Chapter 2
Theory

This chapter introduces the Standard Model (SM), and a brief overview of proton-proton collisions. The most common unit used throughout this dissertation is the GeV (the giga-electronvolt), which corresponds to the kinetic energy of an electron, accelerated through 10^9 V. Units of momentum and mass can be represented as GeV/c and GeV/c^2 respectively, with c the speed of light. To simplify notation, in this chapter and others, natural units are used, corresponding to $c = 1$. This allows energy, momentum and mass to all be measured in the same units, GeV[1], where 1 GeV approximates to the mass of a single proton or neutron (their masses are actually 0.938 and 0.940 GeV respectively).

2.1 Standard Model

The SM of particle physics is a theory that describes the currently known smallest sub-atomic particles and the interactions between them. In the SM, matter is made up of twelve half-integer spin particles called fermions, detailed in Table 2.1. Fundamental fermions can be classified into one of two categories: leptons or quarks. The fundamental fermions are further divided into three generations, with particle masses generally increasing from one generation to the next[2]. For each generation of lepton, there exists one particle with an electric charge of $-e$[3] and one neutral partner, called a neutrino. For each generation of quark, there exists one particle with an electric charge of $+2/3e$ and another with charge $-1/3e$. Each fermion has a partner with identical mass but inverse charge called an antiparticle. Leptons are

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[1] The advantage of this can be seen when considering Einstein's famous equation $E = mc^2$. A particle of rest mass $m = 1$ GeV$/c^2$ has a rest energy of $E = 1$ GeV.

[2] With the exception of neutrinos, which are known to have very small yet unknown absolute masses.

[3] In particle physics, when dealing with charge, it is common to work in units of fundamental charge, e, which has as value 1.602×10^{-19} C.

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Table 2.1 Fermions of the SM, taken from the particle data group summary tables [5]

Generation	Leptons			Quarks		
		Charge [e]	Mass [GeV]		Charge [e]	Mass [GeV]
First	Electron, e	-1	5.11×10^{-4}	Up, u	$+2/3$	≈ 0.002
	e neutrino, ν_e	0	$< 2 \times 10^{-9}$	Down, d	$-1/3$	≈ 0.005
Second	Muon, μ	-1	0.1057	Charm, c	$+2/3$	1.3
	μ neutrino, ν_μ	0	$< 1.9 \times 10^{-4}$	Strange, s	$-1/3$	0.1
Third	Tau, τ	-1	1.777	Bottom, b	$+2/3$	4.2
	τ neutrino, ν_τ	0	$< 1.8 \times 10^{-2}$	Top, t	$-1/3$	173

Table 2.2 Bosons of the SM, taken from the particle data group summary tables [5]

	Mechanism	Boson	Charge [e]	Mass [GeV]
Spin-1	EM force	Photon, γ	0	0
	Strong force	Gluon, g	0	0
		$W^\pm$	± 1	80.4
	Weak force	Z^0	0	91.2
Spin-0	Higgs	Higgs, H^0	0	126

free to exist in nature by themselves, but quarks cannot. Instead they must combine together with other quarks to form hadrons. There exists two types of hadrons: baryons, comprised of three (anti-)quarks and having integer spin (i.e. qqq or $\bar{q}\bar{q}\bar{q}$), and mesons, comprised of a quark-antiquark pair and having half integer spin (i.e. $q\bar{q}$). The matter in our Universe is made up of first generation fermions, since atoms are made of electrons orbiting around a nucleus. Nuclei are composed of protons and neutrons, both of which are baryons, with the proton containing two u-quarks and one d-quark (uud) and the neutron containing two d-quarks and one u-quark (udd).

The SM includes five integer spin bosons, detailed in Table 2.2. Interactions between particles are mediated by four spin-1 bosons, so called "force-carrying" particles, called gauge bosons. These are: the photon (γ), the gluon (g) and the two weak bosons ($W^\pm$ and Z^0), which mediate the electromagnetic (EM), the strong and the weak force, respectively. Each force in turn has its own physical theory that describes how these bosons mediate the interactions between various particles. They are all built from a theoretical framework called quantum field theory, which treats particles as excited states of an underlying quantized field. One spin-0 boson is also included in the SM: the Higgs. It is responsible for giving mass to other particles via the Higgs mechanism. The underlying theories that describe the behaviour of these bosons are introduced briefly below [6–8].

