A Heuristic Approach for Integrated Storage and Shelf-Space Allocation

Nazanin Esmaili, Bryan A. Norman and Jayant Rajgopal

Abstract We address the joint allocation of storage and shelf-space, using an application motivated by the management of inventory items at Outpatient Clinics (OCs). OCs are limited health care facilities that provide patients with convenient outpatient care within their own community, as opposed to having them visit a major hospital. Currently, patients who are prescribed a prosthetics device during their visit to an OC must often wait for it to be delivered to their homes from a central storage facility. An alternative is the use of integrated storage cabinets at the OCs to store commonly prescribed inventory items that could be given to a patient immediately after a clinic visit. We present, and illustrate with an actual example, a heuristic algorithm for selecting the items to be stocked, along with their shelf space allocations. The objective is to maximize total value based on the desirability of stocking the item for immediate dispensing. The heuristic model considers cabinet characteristics, item size and quantity, and minimum and maximum inventory requirements in order to arrive at the best mix of items and their configuration within the cabinet.

Keywords Shelf space allocation · Heuristics · Two-dimensional packing · Healthcare applications

1 Introduction

This paper addresses a combined packing and shelf-space allocation problem. The problem of packing a set of squares or rectangles into a larger square or rectangle has been widely studied. It has been shown that these problems are strongly

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NP-complete (Li and Cheng 1990; Leung et al. 1990), and therefore numerous heuristics have been proposed. General surveys on heuristic method for packing problem can be found in Hopper and Turton (2001), and Ntene and Vuuren (2009). For general surveys on packing problems the reader is referred to Lodi et al. (2002). Similarly, there are many research articles on the shelf space allocation problem to deal with how to optimally allocate shelf space among multiple items so as to maximize profit, minimize inventory costs, or minimize wasted space. For a comprehensive study of shelf space allocation the reader is referred to Yang and Cheng (1999). As noted by Yang (2001) well-managed shelf space can improve the return on inventory investment, raise consumer satisfaction by reducing out-of-stock occurrences, and increase sales and profit margins.

We consider an integrated storage cabinet in the management of prosthetics inventory items at Outpatient Clinics (OCs). OCs are health care facilities that provide patients with a limited suite of services, but the convenience of outpatient care within their own community, obviating the need to visit a relatively distant hospital. Currently, patients who are prescribed a prosthetics device during an OC visit must often wait for it to be delivered to their homes from a central storage facility. An alternative is the use of integrated storage cabinets at the OCs to store commonly prescribed inventory items that could be given to a patient immediately after a clinic visit. The objective is to maximize the total value of items stored in the cabinet, based on the desirability of stocking an item for immediate dispensing and the savings in shipping costs to the patient from the central storage facility.

To our knowledge the specific problem studied has not been considered in the literature and our model and method are new. We consider a restricted version of the combined shelf space allocation and packing problem, where the objective is to pack one or more units of a number of rectangular items into a rectangular cabinet so that the total value of items packed in the cabinet is maximized. We also consider different configurations for each item and propose novel methods to pick the best one for each item. Here, an item's value is defined as the benefit from stocking it in the cabinet as opposed to having it shipped from a central facility. Given that the packing problem by itself is NP-hard in a strong sense, our problem is hard to solve optimally. We therefore develop a new heuristic algorithm to solve it. The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 first introduces three different methods to remove dominated configurations and then presents a heuristic algorithm to solve the problem being modeled. Section 4 illustrates the heuristic with an actual example and examines its performance. Finally, Sect. 5 provides some concluding remarks.

2 Model Description

Integrated storage cabinets come in a wide range of configuration options including open or closed shelving, secure cabinets with several drawers, and specialty storage options. We consider the most general configuration, which is a rectangular form with dimensions H, W and D for the height, width, and depth respectively. The cabinet has a series of shelves arranged vertically, each of which could be of a different height from the set (h, $2h$, $3h$, …). Thus the maximum number of possible shelves within an integrated storage cabinet is given by $J = \left\lfloor \frac{H}{h} \right\rfloor$. Along each shelf are a series of lanes of equal height but possibly varying width that run the depth of the cabinet. Consequently, the maximum number of lanes possible along one shelf is given by $I = \left\lfloor \frac{W}{w} \right\rfloor$, where w is the minimum width of each possible lane. The horizontal coordinate of the bottom left corner of a possible lane is indexed by $i = 0, \ldots, I - 1$, while its vertical coordinate is indexed by $j = 0, \ldots, J - 1$. Note that the each unit on the vertical coordinate is equal to one "shelf height" unit h, and each unit on the horizontal coordinate is equal to one "shelf width" unit w; all of the subsequent discussion will use these units for the height and width of items. Only one item type can be stored in a lane, and units of an item cannot be stacked on top of each other.

