Chapter 2
Observability Property of AC Machines

Abstract In many cases the implementation of control algorithms requires the knowledge of all the components of the state vector. However, because of the high cost of sensors, the reduction of the physical space inside or around the motor, the weight, or the increase of the system complexity, it is often necessary to limit the number of sensors. A similar situation arises when a sensor breaks down. A solution to avoid these difficulties is to eliminate the sensors by replacing them with soft sensors, which are well known as observers in control theory. The soft sensor can also be used to increase the reliability by redundancy with respect to hardware sensors. However, before designing an observer, it is necessary to verify if the system satisfies the observability property. Several techniques and tools have been developed to study whether a nonlinear system is observable or not. Generally, the observability property of a nonlinear system can depend on the inputs. An analysis of the inputs applied to the system is then required to verify if there exist some input that renders the system unobservable. It is clear that in this case the observer may not work correctly. Usually, these inputs are used to control the system, so they are necessary. It is possible to deal with this problem by introducing a class of inputs for which it is conceivable to construct an observer. These inputs are called persistent inputs: inputs with a sufficient quantity of information, so that the observability property is retained. Regarding AC machines, an intrinsic characteristic is that the observability property of the machines is, in most cases, lost at low speed. This phenomenon limits the implementation or degrades the performance of the control algorithms. Then, from the mathematical model of AC machines, a study of the observability property has to be made. If this property is satisfied from the only available measurements, i.e., currents and voltages, the next step is to check if a nonlinear observer can be designed to estimate the nonmeasurable variables, in order to be able to implement the control algorithms.

2.1 Observability Property of AC Machines

The purpose of this chapter is first to introduce definitions and concepts about the observability theory and observer normal forms for nonlinear systems, and then to apply these concepts to AC machines. More precisely, for the PMSM it will be shown

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that if the angle position and/or rotor speed are not measurable, it is necessary to check under which conditions the machine is observable. Similarly, it will be shown that for IM, since the rotor flux is not easily measurable and if the rotor speed measurement is not available, then the observability of the machine is affected. This information can be used to know if it is possible to reconstruct the nonmeasurable components of the state.

A way to reconstruct the state of the system is the use of an *observer*. An observer is a mathematical algorithm (often called *soft sensor*) which is able to reconstruct the state of the system from the limited information obtained from the measured output and the input.

In this chapter, the observation problem of nonlinear systems is presented. Contrary to the linear systems, the observability of the nonlinear systems can depend on the applied input. Taking into account this difficulty, definitions and concepts to determine if a nonlinear system is observable will be introduced.

It is well known that if a linear system is observable, it is possible to design an observer to reconstruct the nonmeasurable state. However, for nonlinear system, even if the system is observable, it is not obvious how to design an observer. To overcome this difficulty some solutions have been proposed. For example, there is a class of nonlinear systems that, by means of a diffeomorphism, can be transformed into a linear system plus an input–output injection, for which it is possible to design an observer, called the nonlinear Luenberger observer.

On the other hand, there is another class of nonlinear systems, such that after a transformation of coordinates, can be represented into a nonlinear system for which the observability property is preserved for any input. For this class of systems several results have been proposed how to design an observer.

By contrast, there is a class of nonlinear systems where the observability depends on the input, i.e., there are inputs rendering the system unobservable. However, for such a class of systems the observability property can be preserved provided the input is persistent [3]. In this case, the observer design is possible for such a class of nonlinear systems, working in the presence of inputs that render the system unobservable.

Taking into account the above, note that there is no normal (canonical) observability form for general nonlinear systems, for which it is possible to construct an observer.

The purpose of this chapter is to analyze the observability property of the PMSM and the IM, and to establish the conditions to reconstruct the nonmeasurable state of these machines.

More precisely, first, an analysis of the observability property for nonlinear systems is presented. After that, since the observability of the system depends on the input, definitions on the different classes of inputs will be introduced. Finally, several structures have been introduced for which it is possible to design an observer.

2.2 Observability

First of all, what is observability? The answer to this question is: *Observability is the possibility to reconstruct the full trajectory of the system from the data obtained from the input and output measurements.*

2.2.1 Observability of Linear Systems

The observability theory of linear systems is well known. The main result is the observability of the linear systems only requires the output measurements and thus does not depend on the input applied to the system. The methodology to verify this property is based on the Kalman criteria of observability. This criteria is verified from the structural representation of the linear system.

The observability of a linear system can be established as follows:

A time invariant linear system is represented by

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{2.1}$$

where $x(t) \in \Re^n$ represents the state, $u(t) \in \Re^m$ is the input and $y(t) \in \Re^p$ is the output; and A, B, and C are matrices of compatible dimensions. System (2.1) is observable, if and only if the observability matrix $\mathcal{O}_{A,C}$

$$\mathcal{O}_{A,C} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{(n-1)} \end{bmatrix}$$

has full rank, i.e., $rank\mathcal{O}_{A,C} = n$, where n is the dimension of the system.

Notice that this condition is independent of the input applied to the system. Furthermore, this result can be extended to the Linear Time-Variant systems.

2.2.2 Observability of Nonlinear Systems

In this section, the observability property of a nonlinear system will be investigated.

Furthermore, since the observability of a nonlinear system can be lost, tools to verify under what conditions a nonlinear system are observable will be introduced.

The observability analysis of a nonlinear system can be divided into two main cases when:

(1) the observability property of the system is independent of the input,
(2) the observability property depends on the input.

For the class of systems where the observability property does not depend on the input, we can find some normal (canonical) forms for which it is possible to design an observer. This class of nonlinear systems, which can be transformed into such a canonical form is called the *u uniformly observable systems class*.

However, if the observability property can be lost when an input is applied to the system, the observer design becomes more difficult and it is necessary to take into account this class of inputs.

On the other hand, several methodologies have been proposed to estimate the state of nonlinear systems. A classical approximate methodology to design an observer is to apply linear techniques to estimate the system state. The first step is the approximate linearization of the nonlinear system around an equilibrium point. The resulting linearized system can be used to design an observer. Of course, this observer can only be efficient around the equilibrium point. Another way to construct an observer is based on the algorithm called the Extended Kalman Filter.

The Extended Kalman Filter is widely used, because its design is relatively simple and this observer gives good results for the nonlinear system observation. However, there is no theoretical justification concerning its effectiveness and no analytic proof of convergence. The observer works in a neighborhood of a particular point, which limits its dynamic performance. Another possibility to design an observer for a nonlinear system is to transform it into another system for which a class of observers is known. For this purpose, several methodologies have been proposed to transform a nonlinear system into particular classes of general nonlinear systems. For example, in [46] for the SISO case and in [75] for the MIMO case, a nonlinear system is transformed into a linear system (or a linear system plus an output injection) for which it is possible to design a linear observer called a *General Luenberger Observer*. When this transformation does not exist, it is possible to search to transform the nonlinear system to a linear time-variant system plus an input–output injection for which an exact Kalman Like Observer can be designed [83].

An Extension of the Kalman Filter (EKF) for the deterministic nonlinear systems is the high gain observer provided that the system can be transformed into a canonical representation, for which the observability property is satisfied for any input.

Before introducing the main results of the observability theory for the nonlinear systems, we introduce some definitions and concepts from the nonlinear control theory [65].