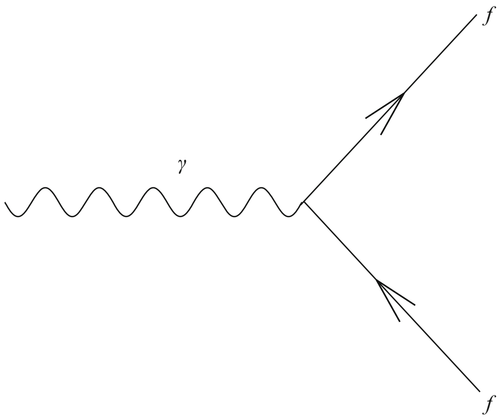

Fig. 2.1 Leading order QED vertex used in Feynman diagrams, where γ is a photon and f is any charged fermion

2.1.1 Quantum Electrodynamics

Quantum Electrodynamics (QED) is the relativistic quantum field theory that describes the interactions between particles that have electric charge. The force, which is mediated by the chargeless photon, has infinite range, but its strength is proportional to r^{-2}.

All interactions between particles are schematically described using Feynman diagrams (described briefly in Sect. 2.1.5) where one builds up particle interactions using the small set of allowable vertices. The leading order vertex for QED is shown in Fig. 2.1. Each vertex has a coupling constant that is proportional to the fine structure constant, $\alpha \approx 1/137$. The term constant is a bit of a misnomer, as it is slightly dependent on the momentum transfer of an interaction. QED is incredibly well understood, and has been extensively tested, with great success.

2.1.2 Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the quantum field theory that describes the strong force, which are interactions involving partons (quarks and gluons). Similar to electric charge, quarks have a "colour charge", which can be one of three values (red, green or blue), and antiquarks have an anti-colour equivalent. Unlike the photon, which does not carry electric charge, the gluon has both a colour and an anti-colour. This implies two unique properties of the strong force: quark colours change when interacting with the gluon, and gluons can interact with itself. Leading order vertices of QCD are shown in Fig. 2.2.

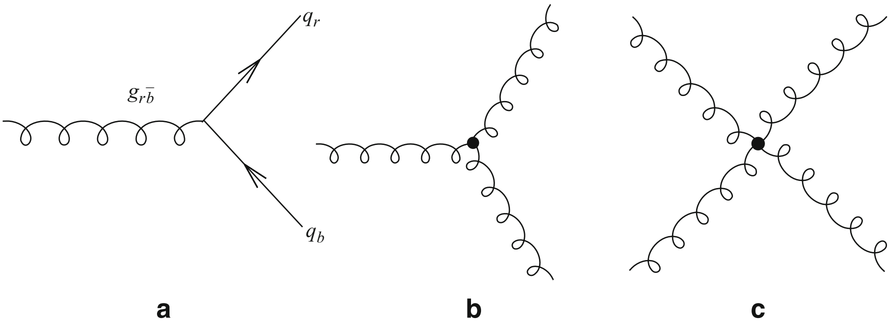

Fig. 2.2 Leading order QCD vertices used in Feynman diagrams: **a** the gluon-quark-quark interaction (note the colour of the gluon, g, is a simplified view of its actual colour), **b** the three-gluon self interaction, and **c** the four-gluon self interaction

Like the photon, gluons are massless, suggesting they can mediate a force of infinite range. However, colour charges are affected by a phenomenon known as confinement, which states that colour charged particles (quarks and gluons) cannot be isolated singularly, and must therefore always be bound in some way in a colourless configuration. These combinations were introduced above: mesons ($q\bar{q}$ where the colour of q is cancelled by the anti-colour of $\bar{q}$) and baryons (qqq where each q has a different colour, thus making it "white"). Due to confinement, when two bound quarks begin to separate, the strong interaction between them (mediated by the gluon) actually becomes stronger. The potential between the two quarks increases until there is sufficient energy to create a quark anti-quark pair. For this reason, the strong force is effectively a short range force.