Given a set of K items, each of which is a rectangular solid of varying dimensions, the packing and space allocation problem is to decide on the number of each item to pack into a cabinet, along with its configuration in the lane(s) it is assigned, so that the total value across all items in the cabinet is maximized. Each item k can be placed on a shelf in any of up to six configurations (two for each pair of facial dimensions). For each $k = 1, \ldots, K$, we are given its minimum (L_k) and maximum (U_k) number required, along with its storage value V_k. Corresponding to configuration c, item k is characterized by height H_k^c, width W_k^c, and depth $D_k^c, k = 1, \ldots, K$ and $c = 1, \ldots, 6$. The minimum required height of a lane for item k is given by $h_k^c = \left\lceil \frac{H_k^c}{h} \right\rceil, h_k^c \leq J$, the minimum required width of a lane for item k is given by $w_k^c = \left\lceil \frac{W_k^c}{w} \right\rceil, w_k^c \leq I$, and the maximum number of item k in one lane is given by $n_k^c = \left\lfloor \frac{D}{D_k^c} \right\rfloor$. Then the maximum number of lanes possible for item k is given by $u_k^c = \left\lceil \frac{U_k}{n_k^c} \right\rceil$, the minimum number of lanes required for item k is given by $l_k^c = \left\lceil \frac{L_k}{n_k^c} \right\rceil$, and the value of having a full lane of item k in the cabinet using configuration c is given by $v_k^c = V_k \times n_k^c$. We assume that a lane dedicated to item k will be filled with $n_k^{c^*}$ units, where $c^* \in (1, \ldots, 6)$ is the final configuration chosen for item k; thus $v_k^{c^*} = V_k \times n_k^{c^*}$. Let us define an integer decision variable x_k as the number of lanes in the cabinet that are to be dedicated to item k. Then the objective of the model is to maximize $\sum_{k=1}^{K} v_k^{c^*} x_k$.

3 Description of Heuristic

We first describe three different methods that reduce the number of configurations the algorithm considers, by eliminating similar or dominated configurations for each item. Then, we propose a heuristic algorithm to solve the problem.

3.1 Methods to Remove Dominated Configurations

Once an item's configuration has been selected, any remaining shelf space above and behind that item is unusable by other items. Consequently, one might as well fill up any extra space in the lane with as much as possible of the same item. Note that alternative configurations of the same item may differ in the amount of space wasted. Therefore, we first evaluate the local space efficiency of a configuration using the following methods to remove inefficient or dominated configurations.

The first method removes similar configurations for each item. The maximum number of possible configurations of an item is 6. But if two dimensions of an item have the same value then the number of possible configurations decreases to 3. If all three dimensions are identical, then there is only one unique configuration.

The second method separately removes dominated configurations for each dimension. We divide the three actual dimensions of an item by the minimum possible shelf height, h and then round these up to determine the minimum required height of a lane for that item with the different configurations. If these three values are all different then we don't have any dominated configurations for the height, but if we have a tie, then the one with highest original dimension dominates others as it causes less wasted space. Thus, any dominated dimension need not be considered as a height in any configuration for that item. We use the same procedure to determine dominated configurations for width with the only difference that instead of h we divide all dimensions by w. We use similar logic to find dominated configurations for depth by dividing the depth of the cabinet by the dimensions of the item, but now rounding down instead of up.

The third method eliminates those configurations that are multiples of some other configuration. Suppose that for some item k, (1) the minimum required height of a lane for configurations c and c' are identical (i.e., $h_k^c = h_k^{c'}$), and (2) the minimum required width of a lane, the maximum number in one lane, and the value of a lane of item k for configuration c are all an integer multiple (say m) of configuration c' (i.e., $\frac{w_k^c}{w_k^{c'}} = \frac{v_k^c}{v_k^{c'}} = m$). Then configuration c is equivalent to m parallel lanes of configuration c'. We may therefore eliminate configuration c.

3.2 Algorithm

The algorithm consists of five steps. First, a preprocessing step checks for feasibility. Second, the configuration selection step chooses the best configuration for each item. Third, the priority step builds two different sets of priority indices. Fourth, the allocation step allocates available space to items one by one according to their priority indices. This step is divided into two sub-steps which respectively ensure that the lower and upper bounds for the amount of item k are not violated. Finally, in the fifth step the objective value of the final solution is calculated. Let

s denote the most recently created new shelf, R_s the index set of items on shelf s, and. Start with $s = 1$.