Let q be a point in E^n, a n-dimensional Euclidean space, and U a neighborhood of q.

Let $\varphi(q) = (x_1(q), \ldots, x_n(q)) : U \to V \subset \Re^n$ be a homeomorphism, that is bijective, with φ and φ^{-1} continuous. (U, φ) is called a coordinate neighborhood or coordinate chart and the real numbers $x_1(q), \ldots, x_n(q)$; which vary continuously are local coordinates of $q \in E^n$, $x_i(q)$ is called the ith coordinate function.

If both φ and φ^{-1} are smooth maps, φ is called a diffeomorphism. If both φ and φ^{-1} are defined in $\Re^n$ and are smooth maps, φ is called a global diffeomorphism.

Given two coordinates neighborhoods (U, φ) and (W, ψ), with $U \cap W \neq 0$ where $\varphi(q) = (x_1(q), \ldots, x_n(q))$, and $\psi(q) = (z_1(q), \ldots, z_n(q))$. The homeomorphism

$$\psi \circ \varphi^{-1} : \Re^n \to \Re^n$$

is a coordinate transformation in $U \cap W$, i.e.,

$$z(x) = \psi \circ \varphi^{-1}(x).$$

If x and z are represented by vectors with n components, namely

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \qquad z = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} \tag{2.2}$$

the coordinate transformations are expressed by n real valued continuous functions defined in $\Re^n$, i.e.,

$$x = \begin{bmatrix} x_1(z_1, \ldots, z_n) \\ \vdots \\ x_n(z_1, \ldots, z_n) \end{bmatrix}, \qquad z = \begin{bmatrix} z_1(x_1, \ldots, x_n) \\ \vdots \\ z_n(x_1, \ldots, x_n) \end{bmatrix}. \tag{2.3}$$

A well-known result from calculus which provides a sufficient condition for a map to be a diffeomorphism is given next.

Theorem 2.1 (Inverse Function) *Let U an open subset of $\Re^n$ and let $\varphi = (\varphi_1, \ldots \varphi_n) : U \to \Re^n$ be a smooth map. If the Jacobian matrix*

$$\frac{\partial \varphi}{\partial x} = \begin{bmatrix} \frac{\partial \varphi_1}{\partial x_1} & \cdots & \frac{\partial \varphi_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial \varphi_n}{\partial x_1} & \cdots & \frac{\partial \varphi_n}{\partial x_n} \end{bmatrix} \tag{2.4}$$

is nonsingular at some point $p \in U$, then there exists a neighborhood $V \subset U$ of q such that $\varphi : V \to \varphi(V)$ is a diffeomorphism.

Let $h : U \subset E^n \to \Re$ be a real-valued function defined on U. Depending on the coordinate neighborhoods (U, φ) chosen, the function h is expressed in local coordinates as

$$h_\varphi = h \circ \varphi^{-1} : \Re^n \to \Re.$$

The expression h_φ depends on the chosen local coordinates.

The differential of a smooth function $h : U \subset E^n \to \Re$ is defined in local coordinates as

$$dh = \frac{\partial h}{\partial x_1} dx_1 + \cdots + \frac{\partial h}{\partial x_n} dx_n \tag{2.5}$$

and may be seen as the product of a row vector with the differential column vector of the state

$$dh = \left[\frac{\partial h}{\partial x_1}, \ldots, \frac{\partial h}{\partial x_n} \right] dx\,. \tag{2.6}$$

Consider the following class of nonlinear systems of the form

$$\begin{cases} \dot{x}(t) = \mathbb{F}(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \tag{2.7}$$

where $x(t) \in \Re^n$ represents the state, $u(t) \in \Re^m$ is the input and $y(t) \in \Re^p$ is the output; $\mathbb{F}$ is a smooth vector field and h is $\mathcal{C}^\infty$ function.

Definition 2.1 ([46]) The Lie derivative of the function h_i along the vector field $\mathbb{F}$ is defined as

$$L_{\mathbb{F}} h_i(x) = \frac{\partial h_i}{\partial x} \mathbb{F}.$$

Furthermore, $dL^j_{\mathbb{F}} h_i$, $i = 1, \ldots, p$; $j = 1, \ldots, m$; are the differentials of the Lie derivative of function h_i along the vector field $\mathbb{F}$, denoted as

$$dL^j_{\mathbb{F}} h_i = \frac{\partial L^{j-1}_{\mathbb{F}} h_i}{\partial x} \mathbb{F}.$$

2.2.2.1 Observability and Classes of Inputs

For a complete study of the observability property, we now introduce some definitions on the observability of nonlinear systems [42].

Definition 2.2 (*Indistinguishability*) For system (2.7), two points x and $\overline{x} \in \Re^n$ are indistinguishable if for every applied input

$$u(t), \qquad \forall T > 0$$

the outputs $h(x(t))$ and $h(\overline{x}(t))$ are identical on $[0, T]$, where x and $\overline{x}$ are the trajectories, issues of x and $\overline{x}$ at time $t = 0$.

Note $\mathbb{I}(x_o)$ the set of all points that are indistinguishable from x_0.

Definition 2.3 (*Observability*)

System (2.7) is observable at x_o, if $\mathbb{I}(x_o) = x_o$.
System (2.7) is observable, if $\mathbb{I}(x) = x$ for all $x \in \Re^n$.

Furthermore, for any pair of distinct points $(x, \overline{x})$, there exists an input which distinguishes them on the interval $[0, T]$, for $T > 0$.

Notice that observability is a global concept. A local concept, which is stronger than the observability, will be defined.

Definition 2.4 Let be $x_0 \in \Re^n$ and $V \subset \Re^n$ a neighborhood of x_0. $x_1 \in V$ is said V-indistinguishable of x_0, if x_1 is indistinguishable of x_0.

A weaker result is that which consists to distinguish a point from its neighborhood.

Definition 2.5 (*Weak Observability*) System (2.7) is said weakly observable if $\forall x_0 \in W$, there exists a neighborhood V of x_0 such that $W \subset V$, $\mathbb{I}_W(x_0) = x_0$.

Definition 2.6 The observation space of a system is defined as the smallest real vector space, denoted by $\mathcal{O}(h)$, of C^∞ functions containing the components of h and closed under Lie derivation along the field $\mathbb{F}(x, u)$ for any constant input u.

For linear systems, Definitions 2.3 and 2.5 are equivalent and result in the algebraic criterion known as the Kalman‘s observability criterion recalled before.

Definition 2.7 (*Observability rank condition* [45]) System (2.7) is said to satisfy the observability rank condition in x if

$$dim\{d\mathcal{O}(h)\} = n.$$

Furthermore, if the observability rank condition holds $\forall x \in \Re^n$, then system (2.7) is observable in the rank sense.

Theorem 2.2 *If system* (2.7) *is observable in the rank sense, then it is weakly observable.*

Additional conditions may be used to design an observer for nonlinear systems. For that, we introduce an important class of inputs for which the observability property is satisfied to design an observer independently of the input.

Definition 2.8 An input is universal on the interval $[0, T]$, for $T > 0$, if it distinguishes all pairs of distinct points on the interval $[0, T]$.

Definition 2.9 A system is uniformly observable if every input is universal.