Another particularity of QCD is that the force between quarks becomes weaker as the distance between them decreases or as the quark energies increase. This is caused by an effective "antiscreening" of colour charge as quark distances decrease. This phenomenon is called asymptotic freedom, and is essential in understanding proton-proton collisions (like those of the Large Hadron Collider (LHC). This also illustrates why protons and neutrons are able to be bound together in such a small space (the nucleus), but not repel each other despite having like electric charge.

As the name suggests, this force is stronger than the EM force, which is most evident when comparing the coupling constant for strong interactions, α_S, to that of EM interactions, α. However, the value of α_s is not constant and is dependent on the energy scale of the interaction, as evidenced by the plot in Fig. 2.3. The strong coupling constant α_s tends to large values at very low energy scales (or inversely, very large length scales), which demonstrates confinement and why quarks cannot exist singularly. Conversely, as energy scales increase (or length scales decrease), α_s decreases, which demonstrates asymptotic freedom and why quarks and gluons within a high energy proton can be treated as free particles.

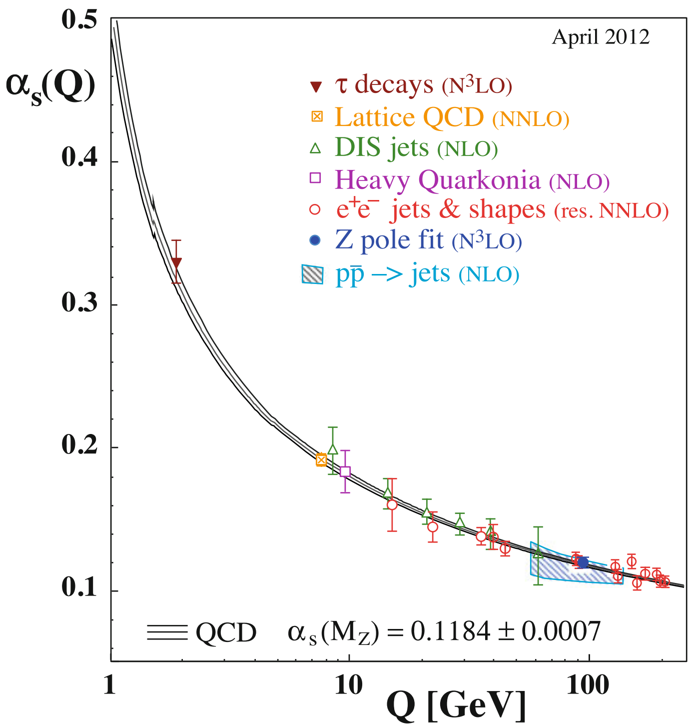

Fig. 2.3 Plot of the strong coupling constant, α_s, as a function of the energy scale, Q. The energy scale can also be considered the inverse length scale: as Q increases, the length scale decreases, and vice versa. Taken from the Particle Data Group reviews [5]

2.1.3 Weak Interaction

The weak interaction is different from both QED and QCD in that the mediating particles, the W and Z bosons, have mass, and the W has either positive or negative electric charge. The weak force is also unique in that quark flavour is not necessarily conserved in an interaction (ie. one quark can change from one flavour to another), and the parity and charge parity symmetries are violated.

At low energy scales, this force is much weaker than the EM and strong forces. The "weak coupling constant" can actually be regarded as greater than that of QED, with $\alpha_W \approx 1/30$; however, the strength of an interaction is suppressed by the massive mediating boson at low energies. As energy scales increase to near the mass of the weak bosons, the weak force becomes comparable to the other forces. All fermions can interact with the weak bosons (including neutrinos) and the W and Z bosons can also interact with itself. Leading order vertices of the weak force are plotted in Fig. 2.4.

