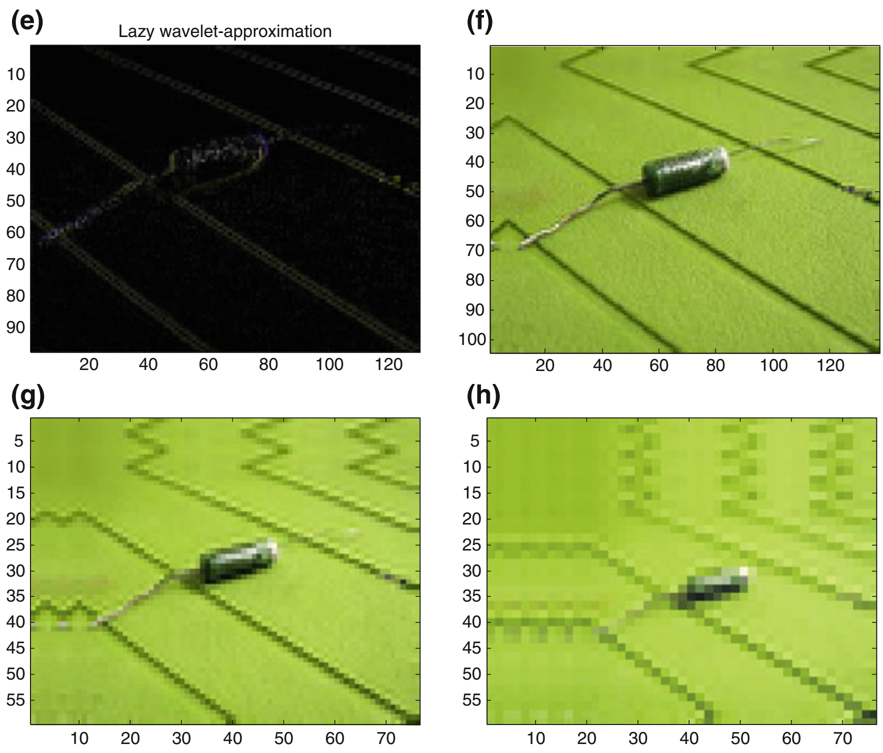

Fig. 2.2 (continued)

For this research, we used classic discrete WT in two dimensions (2D-DWT) with filter implementation, because it was not necessary to use more complex transforms for noise removal.

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