1. *Preprocessing step*: This step checks for problem feasibility by ensuring that the total available space in a cabinet is not less than minimum amount of space required. In particular, $\sum_{k=1}^{K} \min_{c}\{l_k^c\left(w_k^c h_k^c\right)\} > IJ$, then the problem is definitely infeasible, so stop. Otherwise, there may still be a feasible solution so go to the configuration selection step.
2. *Configuration selection step*: In this step, we select the best configuration (c^*) to use for each item in the rest of the algorithm. To do this we execute the next three steps until all items are configured.
   2.1. Consider the ratio $\frac{(H_k^c W_k^c D_k^c)\left\lfloor \frac{D}{D_k^c} \right\rfloor}{(hh_k^c)(ww_k^c)D}$ for each configuration of each item. This ratio gives us the fraction of the available space actually being used in one lane of item k with configuration c. If there is a unique maximum, we denote the corresponding c by c^*.
   2.2. Use Sect. 3.1 to remove any dominated configurations from the ones remaining for each item. If there is only one remaining configuration for the item denote it by c^*, and go on to the priority step; else, go to step 2.3.
   2.3. We still have more than one possible configuration, so assign c^* to the one with minimum $w_k^c \times h_k^c$. If there is a tie assign c^* to the one with minimum $w_k^c + h_k^c$, and if there is still a tie assign c^* to the configuration with the lower index value for c; then move on to the priority step.
3. *Priority step*: In this step we build two different priority indices.
   3.1. Sort the elements of $\{1, \ldots, K\}$ in descending order of the item heights, break any ties based on widths and then on item index k. Define this new sorted set as S_1. Then go to step 3.2.
   3.2. Sort the elements of $\{1, \ldots, K\}$ in descending order of the quantity $\frac{v_k^{c^*}}{w_k^{c^*} h_k^{c^*}}$, and define this new sorted set as S_2, then go to the allocation step.
4. *Allocation step*:
   4.1. For successive $k \in S_1$, allocate the available space in the cabinet from lower left of the bottom shelf to meet the minimum requirement ($l_k^{c^*}$) of item k using a first-fit decreasing height heuristic (FFDH):
      4.1.1. Is S_1 empty? If yes go to step 4.2; else select the next $k \in S_1$ and go to step 4.1.2.
      4.1.2. Is $x_k^{c^*}$ less than its lower bound $l_k^{c^*}$? If yes go to step 4.1.3; else remove item k from set S_1 and go to step 4.1.1.
      4.1.3. Create a lane for item k (left justified) on the first partially filled shelf where it fits. If no shelf can accommodate item k then go to step 4.1.4; else, increment $x_k^{c^*}$ by 1 and go to the step 4.1.2.

4.1.4. Is there any space left to create new shelf for a lane of k? If yes, create new shelf ($s := s + 1$) and create a lane for k (left justified) on it, then increment $x_k^{c^*}$ by 1 and go to step 4.1.2; else stop, the problem is not solvable with this algorithm.

4.2. Create additional lanes (up to $u_k^{c^*}$ of them) for each k if possible: proceeding from the bottom shelf up:

4.2.1. Identify the first available location (left justified) for a new lane. If this corresponds to an existing (partially filled) shelf (s') go to step 4.2.3; otherwise go to step 4.2.2.

4.2.2. This step creates a new shelf if possible:

4.2.2.1. Is S_2 empty? If yes go to step 5; else select the next $k \in S_2$ and check if $x_k^{c^*}$ is less than its upper bound $u_k^{c^*}$. If yes go to step 4.2.2.2; else remove k from set S_2 and repeat this step

4.2.2.2. Is there any space left to create a new shelf for k? If yes, create a new shelf $(s := s + 1)$ and assign the first lane on the shelf to item k, increment $x_k^{c^*}$ by 1 and go to step 4.2.1; else remove k from S_2 and go to step 4.2.2.1.

4.2.3. This step creates a new lane on existing shelf s' if possible:

4.2.3.1. Sort the current elements of S_2 in descending order of the quantity $\frac{v_k^{c^*}}{\sum_{k' \in R_{s'}} (h_{s'} - h_{k'}^{c^*}) w_{k'}^{c^*} + h_{s'} w_k^{c^*}}$, where $h_{s'} = \max_{k' \in R_{s'}} h_{k'}^{c^*}$ is the height of shelf s'. Define this new ordered set as S_3 and go to step 4.2.3.2.