Now, consider the class of multioutput nonlinear systems

$$\begin{cases} \dot{x} = f(x) & x \in \Re^n \\ y = h(x) & y \in \Re^p \end{cases} \tag{2.8}$$

where $h_1, \ldots, h_p$ are smooth functions, $dh_1, \ldots, dh_p$ are linearly independent in $\Re^n$, f is a smooth vector field.

Definition 2.10 A system is locally observable if every state x_o can be distinguished from its neighborhoods by using system trajectories remaining close to x_o.

Theorem 2.3 *System* (2.8) *is locally observable at* x_o *if*

$$rank\{dh_i, \ldots, dL_f^j h_i, i = 1, \ldots, p; \; j \geq 0\} = n \tag{2.9}$$

$\forall x \in U_0 \subset \Re^n$.

Observability indices may be defined for locally observable systems satisfying (2.9).

Definition 2.11 (*Observability Indices* [65]) A set of observability indices $\{k_1, \ldots, k_p\}$ is uniquely associated at x to system (2.8), satisfying (2.9) as follows

$$k_i = card\{\mathcal{S}_j \geq i, j \geq 0\}, i = 1, \ldots, p. \tag{2.10}$$

where

$$\mathcal{S}_0 = rank\{dh_i, i = 1, \ldots, p.\} \tag{2.11}$$

$$\ldots$$

$$\begin{aligned} \mathcal{S}_k = & \; rank\{dh_i, \ldots, dL_f^k h_i, i = 1, \ldots, p.\} \\ & - rank\{dh_i, \ldots, dL_f^{k-1} h_i, i = 1, \ldots, p.\} \end{aligned} \tag{2.12}$$

$$\ldots$$

$$\begin{aligned} \mathcal{S}_{n-1} = & \; rank\{dh_i, \ldots, dL_f^{n-1} h_i, i = 1, \ldots, p.\} \\ & - rank\{dh_i, \ldots, dL_f^{n-2} h_i, i = 1, \ldots, p.\} \end{aligned} \tag{2.13}$$

Then, the observability property can be verified as follows:

Definition 2.12 (*Locally Weakly Observability*) The system (2.8) is locally weakly observable at x^0 if there exists $U(x^0)$, and p integers $\{k_1, \ldots, k_p\}$ that form the smallest p-tuple with respect to the lexicographic ordering, such that

$$\text{(i)} \qquad k_1 \geq k_2 \geq \cdots \geq k_p \geq 0; \tag{2.14}$$

$$\text{(ii)} \qquad \sum_{i=1}^{p} k_i = n; \tag{2.15}$$

$$\text{(iii)} \qquad rank \begin{bmatrix} dh_1 \\ dL_f h_1 \\ \vdots \\ dL_f^{k_1-1} h_1 \\ \vdots \\ dh_p \\ dL_f h_p \\ \vdots \\ dL_f^{k_p-1} h_p \end{bmatrix} = n \tag{2.16}$$

for all $x \in U(x^0)$.

Nonlinear Transformations

Generally, the design of an observer for nonlinear systems is not an easy task. However, it turns out that by means of a change in coordinates (a diffeomorphism), the original nonlinear system can be transformed into another system for which it is easier to design an observer.

Now, some concepts concerning the transformation of a nonlinear system into a special class of system are introduced.

Given r smooth real-valued functions $\{\varphi_1, \ldots, \varphi_r\}$ in U, then

$$rank\{d\varphi_1, \ldots, d\varphi_r\} = r$$

in $q \in U$, is equivalent to

$$rank \begin{bmatrix} \dfrac{\partial \varphi_1}{\partial x_1} & \cdots & \dfrac{\partial \varphi_1}{\partial x_r} \\ \cdots & \cdots & \cdots \\ \dfrac{\partial \varphi_r}{\partial x_1} & \cdots & \dfrac{\partial \varphi_r}{\partial x_r} \end{bmatrix} = r \tag{2.17}$$

for $x = q$.

Theorem 2.4 (Inverse Function Theorem) *If $rank\{d\varphi_1, \ldots, d\varphi_n\} = n$ at some point $q \in U$ an open subset of $\Re^n$, then there exists a neighborhood $V \subset U$ of q such that $\varphi : V \to \varphi(V)$ is a diffeomorphism.*

Definition 2.13 Two systems Σ_1 and Σ_2 are locally diffeomorphic in $x_0 \in \Re^n$, if and only if there exists a diffeomorphism Ψ, defined on a neighborhood of x_0, transforming Σ_1 into Σ_2.

Theorem 2.5 *There exists a set of functions $\phi_1(x), \ldots, \phi_n(x)$ of observation space $\mathcal{O}(\phi)$ such that $\Psi = (\phi_1(x), \ldots, \phi_n(x))^T$ is a diffeomorphism on $\Re^n$, then system (2.7) is observable.*

Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)u, & x \in \Re^n, \quad u \in \Re \\ y = h(x), & y \in \Re. \end{cases} \tag{2.18}$$

Necessary and sufficient conditions are obtained such that an observable nonlinear system of the form (2.18) can be transformed into a system of the form

$$\begin{cases} \dot{x} = Ax + \phi(y, u) \\ y = Cx \end{cases} \tag{2.19}$$

where the term $\phi(y, u)$ is an output–input injection, (see [46] for the SISO case, and [75] for the MIMO case). For this class of system, an extended Luenberger observer can be designed.

Furthermore, the system can be transformed into another nonlinear system for which it is possible to design an observer, for instance, transformed into the state affine system

$$\begin{cases} \dot{x} = A(u)x + \phi(y, u) \\ y = Cx \end{cases} \tag{2.20}$$

or in the general form

$$\begin{cases} \dot{x} = A(u, y)x + \phi(y, u) \\ y = Cx \end{cases} \tag{2.21}$$

where the matrices $A(u)$ and $A(u, y)$ have particular forms (see [3] for more details).

2.3 Permanent Magnet Synchronous Motor Observability Analysis (PMSM)

One of the most important difficulty to control the synchronous motor is when the speed and the position are not available from measurement. This can affect the observability properties of the machine. Significant improvements have been made in the area of the sensorless control of the permanent magnet synchronous motors. However, to implement such a controller, it is necessary to reconstruct the state of the motor. Then, before designing an observer it is necessary to investigate the observability property of the permanent magnet synchronous motor.

It will be shown by the following observability study that an interesting field of research is related to the high-performance sensorless position control of synchronous machines. It involves zero speed control at a determined rotor position. An additional problem is the observer structure is strongly dependent on machine parameters. The position estimation is generally difficult due to scalar speed estimation. Theoretically, the position can be calculated by integrating the speed, but in practice the result will suffer drift problems and moreover the initial position is not always known. There are three main methods to estimate the position: tracking observer based, tracking state filter, and arctangent calculation based. The position estimation of the arctangent direct calculation has no time delay. However, it suffers from large position estimation error due to the noise. The effect of noise can be mitigated by using a state filter but the estimate has lagging fault.

The first methods used to solve the sensorless position estimation are the approaches using the back Electromotive Force (EMF) with fundamental excitation, and spatial salience image, the tracking methods using excitation in addition. The salience tracking methods are suitable for zero-speed operation, whereas the back EMF-based methods fail at low speed.

