

Chapter 2

A Pedagogical Overview of Relevant Literature

2.1 Introduction

There are many approaches that used various research methods to describe and explain teachers' knowledge for teaching mathematics across a range of grade levels (e.g., Klein and Tirosh 1997; Chapman 2004). However, due to limited empirical data, the categories of teachers' knowledge for teaching mathematics at the elementary level and their relations have not been clearly specified. This review of literature is a critical examination of studies in the area of elementary teachers' knowledge for teaching mathematics; it includes a discussion of the implications and implementations emerging from the review. The literature review is divided into four major sections: sociocultural theory, a history of research on teachers' knowledge for teaching mathematics, studies on elementary teachers' knowledge for teaching mathematics, and an interpretive summary and critical analysis. Section 1 is a discussion of sociocultural theory as the lens for this study. This section includes a review of major conceptual arguments and a sample of empirical studies in which sociocultural theory is the focus. Section 2 presents the historical context of teachers' knowledge for teaching mathematics. To illustrate the influence of the research about mathematical knowledge for teaching on teaching practices, a review of major conceptual studies and examples of empirical studies are examined. Section 3 is a descriptive analysis of literature about elementary teachers' knowledge for teaching. Taken together, these three sections of literature provide a historical and theoretical context for this study. The final section summarizes the findings of the research and locates the research question for this study within the context of relevant literature.

2.2 Sociocultural Theory

Sociocultural perspectives focus on social interactions and cultural contexts that influence the psychological development of students. However, it is not just that the child learns from others in social contexts and during social exchange, but rather that the actual means of social interaction are appropriated by the individual to form the intrapsychological tools for higher-level thinking (Wertsch 1985a).

Vygotsky laid the foundation for sociocultural theory, as he emphasized students' learning through social interactions (Albert 2012). In particular, Vygotsky placed more emphasis on the role of communication as a means of social interaction in student learning for all kinds of subjects (Albert 2012; Chaiklin 1986). Teachers' use of language should guide students' creative and critical thinking and lead them to the next learning level (Vygotsky 1978). In this case, language does not simply mean just oral expressions but rather anything that teachers use for communicating and guiding students to promote the understanding of concepts and skills (e.g., classroom materials or drawing models). The significant role of language is also highlighted in mathematics education. Hiebert et al. (1998) suggested that communication is the key for developing students' mathematical understanding. The National Council of Teachers of Mathematics (NCTM 2000) also identified the significance of communication in the learning of mathematics suggesting, "ideas become objects of reflection, refinement, discussion, and amendment through communication" (p. 60).

For mathematics learning, teachers may use communication to create students' zone of proximal development (ZPD), which is another idea espoused by Vygotsky's sociocultural theory (Albert 2000; Steele 2001). The ZPD is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky 1978, p. 86). Based on the concepts of the ZPD, teachers may assist a student in further learning by providing familiar information to assimilate with students' present knowledge (Bruner 1986). The implication of the ZPD is that students are able to learn new skills that go beyond their actual development level. That is, development follows the student's potential to learn (Vygotsky 1978). In addition, sociocultural theory perceives students' mathematical background as a resource for their learning (Moschkovich 2002). Therefore, a teacher should acknowledge students' mathematical background that includes conceptual and procedural mathematical understanding and build new knowledge based on it.

Within the ZPD, which is created through communication and interaction with others, students should *internalize* the new skills individually (Vygotsky 1997). To demonstrate the process of adopting new skills as *internalization*, Vygotsky argues, "Any function in the child's cultural development appears twice or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category" (p. 163). Vygotsky perceives a separate

but related relationship between external social planes and internal psychological planes. However, external and internal processes are not duplicates of one another (Wertsch 1985b). Internalization transforms the external process into the internal, consequently changing both the structure and functions of the process (Vygotsky 1981). Moreover, social interaction plays a pivotal role in determining the nature of internalization. Internalization involves the concept that children's understanding of others is developmentally rooted in their experience of social interaction (Fernyhough 2008).

