

Preface

Courses in modern theoretical physics have to assume some basic knowledge of the theory of generalized functions (in particular distributions) and of the theory of linear operators in Hilbert spaces. Accordingly, the faculty of physics of the University of Bielefeld offered a compulsory course *Mathematische Methoden der Physik* for students in the second semester of the second year, which now has been given for many years. This course has been offered by the authors over a period of about 10 years. The main goal of this course is to provide basic mathematical knowledge and skills as they are needed for modern courses in quantum mechanics, relativistic quantum field theory, and related areas. The regular repetitions of the course allowed, on the one hand, testing of a number of variations of the material and, on the other hand, the form of the presentation. From this course, the book *Distributionen und Hilbertraumoperatoren. Mathematische Methoden der Physik*. Springer-Verlag Wien, 1993 emerged. The present book is a translated, considerably revised, and extended version of this book. It contains much more than this course since we added many detailed proofs, many examples, and exercises as well as hints linking the mathematical concepts or results to the relevant physical concepts or theories.

This book addresses students of physics who are interested in a conceptually and mathematically clear and precise understanding of physical problems, and it addresses students of mathematics who want to learn about physics as a source and as an area of application of mathematical theories, i.e., all those students with interest in the fascinating interaction between physics and mathematics.

It is assumed that the reader has a solid background in analysis and linear algebra (in Bielefeld this means three semesters of analysis and two of linear algebra). On this basis the book starts in Part A with an introduction to basic linear functional analysis as needed for the Schwartz theory of distributions and continues in Part B with the particularities of Hilbert spaces and the core aspects of the theory of linear operators in Hilbert spaces. Part C develops the basic mathematical foundations for modern computations of the ground state energies and charge densities in atoms and molecules, i.e., basic aspects of the direct methods of the calculus of variations including constrained minimization. A powerful strategy for solving linear and non-linear boundary and eigenvalue problems, which covers the Dirichlet problem and

its nonlinear generalizations, is presented as well. An appendix gives detailed proofs of the fundamental principles and results of functional analysis to the extent they are needed in our context.

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