The best example of weak interactions in everyday life is that of beta decay. In this form of radioactivity, a neutron (or proton), within a nucleus, decays to a proton

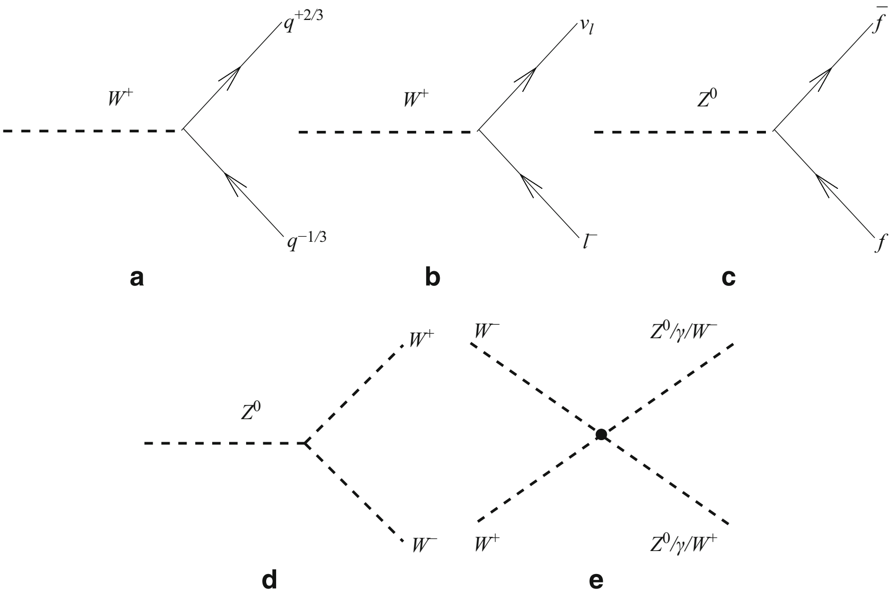

Fig. 2.4 Leading order weak interaction vertices used in Feynman diagrams: **a** $W^{\pm}$–up-type-quark–down-type-quark, **b** $W^{\pm}$–charged-lepton–neutrino, **c** Z^0–fermion–fermion, **d** Z^0–W^+–W^-, and **e** two $W^{\pm}$'s interacting with either two Z^0's, two γ's or two $W^{\pm}$'s

(neutron) and emits both an electron (positron) and an electron antineutrino (electron neutrino). The decay of muons is also mediated by the weak force, which explains its long lifetime.

2.1.4 The Higgs Mechanism

The theories mentioned above describe the fundamental forces of the SM. Since the SM was developed in the 1960s, nearly every prediction has been verified and every theorized particle has been found. However, there is one wrinkle to the SM: the masses of the weak bosons and the fundamental fermions.

In physics, it is generally understood that for each symmetry in nature, there is an associated conservation law. For example, symmetry in time implies a conservation of energy, symmetry in space implies a conservation of momentum, and symmetry in rotation implies a conservation of angular momentum. There exist other symmetries and other conservation laws that are a little more abstract. In QED, the conservation of charge is implied by the symmetry (or invariance) of applying local gauge transformations, which also accounts for the interaction between charged particles and the photon. Similarly, local gauge invariance also accounts for the conservation of colour charge in QCD, and describes the interaction of quarks with gluons.

The weak regime, alone, does not satisfy local gauge invariance. Unlike QED and QCD, which have massless bosons, the weak regime, with its massive bosons, becomes non-unitary within the framework of the SM. This symmetry, however, is restored with the introduction of a scalar field with non-zero vacuum expectation value. As the SM is perturbative in nature, the Lagrangian of the system is transformed to involve expansions about this vacuum expectation value. Although this mechanism does introduce massive gauge bosons, massless "ghost" particles (known as Goldstone particles) also appear. Fortunately, a proper gauge transformation can be chosen, such that the gauge field "eats" up these Goldstone particles. This apparent extra degree of freedom actually accounts for the additional polarization[4] of the massive gauge boson. For a detailed explanation of how a field acquires mass, see Appendix A.