To know the variety of different methods for sensorless control, it is very important to understand the dynamics properties of the electric machines.

2.3.1 IPMSM Observability Analysis

To verify if the Internal Permanent Magnet Synchronous Motor (IPMSM) is observable, it is assumed that the magnetic flux is not saturated, the magnetic field is sinusoidal, and the influence of the magnetic hysteresis is negligible on the IPMSM.

Observation Objective: *by using only the measurement of the currents and voltages, to simultaneously reconstruct (online) the rotor speed, position, load torque, and stator resistance value of the IPMSM.*

Now, we show under which conditions the IPMSM is observable. The observability analysis is made in two steps:

- From the stator currents and its first time derivatives, the observability of the speed and the position will be studied in the (α, β) frame.
- Secondly, to analyze the observability of the system including the stator resistance and the load torque, higher time derivatives of the stator current measurements will be taken into account.

  For simplicity, this second step of the observability analysis is made by using the IPMSM equations in the (d, q) frame.

2.3.1.1 Observability Analysis of the Speed ω and the Position θ_e in the (α, β) Frame

In this section, the IPMSM observability properties will be analyzed in open loop, assuming that all the parameters are known.

Consider the IPMSM electric equations, in the stationary (α, β) frame, given as

$$\begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \end{bmatrix} = (\Lambda_{ss})^{-1} \left\{ - \begin{pmatrix} R_s - 2\omega L_{\alpha\beta} & 2\omega L_1 \cos(2\theta_e) \\ 2\omega L_1 \cos(2\theta_e) & R_s + 2\omega L_{\alpha\beta} \end{pmatrix} \begin{pmatrix} i_{s\alpha} \\ i_{s\beta} \end{pmatrix} - p\Omega\Psi_r \begin{pmatrix} -\sin\theta_e \\ \cos\theta_e \end{pmatrix} + \begin{pmatrix} v_{s\alpha} \\ v_{s\beta} \end{pmatrix} \right\} \tag{2.22}$$

where

$$(\Lambda_{ss})^{-1} = \frac{1}{L_d L_q} \begin{pmatrix} L_\beta & -L_{\alpha\beta} \\ -L_{\alpha\beta} & L_\alpha \end{pmatrix}. \tag{2.23}$$

The determinant $Det(\Lambda_{ss})$ is given as

$$Det(\Lambda_{ss}) = L_\alpha L_\beta - \left(L_{\alpha\beta}\right)^2 = L_0^2 - L_1^2 = L_d L_q, \tag{2.24}$$

where

$$L_\alpha = L_0 + L_1 \cos(2\theta_e), L_\beta = L_0 - L_1 \cos(2\theta_e), L_{\alpha\beta} = L_1 \sin(2\theta_e). \tag{2.25}$$

and

$$L_0 = \frac{L_d + L_q}{2}, \quad L_1 = \frac{L_d - L_q}{2}. \tag{2.26}$$

Moreover, the mechanical equations of the IPMSM are

$$\begin{cases} J\frac{d\Omega}{dt} = -f\Omega + 2pL_1 i_{s\alpha} i_{s\beta} + p(\Psi_{r\alpha} i_{s\beta} - \Psi_{r\beta} i_{s\alpha}) - T_l \\ \frac{d\theta_m}{dt} = \Omega. \end{cases} \tag{2.27}$$

Then, the complete model is given as

$$\begin{cases} \begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \end{bmatrix} = (\Lambda_{ss})^{-1} \left\{ - \begin{bmatrix} R_s - 2\omega L_{\alpha\beta} & 2\omega L_1 \cos(2\theta_e) \\ 2\omega L_1 \cos(2\theta_e) & R_s + 2\omega L_{\alpha\beta} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} - p\Omega\Psi_r \begin{bmatrix} -\sin\theta_e \\ \cos\theta_e \end{bmatrix} \right\} + (\Lambda_{ss})^{-1} \begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} \\ \frac{d\Omega}{dt} = -\frac{f}{J}\Omega + \frac{2pL_1}{J} i_{s\alpha} i_{s\beta} + \frac{p}{J}(\Psi_{r\alpha} i_{s\beta} - \Psi_{r\beta} i_{s\alpha}) - \frac{1}{J}T_l \\ \frac{d\theta_m}{dt} = \Omega \end{cases}$$

which is of the general form

$$\begin{cases} \dfrac{dX_{\alpha\beta}}{dt} & = \mathbb{F}(X_{\alpha\beta}, v_{\alpha\beta}) \\ y & = h(X_{\alpha\beta}) \end{cases} \tag{2.28}$$

where

$$X_{\alpha\beta} = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \Omega \\ \theta_m \end{bmatrix}, \quad v_{\alpha\beta} = \begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix}, \quad h(X_{\alpha\beta}) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix},$$

and $X_{\alpha\beta}$ is the state, $v_{\alpha\beta}$ is the input, and $h(X_{\alpha\beta})$ is the measurable output whose components are the stator currents $i_{s\alpha}$ and $i_{s\beta}$.

The observation space $\mathcal{O}_{\alpha\beta}(X_{\alpha\beta})$ containing the components of h_1, h_2; and closed under Lie derivation along the field $\mathbb{F}$, is given by (see [46])

$$\mathcal{O}_{\alpha\beta}(X_{\alpha\beta}) = \{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2\}.$$

Then, the observability analysis of the IPMSM is made by verifying if the matrix

$$d\mathcal{O}_{\alpha\beta}(X_{\alpha\beta}) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \end{bmatrix} \tag{2.29}$$

satisfies the condition of Theorem 2.3, i.e., the rank of $d\mathcal{O}_{\alpha\beta}(X_{\alpha\beta})$ is equal to $n = 4$.

It is equivalent to determine if matrix $d\mathcal{O}_{\alpha\beta}$ is nonsingular, which implies to evaluate the determinant of the matrix $d\mathcal{O}_{\alpha\beta}$ given by:

$$\begin{aligned} Det(d\mathcal{O}_{\alpha\beta}) = {} & \frac{2L_1\Psi_r(L_0 + L_1)}{Det(\Lambda_{ss})^2}(v_{s\alpha}\sin\theta_e - v_{s\beta}\cos\theta_e) \\ & - \frac{2R_sL_1\Psi_r(L_0 + L_1)}{Det(\Lambda_{ss})^2}(i_{s\alpha}\sin\theta_e - i_{s\beta}\cos\theta_e) \\ & + \frac{\Psi_r^2\omega(L_0 + L_1)^2}{Det(\Lambda_{ss})^2} + \frac{4L_1^2L_0}{Det(\Lambda_{ss})^2}(i_{s\beta}v_{s\alpha} - i_{s\alpha}v_{s\beta}) \\ & + \frac{8L_1L_0\Psi_r\omega(L_0 + L_1)(i_{s\alpha}\cos\theta_e + i_{s\beta}\sin\theta_e)}{Det(\Lambda_{ss})^2} \\ & + \frac{4L_1^3 i_{s\beta}}{Det(\Lambda_{ss})^2}(v_{s\alpha}\cos 2\theta_e + v_{s\beta}\sin 2\theta_e) \\ & + \frac{4L_1^3 i_{s\alpha}}{Det(\Lambda_{ss})^2}(v_{s\alpha}\sin 2\theta_e - v_{s\beta}\cos 2\theta_e) \end{aligned}$$

$$+\left[\frac{8L_1^2L_0^2\omega - 4R_sL_1^3\sin 2\theta_e + 8L_1^3L_0\omega\cos 2\theta_e}{Det(\Lambda_{ss})^2}\right](i_{s\alpha}^2 + i_{s\beta}^2)$$
$$+\frac{2L_1^2\Psi_r\omega(L_0 + L_1)}{Det(\Lambda_{ss})^2}(i_{s\alpha}\cos\theta_e - i_{s\beta}\sin\theta_e).$$

Analyzing the expression of $Det(d\mathcal{O}_{\alpha\beta})$, it can be remarked that, for $\omega \neq 0$, $Det(d\mathcal{O}_{\alpha\beta})$ cannot be null. Thus, we can first conclude that the IPMSM is observable if $\omega \neq 0$.