4.2.3.2. Is S_3 empty? If yes go to step 4.2.1; else select the next $k \in S_3$ and check if $x_k^{c^*}$ is less than its upper bound $u_k^{c^*}$? If yes go to step 4.2.3.3; else remove item k from set S_2, S_3 and repeat this step

4.2.3.3. Try to create a lane for item k (left justified) in the available space on shelf s'. If it fits then increment $x_k^{c^*}$ by 1 and go to the step 4.2.1; else, remove k from S_3 and go to step 4.2.3.2

5. *Termination step*: Calculate the objective function $\sum_{k=1}^{K} v_k^{c^*} x_k^{c^*}$

4 Numerical Example

We now solve an actual example from a clinical setting with the proposed heuristic algorithm. We are given a cabinet with height 74 in., width 22 in. and depth 22 in. The height of each possible shelf and the width for each possible lane are 4.66 and 1.00 in. respectively. Therefore the cabinet can have up to 15 shelves, and up to 22 lanes per shelf (each with depth 22 in.). Table 1 provides the item characteristics for the items that we want to allocate in this cabinet.

Table 1 Items parameters

Item name	No.	$H_k^{c^*}$	$W_k^{c^*}$	$D_k^{c^*}$	V_k	$L_k^{c^*}$	$U_k^{c^*}$	$h_k^{c^*}$	$w_k^{c^*}$	$v_k^{c^*}$	$l_k^{c^*}$	$u_k^{c^*}$
Accucheck	1	3.75	6.50	5.50	11	10	30	1	7	44	3	8
BP cuff	2	8.25	4.00	5.50	8	10	20	2	4	32	3	5
Mask	3	7.00	6.50	2.00	2	10	20	2	7	22	1	2
Nebulizer	4	9.25	8.00	8.00	20	5	10	2	8	40	3	5
Thermopress	5	10.00	6.00	2.00	15	7	20	3	6	165	1	2
Heating pad	6	12.00	8.00	3.00	10	7	20	3	8	70	1	3
Humidifier	7	13.00	11.50	8.00	25	1	2	3	12	50	1	1

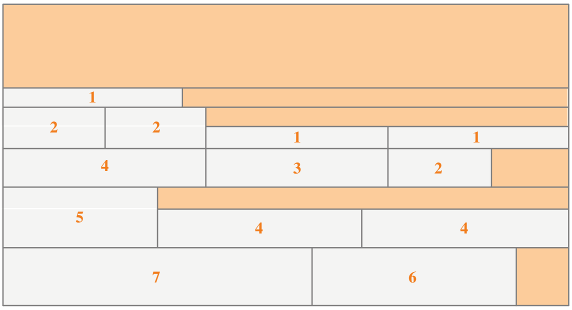

Fig. 1 Example configuration after step 4.1

Solving this example with the proposed algorithm yields an objective value of 1180. Figure 1 shows the algorithm at the end of step 4.1 when the minimum allocations have been completed and Fig. 2 shows the final allocation.

Since we do not know the optimal solution we propose an upper bound for empirically evaluating the heuristic solution. This bound, z_{up} is obtained by solving the following knapsack problem considering the overall volume of the cabinet $\max\{\sum_{k=1}^{K} v_k^{c^*} x_k^{c^*} : \sum_{k=1}^{K} h_k^{c^*} w_k^{c^*} x_k^{c^*} \leq IJ, l_k^{c^*} \leq x_k^{c^*} \leq u_k^{c^*}\ \forall k, x_k^{c^*} \in \mathbb{Z}_+\}$. For our problem, $z_{up} = 1292$; therefore, it follows that the optimum value lies in the interval $[1180, 1292]$. The ratio $1292/1180 = 1.09$ indicates that the best we could possibly do is about 9 % better than our heuristic solution. In fact, it is likely that the

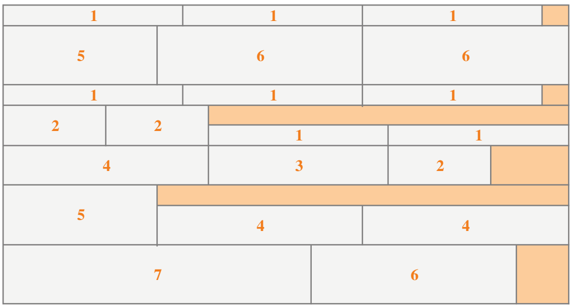

Fig. 2 Example final configuration

improvement that we could get with the optimal solution is actually much smaller since the upper bound is approximate and not necessarily a very tight one.

5 Concluding Remarks

This paper proposes a restricted version of the combined shelf space allocation and packing problem for integrated storage cabinets. Using an FFDH approach, consideration of different configurations and a new prioritization scheme, a heuristic algorithm is proposed to solve this problem. We also introduce different methods to choose the best configuration for each item. We empirically evaluate the heuristic solution by proposing an upper bound. This bound is obtained by considering the overall volume of the cabinet and solving a knapsack problem. The algorithm is illustrated with an actual example from a clinical setting and the solution is shown to be a very good one. We are currently in the process of evaluating the algorithm on a wider range of problems and on developing an integer programming formulation to find the optimal solution for this class of problems.

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