This mechanism is known as the "Higgs mechanism" [9–14], named after the writer of one of the original papers, Peter Higgs. Although the explanation above (along with the example in Appendix A) is a simplified description of the method, the electroweak ($SU(2)_L \times U(1)$) gauge symmetry is restored with the addition of this new scalar field.

The Higgs field can be used to generate masses for fermions as well, using the same mechanism. The coupling of a quark or lepton field with the Higgs field (generally known as a Yukawa interaction), along with an appropriate gauge transformation, causes the fermions to acquire mass, just as the massive bosons did above.

The Higgs field, in turn, has its own boson (the Higgs boson), which couples to the massive particles of the SM, as plotted in Fig. 2.5, with the strength of the coupling directly proportional to the mass of the other particle. As the Higgs boson itself has mass, it also couples to itself.

2.1.5 Interactions of the Standard Model

As mentioned in each of the previous sections, all the particles of the SM interact with other particles in very specific ways. These interactions can be predicted using the fundamental theory of the relevant force. To calculate the predictions at leading order of a certain process of the SM, the most basic combination of vertices (using the ones pictured above) is used to build the given process. Predictions are calculated using the Feynman calculus, a set of rules based off the number and types of particles and vertices. It can be used to calculate particle lifetimes or reaction cross sections. Typically denoted as σ, cross section is a measure of the likelihood of an interaction to occur between two particles. The actual unit of σ is area, and was derived from the classical picture of point-like particles being fired at an area that includes a solid target. The point-like particles will either scatter or not, depending on whether it hits

[4] Massless particles have two polarizations, in the transverse plane. Massive particles, however, have an additional polarization, in the longitudinal direction.

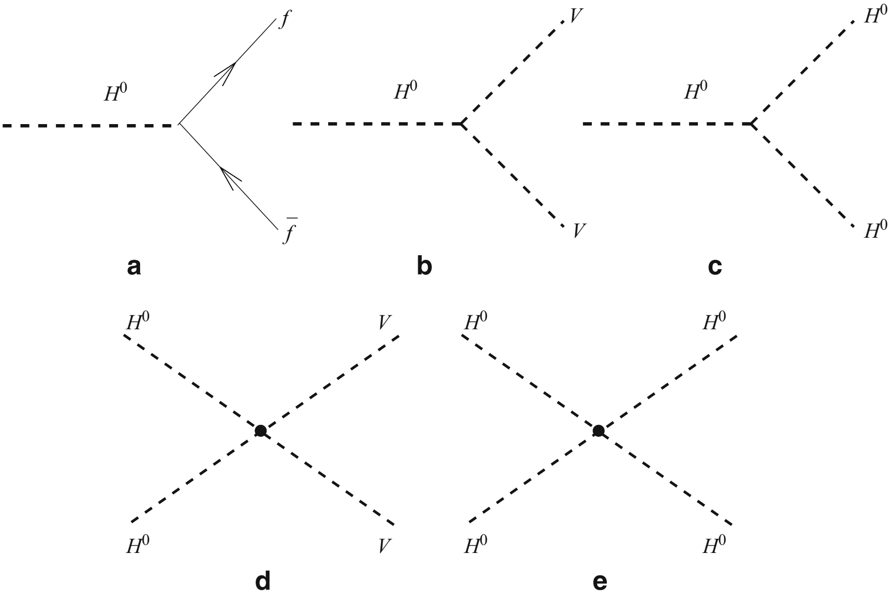

Fig. 2.5 Leading order vertices involving the SM Higgs boson to massive particles: **a** fermions, f, **b** vector bosons, V, **c** Higgs self-coupling, **d** di-Higgs to dibosons, and **e** four Higgs self coupling

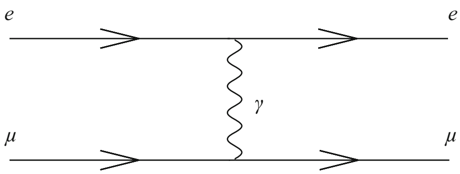

Fig. 2.6 Most basic Feynman diagram of electron-muon scattering

the solid target. The "interaction" probability is the ratio of cross sectional area of the solid target and the total area in which the targets are being fired. In particle physics, the cross sections are so small (on the order of the size of subatomic particles), that the more common unit to use is the "barn", where 1 b $= 10^{-24}$ cm^2.