For $\omega = 0$, a complementary study is now developed. Using the following transformation

$$\begin{aligned} v_{sq} &= -v_{s\alpha}sin\theta_e + v_{s\beta}cos\theta_e \\ i_{sq} &= -i_{s\alpha}sin\theta_e + i_{s\beta}cos\theta_e, \end{aligned} \tag{2.30}$$

it is possible to study the observability condition. $Det(d\mathcal{O}_{\alpha\beta})$ can be written at zero speed as

$$Det(P_{\alpha\beta}) = \frac{2L_1\Psi_r(L_0 + L_1)L_q}{Det(\Lambda_{ss})^2}\frac{di_{sq}}{dt} + \left[\frac{4L_1^2}{Det(\Lambda_{ss})^2}(L_1 + L_0)\right](v_{sd}i_{sq})$$
$$-\left[\frac{8L_1^2}{\Psi_r Det(\Lambda_{ss})^2}(L_1 + L_0)\right](v_{sq}i_{sq}^2).$$

Proposition 2.1 *The state of the IPMSM is observable at zero speed ($\Omega = p\omega = 0$) if*

$$L_1[-\frac{4L_1^2}{\Psi_r}v_{sq}i_{sq}^2 + (2L_1v_{sd} - \Psi_rR_s)i_{sq} + \Psi_rv_{sq}] \neq 0, \tag{2.31}$$

or equivalently, if one of the following conditions are not satisfied:

(i) if $v_{sq} = 0$ and $v_{sd} \neq \dfrac{\Psi_rR_s}{2L_1}$, then, $i_{sq} = 0$ and $T_e = 0$.

(ii) if $v_{sq} = 0$ and $v_{sd} = \dfrac{\Psi_rR_s}{2L_1}$.

(iii) if $v_{sq} \neq 0$ and $v_{sd} = \dfrac{\Psi_rR_s}{2L_1}$, then $i_{sq} = \dfrac{\Phi_r}{2L_1}$ and $T_e = \dfrac{p\Psi_r^2}{L_1}$

(iv) if $v_{sq} \neq 0$ and $v_{sd} \neq \dfrac{\Psi_rR_s}{2L_1}$, then

$$i_{sq} = \frac{-(2L_1v_{sd} - \Psi_rR_s) \pm [(2L_1v_{sd} - \Psi_rR_s)^2 + (16L_1^2v_{sq}^2)]^{1/2}}{-8L_1^2v_{sq}/\Psi_r},$$

and $T_e \neq 0$.

Remark 2.1 The four cases can be checked by using the parameters values for a given machine. The condition (i) can only be verified at standstill (the currents and the voltages are zero). This particular case is easily detected by the electrical measurements. The physical meaning of the case (iv) is that a nonzero load torque exists at zero speed.

On the other hand, taking into account that the parameters of the motor given in Sect. 1.6.2, the cases (ii), (iii), and (iv) are unrealistic, i.e., these cases cannot occur in the IPMSM physical operation domain.

2.3.1.2 IPMSM Observability Analysis for the Stator Resistance R_s and the Load Torque T_l in the (d, q) Frame

Next, a sufficient condition for the observability of the IPMSM, including the stator resistance and the load torque, is given. For computational simplicity, we analyze this observability in the (d, q) frame by using higher time derivatives of the measured output.

Consider the extended model of (1.70), where the rotor resistance R_s and the load torque T_l are the components of the extended state vector, and described by

$$\begin{cases} \dfrac{di_{sd}}{dt} &= -\dfrac{R_s}{L_d} i_{sd} - p\Omega \dfrac{L_q}{L_d} i_{sq} + \dfrac{v_q}{L_d} \\ \dfrac{di_{sq}}{dt} &= -\dfrac{R_s}{L_q} i_{sq} + p\Omega \dfrac{L_d}{L_q} i_{sd} + \dfrac{v_d}{L_q} - p\Omega \dfrac{\Psi_r}{L_q} \\ \dfrac{d\Omega}{dt} &= -\dfrac{f}{J}\Omega + \dfrac{1}{J} p(L_d - L_q) i_{sd} i_{sq} + p\Psi_r i_{sq} - \dfrac{1}{J} T_l \\ \dfrac{dT_l}{dt} &= 0 \\ \dfrac{dR_s}{dt} &= 0 \end{cases} \tag{2.32}$$

which is of the general form

$$\begin{cases} \dfrac{dX_{dq}}{dt} &= \mathbb{F}(X_{dq}, v_{dq}) \\ y &= h(X_{dq}) \end{cases} \tag{2.33}$$

where X_{dq} is an extended state vector and $y = h(X_{dq})$ is the measurable output, that are given by

$$X_{dq} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \Omega \\ R_s \\ T_l \end{bmatrix}, \qquad h(X_{dq}) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}.$$

The observation space $\mathcal{O}_{dq}$ defined by the vector space of the functions constituted by the measurements of the stator currents i_{sd} and i_{sq} and closed under Lie derivatives along the field $\mathbb{F}$, is given by $\{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}^{(2)}h_2\}$.

Following the same procedure as before, the observability analysis is made by verifying the condition of Theorems 2.3 and 2.5, i.e., by analyzing the rank of the matrix

$$d\mathcal{O}_{dq}(X_{dq}) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}^{(2)}h_2 \end{bmatrix}.$$

This is equivalent to determine if the determinant

$$Det(d\mathcal{O}_{dq}) = ai_{sq}^6 + bi_{sq}^4 + ci_{sq}^2,$$

is different to zero where

$$a = -\frac{p^2(L_d - L_q)^3}{JL_q^2\phi_f^2}, b = -\frac{p^2(L_d - L_q)}{JL_q\phi_f} \text{ and } c = -\frac{p^2(L_d - L_q)}{JL_q\phi_f} + \frac{p^2\phi_f(L_d - L_q)}{JL_q^2L_d^2}.$$

From $Det(d\mathcal{O}_{dq})$, it is clear that the rank condition is not satisfied when $i_{sq} = 0$. Then, we can establish the following result.

Proposition 2.2 *Consider the IPMSM model (2.32) and assume that the stator currents are measurable. Then, the rotor speed Ω, the stator resistance R_s and the load torque T_l are observable if and only if*

$$i_{sq} \neq 0.$$

Remark 2.2 In this case, the motor does not produce any torque, i.e., it does not play a role with respect to the load.