By embracing sociocultural theory as a framework for the research described in this book, we operated under several assumptions. It is explicitly viewed that teachers' use of language plays a pivotal role in students' mathematical understanding and their success in learning mathematics. In this case, all forms of communication applied in the classroom can mediate student thinking and understanding of the mathematics content. Therefore, a major purpose of the study is to understand elementary teachers' knowledge for teaching mathematics by analyzing the teachers' communication during the lessons by observing their teaching. In addition, sociocultural theory views the social context as a significant aspect of learning. Thus, the teachers' classroom environment is the social context, and their language is key in constructing the social context that may influence students' internalization of mathematical concepts and skills. Consequently, this study considered how teachers create various social contexts for students to learn mathematics concepts. Teachers' pedagogical intentions for developing learning contexts will be investigated through observations, interviews, and analysis of their lesson plans. To examine the learning context influences on students' internalization of mathematical concepts, these data sources are connected to the social contexts that the teachers construct for their students. Taken together, sociocultural theory offers a holistic perspective on the relationship between elementary teachers' knowledge for teaching mathematics and teachers' use of language or communicative process for supporting students' development of mathematical understanding.

2.3 History of Research on Teachers' Knowledge for Teaching Mathematics

An important aspect for understanding the importance of sociocultural theory relation to this study is to consider the history of teachers' knowledge for teaching mathematics. First, this section examines the historical context concerning the research on teachers' knowledge for teaching mathematics. Second, this discussion includes an exploration of the categories of teachers' knowledge for teaching mathematics. Also, it is essential to briefly examine reform in South Korea mathematics education because it serves as the larger sociocultural context for the present study.

2.3.1 *Research on Teachers' Knowledge for Teaching Mathematics*

The arrival of the common school in the 1830s in the USA initiated a process of simplifying a wide variety of educational settings and generated a demand for highly qualified teachers (Labaree 2008). *The Equality of Educational Opportunity Study* (EEOS) in 1966, conducted in response to provisions of the Civil Rights Act of 1964, marked a watershed moment for studies about teacher quality (Lagemann 2000). This report, also known as the Coleman Study, was about educational equality in the USA and had more than 650,000 students in the sample (Borman and Dowling 2010). According to this report, teachers were the most significant factors in determining educational outcomes (Lagemann 2000). After the EEOS was released, studies based on observation of classroom processes and reliable measures of student achievement began to appear and to increase in the late 1960s and 1970s (Needels 1988). Shulman (1986) named this stream the process–product paradigm. This paradigm has enhanced our understanding of teaching greatly and provoked debate about school effects (Hanushek 1998).

In mathematics education, the emphasis on teacher quality arose with a focus on the process–product paradigm. In particular, *A Nation at Risk* (The National Committee of Excellence in Education 1983), which revealed the steady decline in student academic achievement scores, highlighted, “not enough of the academically able students are being attracted to teaching; that teacher preparation programs need substantial improvement” (p. 20). In a similar manner, *The Underachieving Curriculum* (McKnight et al. 1987), which analyzed US performance on the Second International Mathematics Study (SIMS), emphasized that “professional development programs for mathematics teachers must be improved” (p. 115). The claims underscored in both of these documents were supported by research findings; studies found that there are positive influences on what teachers know regarding how they teach (Darling Hammond 2000). That is, one of the most significant factors that influence teachers’ quality is their knowledge for teaching.

Considering this perspective, the NCTM published the *Curriculum and Evaluation Standards for School Mathematics* in 1989 and *Professional Standards for Teaching Mathematics* in 1991 in response to *A Nation at Risk* and *The Underachieving Curriculum*. The professional standards for teaching mathematics highlight mathematics teachers’ knowledge by “present[ing] a vision of what teaching should entail to support the changes in curriculum set out in the *Curriculum and Evaluation Standards*. This document spells out what teachers need to know to teach toward new goals for mathematics education and how teaching should be evaluated for the purpose of improvement” (NCTM 1991, p. vii).

In 2000, NCTM published *Principles and Standards for School Mathematics*. This document updated the *Curriculum and Evaluation Standards for School Mathematics* and includes *Professional Standards for Teaching Mathematics* and

Assessment Standards for Teaching Mathematics. According to these *standards*, teachers are required to be well prepared to teach mathematics (NCTM 2000). The *standards* provide outlines for teachers' knowledge for teaching mathematics as well as general goals for pre K–12 mathematics education. Specifically, the *standards* describe a set of principles that define what teachers should know in order to teach mathematics; teachers are required to know and understand mathematics, students as learners, and pedagogical strategies as well as how to challenge and support the classroom learning environment. In addition, the *standards* propose, “teachers need to know and use mathematics for teaching that combines mathematical knowledge and pedagogical knowledge...they must continue to learn new or additional mathematics content and study how students learn mathematics” (NCTM 2000, p. 370). This vision emphasizes the significance of teachers' knowledge for teaching and suggests teachers begin to view themselves as lifelong learners (Graham 2001).