The use of Feynman diagrams can be demonstrated using electron-muon scattering, pictured in Fig. 2.6. The plot shows the lowest order diagram of electron-muon scattering, as it is drawn using the smallest number of vertices. However, more complex diagrams exist that have the same initial and final particles, but with additional vertices. Examples of these higher order Feynman diagrams are drawn in Fig. 2.7. In theory, each of these additional diagrams contributes to the scattering calculation. However, each additional QED vertex introduces a factor of $\alpha \approx 1/137$, meaning they contribute much less to the overall scattering cross section calculation. Thus, to first-order, Fig. 2.6 alone can be used to predict electron-muon scattering. These

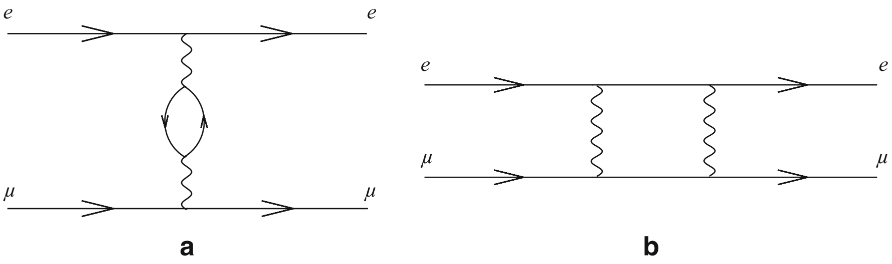

Fig. 2.7 Examples of higher order diagrams that contribute to the electron-muon scattering process

higher order diagrams are often omitted because they add much more complexity to the scattering calculation; however, approximate methods (e.g. perturbation theory) may be used to add these diagrams to the calculation, thus achieving a more precise prediction. The process can be further complicated by the addition of other final state particles (for example the production of an additional photon in electron-muon scattering). These are examples of next-to-leading order diagrams, and, though they marginally contribute to the final cross section, must be considered to derive more precise calculations.

2.2 Proton–Proton Collisions

Most of the particles of the SM have short lifetimes and cannot be found by themselves in nature. This makes the study of these particles rather difficult. In order to more thoroughly understand and study the particles of the SM, it is necessary to produce them artificially. This can be done in one of several ways; however, the most common way is by colliding stable energetic particles together that have accessible energies greater than the rest mass of the particle of interest. The available energy in a collision between two particles is often characterized using one of the Mandelstam variables, s [15]:

$$s \equiv (p_\mu^A + p_\mu^B)^2 = (E_A + E_B)^2 - (\mathbf{p_A} + \mathbf{p_B})^2 \tag{2.1}$$

where p_μ^A is the four-momentum[5] of particle A before collision, E_A is its energy, and $\mathbf{p_A}$ is its momentum vector. The available total energy is maximized in the centre-of-mass frame ($\mathbf{p_A} = -\mathbf{p_B}$), in which case the total centre-of-mass energy is:

$$E_{CM}^{TOT} = \sqrt{s} = E_A + E_B. \tag{2.2}$$

[5] Four-momentum is simply an extension of the classical three-dimensional momentum, $\mathbf{p} = (p_x, p_y, p_z)$, to also contain energy, $p_\mu \equiv (E, \mathbf{p}) = (E, p_x, p_y, p_z)$.

This energy can then be converted into a massive particle of mass up to $\sqrt{s}$. For example at LEP, in Geneva, Switzerland in the late 1980s, by colliding a beam of electrons of 45 GeV with a beam of positrons of same energy (amounting to $\sqrt{s} \approx 90$ GeV), physicists were able to effectively produce and study the Z^0 boson.