2.3.2 SPMSM Observability Analysis

Now, the observability property of the SPMSM will be studied.

Consider model (2.28) and remark that the inductances are such that:

$$L_s := L_d = L_q = L_0, \quad \text{and} \quad L_1 = 0.$$

It follows that the model of the SPMSM, in the (α, β) frame, is given by

$$\begin{cases} \dfrac{di_{s\alpha}}{dt} = -\dfrac{R_s}{L_s} i_{s\alpha} + p\Omega\Psi_r \sin(p\theta_m) + \dfrac{1}{L_s} v_{s\alpha} \\ \dfrac{di_{s\beta}}{dt} = -\dfrac{R_s}{L_s} i_{s\beta} - p\Omega\Psi_r \cos(p\theta_m) + \dfrac{1}{L_s} v_{s\beta} \\ \frac{d\Omega}{dt} = -\dfrac{f}{J}\Omega + \dfrac{p}{J}\sqrt{\dfrac{2}{3}}\Psi_r (i_{s\beta} \cos(p\theta - m) - i_{s\alpha} \sin(p\theta_m)) - \dfrac{1}{J} T_l \\ \dfrac{d\theta_m}{dt} = \Omega \end{cases} \tag{2.34}$$

which is of the general form

$$\frac{dX_{\alpha\beta}}{dt} = \mathbb{F}(X_{\alpha\beta}, v_{\alpha\beta}) \tag{2.35}$$

$$y = h(X_{\alpha\beta}) \tag{2.36}$$

where

$$X_{\alpha\beta} = \begin{pmatrix} i_{s\alpha} \\ i_{s\beta} \\ \Omega \\ \theta_m \end{pmatrix}, \quad v_{\alpha\beta} = \begin{pmatrix} v_{s\alpha} \\ v_{s\beta} \end{pmatrix}, \quad h(X_{\alpha\beta}) = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} i_{s\alpha} \\ i_{s\beta} \end{pmatrix},$$

with $X_{\alpha\beta}$ is the state, $v_{\alpha\beta}$ is the stator voltages vector and is the system input; $h(X_{\alpha\beta})$ components are the measurable outputs: the stator currents $i_{s\alpha}$ and $i_{s\beta}$.

2.3.2.1 Observation Objective

Consider that in the (α, β) frame, the stator currents $i_{s\alpha}$ and $i_{s\beta}$ are the measurable outputs, the stator voltages $v_{s\alpha}$ and $v_{s\beta}$ are the control inputs of the motor.

The objective is to reconstruct the rotor speed Ω and the position θ_m assuming that they are not available by measurement and moreover under the fact that the stator-winding resistance R_s and the stator-winding inductance L_s are inaccurately known.

The property of observability of the SPMSM is determined by using first Definition 2.6, where the observation space $\mathcal{O}_1$ is constituted of measured outputs and their Lie derivatives along the vector field $\mathbb{F}$, i.e., $\mathcal{O}_1 = \{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2\}$ and the measured output is

$$h(x) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

From Theorem 2.3, it follows that

$$d\mathcal{O}_1(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \end{bmatrix}. \tag{2.37}$$

Then, by evaluating the determinant of matrix $d\mathcal{O}_1(x)$, we obtain

$$Det(d\mathcal{O}_1) = \frac{\Psi_r^2 \omega}{L_s^2}.$$

Proposition 2.3 *Consider that the magnet flux Ψ_r and the inductance L_0 are different from zero. The SPMSM is observable if and only if its electrical speed is not null, i.e., if $\omega \neq 0$.*

Remark 2.3 Notice that even by using higher order derivatives of the measured outputs to study the observability property, no additional information for the observability analysis is obtained.

2.4 Induction Motor Observability Analysis

The purpose of this section is to analyze the observability of the induction motor in order to reconstruct the nonmeasurable components of the state vector, i.e., the rotor flux, the rotor speed, and also unknown parameters: the load torque and the rotor resistance.

2.4.1 Mathematical Model in the (d, q) Rotor Flux Frame

Consider the mathematical model of the induction motor, in a state-space representation (1.108) and (1.128) written in the (d, q) frame depending on the stator pulsation ω_s, where

$$\phi_{rq} = \dot{\phi}_{rq} = 0. \tag{2.38}$$

From (1.108) and (2.38), the flux angle ρ is given by

$$\dot{\rho} = \omega_s = p\Omega + \frac{aM_{sr}}{\phi_{rd}} i_{sq}. \tag{2.39}$$

Furthermore, the Electromagnetic Torque equation is given by

$$T_e = \frac{pM_{sr}}{L_r} \phi_{rd} i_{sq}. \tag{2.40}$$

Replacing the stator pulsation ω_s and the differential equation of ϕ_{rq}, by those of the flux angle ρ obtained from (2.39) in the nonlinear model of the induction motor (1.108), it follows that

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{\phi}_{rd} \\ \dot{\rho} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} -\gamma i_{sd} + ab\phi_{rd} + p\Omega i_{sq} + a\dfrac{M_{sr}}{\phi_{rd}} i_{sq}^2 \\ -\gamma i_{sq} - bp\Omega\phi_{rd} - p\Omega i_{sd} - a\dfrac{M_{sr}}{\phi_{rd}} i_{sd} i_{sq} \\ -a\phi_{rd} + aM_{sr} i_{sd} \\ p\Omega + a\dfrac{M_{sr}}{\phi_{rd}} i_{sq} \\ m\phi_{rd} i_{sq} - c\Omega - \frac{1}{J} T_l \end{bmatrix} + \begin{bmatrix} m_1 & 0 \\ 0 & m_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}. \tag{2.41}$$

Remark 2.4 From (2.39), the slip pulsation is given by $\omega_r = \omega_s - p\Omega$, where

$$\omega_r = \frac{aM_{sr}}{\phi_{rd}} i_{sq}. \tag{2.42}$$

2.4.2 Introduction to the Sensorless IM Observability

Several works have studied the observability of the induction motor (see [11, 32, 44]). In [32], sufficient conditions under which the induction motor loses the observability property have been presented. This study has been realized using model (1.121). In this subsection, we present a similar study using the model (1.115). To analyze the observability of the induction motor, the criteria of the observability rank will be applied (see [30]).

2.4.3 Induction Motor Observability with Speed Measurement

Consider the induction motor model (1.114). For the analysis of the observability of the induction motor, firstly assume that the rotor speed is measured.