During this same time, the No Child Left Behind (NCLB) Act was passed. It required schools' and teachers' accountability for student academic achievement in reading/language arts and mathematics. Schools were required to present *adequate yearly progress* (AYP) in state standardized test scores. Therefore, a major effect of NCLB is linking student mathematics achievement to state standards to identify school failing (Dee et al. 2011). For this reason, policy makers and national organizations focused more on how teachers might help promote students' high academic achievement (US Department of Education et al. 2003).

The Common Core State Standards (CCSS) released in 2010 was developed to narrow the discrepancy among content guidelines between states in the area of English language arts and mathematics. The CCSS is poised to be widely adopted and to become entrenched in state education policy (Porter et al. 2012). The CCSS intended to influence the assessment and implementation of the curricula. At the same time, the CCSS (2010) also proposes that mathematics teachers are required to know both procedural skills and conceptual understanding to make sure students are learning and understanding the critical information they need to succeed at higher levels. That is, teachers need to develop not just a deeper knowledge of mathematics content but also an understanding of the mathematical process of inquiry and problem solving to enrich their teaching practices and to encourage critical thinking skills development in their students (Albert 2012).

As noted above, diverse studies reveal that teacher quality in education ultimately determines the success or failure of school education and this is the reason every document focusing on standards mentions teachers' knowledge or understanding of mathematics in the new standards. The significance of teachers' mathematics knowledge for teaching has been emphasized along with the importance of teacher quality in education. Mathematics teachers' knowledge of what constitutes good mathematics instruction poses a great influence on the type of mathematics pedagogy teachers will deliver in their own classrooms (Hill 2004).

2.3.2 *Historical Context of Research on Teachers' Knowledge for Teaching Mathematics in South Korea*

The emphasis on mathematics teachers' quality is not unique to the US education system. In addition, teachers' effort to improve student mathematical achievement has become a global concern, including in South Korea (Park 2010). The South Korean government's laws have led the reform movement on teachers' quality in South Korea rather than research or standards. Thus, this section concentrates on major reforms and laws regarding teachers' quality in order to provide a broader understanding of the social context of South Korea. In particular, this section focuses primarily on elementary education, and not secondary education, because the purpose of this study is elementary school teachers' knowledge for teaching mathematics. The examination of national concerns about South Korean teachers' quality may provide insights into how to apply the findings regarding improving the quality of elementary mathematics teachers in South Korea.

South Korea was an absolute monarchy until Korea was annexed by Japan in 1910. After Japan invaded Korea, Japan established a modern education system in Korea based on colonial education. At that time, modern schools were founded, and the education system was controlled by Japan. However, this period cannot be regarded as the beginning of modern education in Korea. Although the surface of the education system was modernized, the inner side of the education system was still despotic. After its liberation from the Japanese in 1945, the South Korean government required 6 years of compulsory elementary schooling according to *The Education Law*, which was enacted in 1949. However, the efforts to establish a modernized education system had not succeeded in South Korea until 1954, due to the outbreak of the 1950–1953 Korean War. In 1954, the South Korean government announced a *6-year plan for accomplishing compulsory schooling*, including mathematics education, which aimed to increase school attendance of students and to secure the infrastructure for the education system. After the plan was enacted, the educational demand to teach children rapidly increased in South Korea, which inevitably increased the need for more teachers. In the 1950s, the South Korean government administrated 24 *provisional elementary teacher training schools*, which provided a 2-month elementary teacher certification program. With the certification, elementary teachers were eligible to teach all kinds of subjects in public schools.

In the 1960s, after balancing the supply with demand for elementary teachers, the South Korean government became interested in the quality of the elementary teachers in terms of their knowledge. The government believed that teachers' knowledge for teaching was a significant factor in the quality of instruction and concluded that a 2-month training program for teachers was insufficient (The Ministry of Education 2013). Based on the Special Act on Education in 1961, the government established 16 specialized universities, which provided a 2-year elementary school teacher certification program. The specialized universities offered curriculum that focused on knowledge elementary teachers should have in order to teach at the

elementary level, including educational theories, teaching methods for each subject, and student development.

However, the demands set for high-quality elementary teachers have been part of an ongoing process. Thus, the government expanded the years of the elementary teacher education program from 2 to 4 years according to *The Revised Education law* in 1981. Preservice teachers must earn 140 credits in order to acquire elementary teacher certification from one of the specialized universities, including five credits related to elementary mathematics education (Seoul National University of Education 2012). Over time, based on the decreasing demand for teachers, the numbers of specialized universities preparing elementary teachers decreased from 16 to 11, the number of current universities today.

