# Preface

In recent years, the field of partial differential equation (PDE)-constrained optimization has received a significant impulse with large research projects being funded by different national and international agencies. A key ingredient for this success is related to the wide applicability that the developed results have (e.g., in crystal growth, fluid flow, or heat phenomena). In return, application problems gave rise to further deep theoretical and numerical developments. In particular, the numerical treatment of such problems has motivated the design of efficient computational methods in order to obtain optimal solutions in a manageable amount of time.

Although some books on optimal control of PDEs have been edited in the past years, they are mainly concentrated on theoretical aspects or on research-oriented results. At the moment, there is a lack of student accessible texts describing the derivation of optimality conditions and the application of numerical optimization techniques for the solution of PDE-constrained optimization problems. This text is devoted to fill that gap.

By presenting numerical optimization methods, their application to PDE-constrained problems, the resulting algorithms and the corresponding MATLAB codes, we aim to contribute to make the field of numerical PDE-constrained optimization accessible to advanced undergraduate students, graduate students, and practitioners.

Moreover, recent results in the emerging field of nonsmooth numerical PDE-constrained optimization are also presented. Up to the author's knowledge, such results are not part of any monograph yet. We provide an overview on the derivation of optimality conditions and on some solution algorithms for problems involving bound constraints, state constraints, sparsity enhancing cost functionals, and variational inequality constraints.

After an introduction and some preliminaries on the theory and approximation of partial differential equations, the theory of PDE-constrained optimization is presented.

Existence of optimal solutions and optimality conditions are addressed. We use a general framework that allows to treat both linear and nonlinear problems. First order optimality conditions are presented by means of both a reduced approach and a Lagrange multiplier methodology. The derivation is also illustrated with several examples, including linear and nonlinear ones. Also sufficient second-order conditions are developed and the application to semilinear problems is explained.

The next part of the book is devoted to numerical optimization methods. Classical methods (descent, Newton, quasi-Newton, sequential quadratic programming (SQP)) are presented in a general Hilbert-space framework and their application to the special structure of PDE-constrained optimization problems explained. Convergence results are presented explicitly for the PDE-constrained optimization structure. The algorithms are carefully described and MATLAB codes, for representative problems, are included.

The box-constrained case is addressed thereafter. This chapter focuses on bound constraints on the design (or control) variables. First- and second-order optimality conditions are derived for this special class of problems and solution techniques are studied. Projection methods are explained on basis of the general optimization algorithms developed in Chap. 4. In addition, the nonsmooth framework of primal-dual and semismooth Newton methods is introduced and developed. Convergence proofs, algorithms, and MATLAB codes are included.

In the last chapter, some representative nonsmooth PDE-constrained optimization problems are addressed. Problems with cost functionals involving the $L^1$-norm, with state constraints, or with variational inequality constraints are considered. Numerical strategies for the solution of such problems are presented together with the corresponding MATLAB codes.

This book is based on lectures given at the Humboldt-University of Berlin, at the University of Hamburg, and at the first *Escuela de Control y Optimización (ECOPT)*, a summer school organized together by the Research Center on Mathematical Modeling (MODEMAT) at EPN Quito and the Research Group on Analysis and Mathematical Modeling Valparaíso (AM2V) at USM Chile.

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