The induction motor model is:

$$\begin{bmatrix} \dfrac{di_{sd}}{dt} \\ \dfrac{di_{sq}}{dt} \\ \dfrac{d\phi_{rd}}{dt} \\ \dfrac{d\phi_{rq}}{dt} \\ \dfrac{d\Omega}{dt} \\ \dfrac{dT_l}{dt} \end{bmatrix} = \begin{bmatrix} -\gamma i_{sd} + \omega_s i_{sq} + ba\phi_{rd} + bp\Omega\phi_{rq} + m_1 v_{sd} \\ -\omega_s i_{sd} - \gamma i_{sq} - bp\Omega\phi_{rd} + ba\phi_{rq} + m_1 v_{sq} \\ aM_{sr} i_{sd} - a\phi_{rd} + (\omega_s - p\Omega)\phi_{rq} \\ aM_{sr} i_{sq} - (\omega_s - p\Omega)\phi_{rd} - a\phi_{rq} \\ m(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) - c\Omega - \dfrac{1}{J} T_l \\ 0 \end{bmatrix} \tag{2.43}$$

where the state, the input and the measurable output are given by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \\ \Omega \\ T_l \end{bmatrix} \in \Re^6, \quad u = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} \in \Re^2,$$

$$h(x) = \begin{bmatrix} h_1 \\ h_2 \\ h_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \Omega \end{bmatrix} \in \Re^3,$$

and the vector field is given by

$$\mathbb{F}(x,u) = \begin{bmatrix} -\gamma x_1 + \omega_s x_2 + bax_3 + bpx_5x_4 + m_1 v_{sd} \\ -\gamma x_2 - \omega_s x_1 + bax_4 - bpx_5x_3 + m_1 v_{sq} \\ -ax_3 + (\omega_s - px_5)x_4 + aM_{sr}x_1 \\ -ax_4 - (\omega_s - px_5)x_3 + aM_{sr}x_2 \\ m(x_3x_2 - x_4x_1) - cx_5 - \frac{1}{J}x_6 \\ 0 \end{bmatrix}.$$

Notice that the rotor speed is considered as an output as well as the stator currents.

Consider the observation space $\mathcal{O}_{IM,0}(x)$ of functions containing the components of h and closed under Lie derivation along the vector field $\mathbb{F}$, i.e., $\mathcal{O}_{IM0}(x) = \{h_1, h_2, h_5, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}h_5\}$.

To verify the observability rank condition, it is sufficient to check that the rank of matrix

$$d\mathcal{O}_{IM,0}(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dh_5 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}h_5 \end{bmatrix}$$

is equal to $n = 6$.

The matrix $d\mathcal{O}_{IM,0}(x)$ characterizing the observability of the system (2.43) in the rank sense, is given by

$$d\mathcal{O}_{IM0}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\gamma & \omega_s & ba & bpx_5 & bpx_4 & 0 \\ -\omega_s & -\gamma & -bpx_5 & ba & -bpx_3 & 0 \\ -mx_4 & mx_3 & mx_2 & mx_1 & -c & -\frac{1}{J} \end{bmatrix}.$$

Next, computing the determinant $d\mathcal{O}_{IM,0}(x)$, it follows that

$$Det(d\mathcal{O}_{IM,0}(x)) = -\frac{b^2}{J}(a^2 + (px_5)^2).$$

Notice that the determinant $Det(d\mathcal{O}_{IM,0}(x))$ is different from zero for any value of the rotor speed. Then, the matrix $d\mathcal{O}_{IM,0}(x)$ is full rank. As a consequence, using the rotor speed and the stator currents measurements, we can conclude that the IM is observable.

Remark 2.5 The determinant $Det(d\mathcal{O}_{IM0}(x))$ is independent of the stator pulsation ω_s. An identical result is obtained in [32] from the model (1.121).

2.4.4 Observability of the Induction Motor: Sensorless Case

In the sequel, the observability study will be determined assuming that the rotor speed Ω is not available from measurement.

In the (mechanical) sensorless case, only the stator currents are measured. From (1.115), the model of the IM, for the sensorless case, is

$$\begin{cases} \dot{x} = \mathbb{F}(x, u) \\ y = h(x) \end{cases} \tag{2.44}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \\ \Omega \\ T_l \end{bmatrix} \in \Re^6, \quad u = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} \in \Re^2,$$

$$h(x) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} \in \Re^2$$

and the vector field $\mathbb{F}$ is given by

$$\mathbb{F}(x, u) = \begin{bmatrix} -\gamma x_1 + \omega_s x_2 + bax_3 + bpx_5 x_4 + m_1 v_{sd} \\ -\gamma x_2 - \omega_s x_1 + bax_4 - bpx_5 x_3 + m_1 v_{sq} \\ -ax_3 + (\omega_s - px_5)x_4 + aM_{sr}x_1 \\ -ax_4 - (\omega_s - px_5)x_3 + aM_{sr}x_2 \\ m(x_3 x_2 - x_4 x_1) - cx_5 - \frac{1}{J}x_6 \\ 0 \end{bmatrix}.$$

From Definition 2.6, the observation space $\mathcal{O}_{IM,1}(x)$ constituted by the components of the output and closed under Lie derivation is given by:

$$\{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}^2h_1, L_{\mathbb{F}}^2h_2\}.$$

Then, from Theorems 2.3 and 2.5, and to verify the observability rank condition, it can be checked that the matrix $d\mathcal{O}_{IM,1}(x)$

$$d\mathcal{O}_{IM,1}(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}^2h_1 \\ dL_{\mathbb{F}}^2h_2 \end{bmatrix}.$$

satisfies the observability rank condition if the determinant of

$$d\mathcal{O}_{IM,1}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\gamma & \omega_s & ba & bpx_5 & bpx_4 & 0 \\ -\omega_s & -\gamma & -bpx_5 & ba & -bpx_3 & 0 \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{bmatrix}$$

is different from zero, where

$$\begin{aligned}
a_1 &= \gamma^2 - bM_{sr}a^2 - bpmx_4^2 - \omega_s^2 \\
a_2 &= bpmx_3x_4 + bpaM_{sr}x_5 + \dot{\omega}_s - 2\gamma\omega_s \\
a_3 &= -ba^2 + bpmx_2x_4 + bp^2x_5^2 - \gamma ba - 2bpx_5\omega_s \\
a_4 &= -2bapx_5 + bp(mx_2x_3 - mx_4x_1 - cx_5 - \frac{x_6}{J}) - \gamma bpx_5 - bpmx_4x_1 - 2ba\omega_s \\
a_5 &= -bapx_4 - bpcx_4 + bp(-ax_4 + px_5x_3 + aM_{sr}x_2) \\
&\quad - \gamma bpx_4 + bp^2x_5x_3 - 2bpx_3\omega_s, \\
a_6 &= -\frac{bp}{J}x_4
\end{aligned}$$

and

$$\begin{aligned}
b_1 &= bpmx_3x_4 - bpaM_{sr}x_5 - \dot{\omega}_s + 2\gamma\omega_s \\
b_2 &= \gamma^2 + bM_{sr}a^2 - bpmx_3^2 - \omega_s^2 \\
b_3 &= 2bapx_5 - bp(mx_2x_3 - mx_4x_1 - cx_5 - \frac{x_6}{J}) + \gamma bpx_5 - bpmx_2x_3 - 2ba\omega_s \\
b_4 &= -ba^2 + bpmx_1x_3 + bp^2x_5^2 - \gamma ba - 2bpx_5\omega_s
\end{aligned}$$

$$
\begin{aligned}
b_5 &= bapx_3 + bpcx_3 - bp(-ax_3 - px_5x_3 + aM_{sr}x_1) \\
&\quad - \gamma bpx_3 + bp^2x_5x_3 - 2bpx_4\omega_s, \\
b_6 &= \frac{bp}{J}x_3.
\end{aligned}
$$

More precisely, the determinant of matrix $d\mathcal{O}_{IM,1}(x)$ is given by

$$
\begin{aligned}
Det(d\mathcal{O}_{IM,1}(x)) = -\frac{b^3p^2}{J}[&-(px_5x_3 + ax_4)(x_3a_3 + x_4b_3) \\
&+ (ax_3 - px_4x_5)(x_3a_4 + x_4b_4) + (\frac{a}{p^2} - px_5^2)(x_3a_5 + x_4b_5)].
\end{aligned}
$$

From the complexity of a_3, a_4, a_5, b_3, b_4 and b_5, the determinant of the matrix $d\mathcal{O}_{IM,1}(x)$ is difficult to directly analyze.