The effort of the government to prepare highly qualified elementary teachers who have in-depth knowledge for teaching also affected the teacher recruitment system in South Korea. *The Comprehensive Plan for Elementary and Secondary Teachers*, which was suggested by the *Advisory Committee of Educational Policy* in 1988, included policies for improving the teacher certification program and recruitment system. Until 1990, preservice teachers who had an elementary teacher certification could work in a public school without taking a national exam. However, the government revised the education law regarding the teacher recruitment system in 1990; to become an elementary teacher in a public school, preservice teachers who acquired their teacher certification from a specialized university must also pass the *national examination*.

In 1995, the government turned its attention from the quality of preservice teacher education programs to in-service teacher education programs by announcing the *Educational Reform Plan*, which included plans for improving in-service teachers' knowledge for teaching. The plan called for reinforcing in-service teacher education programs and supporting professional development that allowed in-service teachers to conduct educational research. For example, in-service teachers may conduct their own research during the school year as individuals or as a group, and the government provides financial support based on the research plan. The government provided in-service teacher education professional development programs, and elementary teachers in South Korea were encouraged to participate in professional development activities or programs for at least 60 h per year. The government's support for elementary teachers to research educational phenomena and new teaching methods also was critical to improving teachers' knowledge for teaching. Therefore, there are different ways for elementary teachers to participate in educational research in South Korea (e.g., research schools, teacher research teams). In addition, the government allowed the specialized universities to offer master's degree programs, which focused on each subject in elementary education such as *elementary mathematics education master's program* in order to improve teacher professionalism based on the 1996 plan. The master's program was designed for in-service teachers to become specialists in each subject. Starting in 2013, the specialized universities began to offer doctorate programs, which also focused on each subject taught in elementary education.

Although the South Korean government did not enact specific laws or regulations regarding the quality of elementary mathematics teachers, the government regulated general teachers' quality including their ability to provide excellent mathematics instruction. In addition, despite the lack of studies regarding teachers' knowledge for teaching mathematics in South Korea, the South Korean government provides pre- and in-service elementary level programs in order to improve teachers' knowledge for teaching.

In both countries (the USA and South Korea), high expectations are being placed on teachers' knowledge. Thus, the findings of this study may provide meaningful implications to both countries. In particular, each country has its own strengths and weaknesses in terms of teachers' knowledge. Although the USA has advanced many research studies on how to improve teachers' knowledge for teaching mathematics, the South Korean government might have a more practical system for improving teachers' knowledge, such as specialized degree programs at elementary level and support systems for teachers' research, than does the USA. Thus, studying South Korean elementary teachers' knowledge for teaching mathematics, with the research resources in America, provides an opportunity to inform the preparation and professional development of elementary teachers in both countries.

2.4 Research on Teachers' Knowledge for Teaching Mathematics

In the previous section, this review examined the historical context regarding research on teachers' knowledge for teaching mathematics to understand the significance of this study. To obtain perspective about elementary teachers' knowledge for teaching, this section investigates the major conceptual studies about elementary teachers' knowledge for teaching mathematics. The discussion in this section may help develop an overall understanding of elementary teachers' knowledge for teaching mathematics.

For all the significance of teachers' knowledge for teaching mathematics, there are not many discussions about categories of teachers' knowledge for teaching mathematics. While the characteristics of the general knowledge needed for teaching was regarded, "A body that encompasses both knowledge of general pedagogical principles and skills and knowledge of the subject matter to be taught" traditionally is needed (Grossman and Richert 1988, p. 54). For the past two decades, most studies have focused on teachers' transformation of mathematics concepts or on teachers' decision-making methods used in mathematics classrooms (Ponde and Chapman 2006). However, the purpose of the present study is to identify the categories and their relationships to South Korean elementary teachers' knowledge for teaching mathematics. Therefore, this section focuses on the studies about the categories or domains of teachers' knowledge. To provide a broader understanding of teachers' knowledge, it begins with research in which the focus is not mathematics teachers (e.g., Shulman 1986).