Electron-positron collisions may be the simplest to understand as they annihilate with each other. However, any collision of a pair of particles is also capable of producing more massive particles if their centre-of-mass energy, $\sqrt{s}$, is high enough. The LHC collides protons due to their relative ease to obtain, their ability to achieve high rates of collisions, and their minimal energy loss due to radiation when accelerated in a circle (ie. the LHC tunnel), compared to electrons. However, several additional issues do arise when colliding protons. Protons are not fundamental particles, but rather are made up of three valence quarks, that are embedded in a "sea" of quark-antiquark pairs, generated by the gluons exchanged between the quarks of the proton. Fortunately, due to asymptotic freedom, as the proton energy increases, the coupling between the partons within decreases, allowing the partons to basically move freely and independently of each other within the proton. The partons of the proton are thus better described using the variable x, the fraction of the total momentum of the proton. The momentum distribution functions of the parton are then described using Parton Distribution Functions (PDFs). The PDFs are basically a probability density to find a parton carrying a momentum fraction x, at a given energy scale, Q^2 [6].

Each parton type has its own PDF, though their general shape is higher at low x, and dropping down to 0 at high x. However, as Q^2 increases, the valence quarks are no longer dominant, with more quark-antiquark pairs created within the proton. Therefore, as Q^2 increases, the densities increase at low x, since the proton's momentum becomes more spread through the partons of the proton. This is seen clearly in Fig. 2.8, which show Next-to-Leading Order (NLO) PDFs used at the LHC, derived using data from several deep inelastic scattering experiments over the last 20 years. It is also interesting to note that the quarks and antiquarks typically carry about 50 % of the proton's momentum, with gluons carrying the other half. The gluon fraction also tends to increase with Q^2.

PDFs are very important in proton-proton collisions, since they are essential in predicting scattering processes. For example, calculating the cross section for a specific process, $a + b \rightarrow X$, in a proton-proton collision, is accomplished by evaluating the following:

$$\sigma_{ab\rightarrow X} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, Q^2), \tag{2.3}$$

[6] Q^2 by definition is equal to $-t$, where t is another Mandelstam variable, defined as $t \equiv (p^A_{\mu,i} + p^A_{\mu,f})^2 = (E_{A,i} + E_{A,f})^2 - (\mathbf{p_{A,i}} + \mathbf{p_{A,f}})^2$, where the variables are now the initial and final energies and momenta of the same article. This is essentially a measure of momentum transfer in a collision, where a "soft", glancing collision leads to a small Q^2, whereas a "hard", more direct collision leads to a larger Q^2.

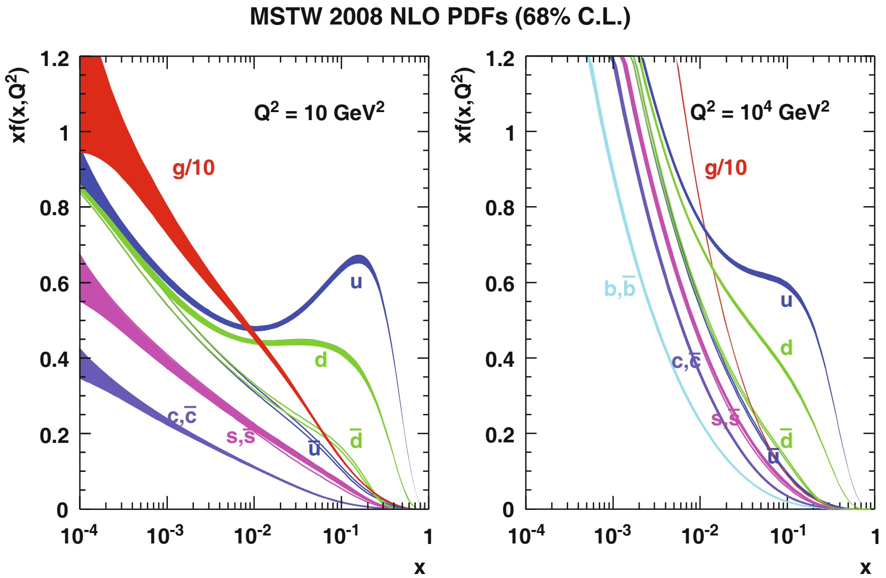

Fig. 2.8 Parton distribution functions of the quarks and gluons within a proton at a lower energy scale ($Q^2 = 10$ GeV^2, *left*) and at a higher energy scale ($Q^2 = 10^4$ GeV^2, *right*) at NLO. The vertical axis is $x \cdot f(x, Q^2)$, the parton fraction, x, times the distribution function. The gluon distribution is scaled by a factor of 1/10 for display purposes. Taken from [16]

where i, j are the possible initial state partons (gluon, up quark, etc.), f_i and f_j are the PDFs for the given parton, P_1 and P_2 are are the initial momenta of the protons, and $\hat{\sigma}_{ij}$ is the cross section for the specific process, derived as a function of initial state momenta and energy scale, Q^2. The cross section is typically derived using the Feynman calculus, as described in Sect. 2.1.5. This method is only valid for hard scattering processes (those at high energy scales, Q^2, or at small length scales), where a perturbative approach can be used to calculate $\hat{\sigma}$. Soft processes (low energy scales or long length scales) are dominated by non-perturbative QCD effects, which are much less well understood [17].

2.2.1 Coordinate System in Hadron Collisions

The coordinate system used in most hadron collisions is a right-handed cylindrical system, where the origin lies at the interaction point, with the z-axis running along the beam line. The plane that is transverse to the beam, or x–y, is very important in hadron collisions. As seen in Eq. 2.3, the cross section for a process is calculated by integrating over all *possible* values of x, as plotted, for example, in Figure 2.8. For any given collision at the LHC, however, the value of x for either colliding parton is unknown. Thus, it is not known, *a priori*, whether the collision occurred at centre-of-mass (ie. with momenta summing to zero) or not. As the protons collide head-on along the z-axis, the initial momenta of the interacting partons in the transverse plane, x–y, are negligible. Thus, for any hard process, the vector sum of the transverse momenta of final state particles must add to zero. Quantities such as transverse momentum:

$$p_T \equiv \sqrt{p_x^2 + p_y^2} \tag{2.4}$$

and transverse energy:

$$E_T \equiv \sqrt{m^2 + p_T^2} \tag{2.5}$$

are often used. It also becomes convenient to use an adapted four-momentum notation:

$$p_\mu = (E, p_x, p_y, p_z) \tag{2.6}$$

$$= (m_T \cosh y, p_T \cos\phi, p_T \sin\phi, m_T \sinh y) \tag{2.7}$$

where ϕ is the azimuthal angle, $m_T = \sqrt{E^2 - p_T^2}$ is the transverse mass, and y is the rapidity:

$$y \equiv \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right). \tag{2.8}$$

The advantage of using these quantities are that p_T, ϕ and differences in y are invariant under longitudinal boosts.

For particles travelling close to the speed of light (or when $|\mathbf{p}| \gg m$), rapidity can be approximated to pseudorapidity, η:

$$\eta \equiv \frac{1}{2} \ln\left(\frac{|\mathbf{p}| + p_z}{|\mathbf{p}| - p_z}\right) = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right], \tag{2.9}$$

where θ is the polar angle in the cylindrical coordinate system. To help visualize this quantity, $\eta = 0$ corresponds to a vector pointing in the transverse plane, whereas $\eta = +(-)\infty$ corresponds to vectors in the positive (negative) beam axis. Another advantage of this quantity is that for minimum bias events [7], the particle multiplicity per unit of rapidity is approximately constant.

A useful quantity to measure angular separation in the detector is ΔR:

$$\Delta R = \sqrt{(\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2}. \tag{2.10}$$

This, along with the other quantities above, are used throughout this thesis.

[7] Minimum bias events are ones that would be collected with a totally inclusive trigger, and would include both diffractive and non-diffractive events. Diffractive events occur when the protons are not, or just barely, broken up; non-diffractive events occur when the protons are broken up and hit the detector. Diffractive events are experimentally difficult to measure, thus most minimum bias events are inelastic non-diffractive collisions.

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