2.4.1 *Shulman's Research on Teachers' Knowledge for Teaching*

In 1986, Shulman provided a framework for teachers' knowledge for teaching, and his notion has remained mostly unchanged despite the increasing number of studies on teachers' knowledge for teaching (Bullough 2001; Kinach 2002; Segall 2004). It has had a huge effect on understanding the categories of teachers' knowledge for teaching mathematics, although Shulman did not specify categories of teacher knowledge for teaching mathematics (e.g., Ball et al. 2008; Rowland et al. 2005). Shulman's key categories of teachers' knowledge for teaching are presented in Box 2.1.

Box 2.1 Shulman's key categories of teachers' knowledge

- General pedagogical knowledge
- Knowledge of learners
- Knowledge of education context
- Knowledge of educational, philosophical, and historical grounds
- Content knowledge
- Curriculum knowledge
- Pedagogical content knowledge

General pedagogical knowledge indicates pedagogy in general regardless of the content knowledge teachers are to be specialized in (Shulman 1986). This category empowers teachers to be aware of the educational system as a whole with a focused comprehension of their students through research in psychology and pedagogy (Richards and Farrell 2005).

Knowledge of learners demonstrates that teachers have a specific understanding of the learners' characteristics and how these characteristics can be used to specialize and adjust instruction (Shulman 1986). According to Rahman et al. (2010), knowledge of learners consists of empirical and cognitive knowledge; empirical or social knowledge is knowledge of children of a particular age range, and cognitive knowledge demonstrates understanding of child development. Possessing knowledge of learners indicates that teachers understand skills and processes for adapting activities and representations to meet the needs of particular learners, including differentiation for diverse abilities. Effective teachers make successful decisions during the instruction based on their understanding of learners (Wiseman et al. 1999).

Knowledge of educational context encompasses teachers' knowledge of how sociocultural and institutional contexts affect learning and teaching, and thus teachers need to know what is appropriate in one's educational system (Shulman 1986). Barnett and Hodson (2000) claim that internal and external sources consist of knowledge of educational context. They argue, "internal sources include reflection on personal experiences of teaching, including feelings about responses of students,

parents and other teachers to one's reaction; external sources include Subject Matter Knowledge, governmental regulation, school policies, and the like" (p. 436).

Knowledge of educational, philosophical, and historical grounds provides the foundation for accumulating knowledge for teaching (Shulman 1986). Shulman offers this example: "he or she [English teacher] should be familiar with the critical literature that applies to particular novels or epics that are under discussion in class. Moreover, the teacher should understand alternative theories of interpretations and criticism, and how these might relate to issues of curriculum and of teaching" (p. 9). In this case, educational, philosophical, and historical grounds offer a method or activity of analysis and clarification of knowledge (Hardie 1942).

Content knowledge includes knowledge of the subject and its organizing structures (Grossman et al. 1989; Wilson et al. 1987). Specifically, content knowledge comprises substantive and syntactic knowledge (Shulman 1986). Substantive knowledge can be characterized as knowledge of facts and concepts and the ways that they are organized, while syntactic knowledge is about the nature of inquiry in the field and the mechanisms through which new knowledge is introduced and accepted in that community (Schwab 1964). In other words, content knowledge encompasses structure knowledge, which indicates the theories, principles, and concepts of a particular discipline (Shulman 1992).

Shulman (1986) defines curricular knowledge as "the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those program, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10). Shulman also suggests two subcategories of curricular knowledge, *lateral curriculum knowledge* and *vertical curriculum knowledge* (p. 10). Lateral knowledge indicates the relationship among curriculum in different subject areas, while vertical knowledge demonstrates familiarity with the topics and issues within the same subject. Shulman points out that curricular knowledge is the base knowledge for *PCK*.

Effective teachers acquire in-depth knowledge of how to represent the subject matter to students (Parker and Heywood 2000). Shulman (1986) named this profound knowledge as *PCK*. *PCK* is teachers' special form of professional understanding, which provides a special amalgam of content and pedagogy (Shulman 1986). However, *PCK* is not limited to useful representations, unifying ideas, clarifying examples and counterexamples, helpful analogies, important relationships, and connections among ideas, although all of these are included (Grouws and Schultz 1996). *PCK* should also contain the knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes, and values and their philosophical and historical bases (Shulman 1987), and knowledge of how to transform content into forms that are adaptive to the variations in ability and background presented by the students based on the components of *PCK* (An et al. 2004).

Shulman's categories of teachers' knowledge broaden the perspective on teachers' knowledge. Specially, *PCK* has had a huge influence on research about teachers' knowledge of their subject matter and the importance of this knowledge for

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