Consequently, to study the observability of the induction motor without mechanical speed sensor, the following subcases will be analyzed.

(i) Case 1: $\dot{\Omega} = 0$, i.e., constant rotor speed.
(ii) Case 2: $\omega_s = 0$.
(iii) Case 3: $\dot{\phi}_{rd} = \dot{\phi}_{rq} = \omega_s = 0$.
(iv) Case 4: $\dot{\phi}_{rd} = \dot{\phi}_{rq} = \omega_s = 0$ and $\dot{\Omega} = 0$.

2.4.4.1 Case 1: $\dot{\Omega} = 0$, i.e., Constant Rotor Speed

Consider the case where the IM rotor speed is constant, then the resulting model (1.114) is simplified:

$$
\begin{cases}
\dot{x} = \mathbb{F}(x, u) \\
y = h(x)
\end{cases}
\tag{2.45}
$$

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \\ \Omega \end{bmatrix} \in \Re^5, \quad u = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}, \quad h(x) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}
$$

and the vector field is given by

$$
\mathbb{F}(x, u) = \begin{bmatrix} bax_3 + bpx_5x_4 - \gamma x_1 + \omega_s x_2 + m_1 v_{sd} \\ bax_4 - bpx_5x_3 - \gamma x_2 - \omega_s x_1 + m_1 v_{sq} \\ -ax_3 + (\omega_s - px_5)x_4 + aM_{sr}x_1 \\ -ax_4 - (\omega_s - px_5)x_3 + aM_{sr}x_2 \\ 0 \end{bmatrix}.
$$

Consider the following two observation spaces $\mathcal{O}_{IMS,1,\dot{\Omega}}(x)$ and $\mathcal{O}_{IMS,2,\dot{\Omega}}(x)$ of functions containing the components of h and closed under Lie derivation given by $\mathcal{O}_{IMS,1,\dot{\Omega}}(x) = \{h_1, L_{\mathbb{F}}h_1, L^2_{\mathbb{F}}h_1, h_2, L_{\mathbb{F}}h_2\}$ and $\mathcal{O}_{IMS,2,\dot{\Omega}}(x) = \{h_1, L_f h_1, h_2, L_{\mathbb{F}}h_2, L^2_{\mathbb{F}}h_2\}$, respectively.

From Theorems 2.3 and 2.5, and to verify the observability rank condition, the matrices $d\mathcal{O}_{IMS,1,\dot{\Omega}=0}(x)$ and $d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x)$ are computed

$$d\mathcal{O}_{IMS,1,\dot{\Omega}=0}(x) = \begin{bmatrix} dh_1 \\ dL_{\mathbb{F}}h_1 \\ dL^2_{\mathbb{F}}h_1 \\ dh_2 \\ dL_{\mathbb{F}}h_2 \end{bmatrix}, \quad d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x) = \begin{bmatrix} dh_1 \\ dL_{\mathbb{F}}h_1 \\ dh_2 \\ dL_{\mathbb{F}}h_2 \\ dL^2_{\mathbb{F}}h_2 \end{bmatrix}.$$

The matrices $d\mathcal{O}_{IMS,1,\dot{\Omega}=0}(x)$ and $d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x)$ are expressed in terms of the induction motor dynamics and then,

$$d\mathcal{O}_{IMS,1,\dot{\Omega}=0}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\gamma & \omega_s & ba & bpx_5 & bpx_4 \\ \gamma^2 + ba^2M_{sr} - \omega_s & bM_{sr}apx_5 - 2\omega_s\gamma & b_7 & b_8 & b_9 \\ 0 & 1 & 0 & 0 & 0 \\ -\omega_s & -\gamma & -bpx_5 & ba & -bpx_3 \end{bmatrix}$$

$$d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\gamma & \omega_s & ba & bpx_5 & bpx_4 \\ 0 & 1 & 0 & 0 & 0 \\ -\omega_s & -\gamma & -bpx_5 & ba & -bpx_3 \\ -bM_{sr}apx_5 + 2\omega_s\gamma & \gamma^2 + ba^2M_{sr} - \omega_s^2 & b_{10} & b_{11} & b_{12} \end{bmatrix}$$

where

$$\begin{aligned} b_7 &= -ba^2 + bp^2x_5^2 - \gamma ba - 2bp\omega_s x_5 \\ b_8 &= -2bapx_5 - bp\gamma x_5 + 2ba\omega_s \\ b_9 &= -bpax_4 + bp\dot{x}_4 + bp^2x_5x_3 - \gamma bpx_4 - bp\omega_s x_3 \\ b_{10} &= 2bapx_5 + bp\gamma x_5 - 2ba\omega_s \\ b_{11} &= -ba^2 + bp^2x_5^2 - \gamma ba - 2bp\omega_s x_5 \\ b_{12} &= bpax_3 - bp\dot{x}_3 + bp^2x_5x4 + \gamma bpx_3 - bp\omega_s x_4 \end{aligned}$$

must be of dimension equal to 5, respectively.

It can be directly verified that the determinants are

$$Det(d\mathcal{O}_{IMS,1,\dot{\Omega}=0}(x)) = -b^3p^3(\dot{x}_4 + \omega_s x_3)(\frac{a^2}{p^2} + x_5^2),$$

$$Det(d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x)) = b^3 p^3(\dot{x}_3 - \omega_s x_4)(\frac{a^2}{p^2} + x_5^2).$$

From the determinants $Det(d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x))$ and $Det(d\mathcal{O}_{IMS,2,\dot{\Omega}=0}(x))$ it can be remarked that $\dot{x}_4 = -\omega_s x_3$, $\dot{x}_3 = \omega_s x_4$ or $\dot{x}_4 = \dot{x}_3 = \omega_s = 0$, represent the observability singularities for the case 1. Then, for these particular dynamics, the observability rank condition is not satisfied.

2.4.4.2 Case 2: $\omega_s = 0$.

Consider that the synchronous speed $\omega_s = 0$, the load torque T_l and the rotor speed Ω are not available by measurement. The resulting model of the Induction Motor (1.114) used to analyze the observability properties is then defined in terms of the state, input and measurable output as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \\ \Omega \\ T_l \end{bmatrix} \in \Re^6, \quad u = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}, \quad h(x) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}$$

and

$$\mathbb{F}(x, u) = \begin{bmatrix} bax_3 + bpx_5x_4 - \gamma x_1 + m_1 v_{sd} \\ bax_4 - bpx_5x_3 - \gamma x_2 + m_1 v_{sq} \\ -ax_3 - px_5x_4 + aM_{sr}x_1 \\ -ax_4 + px_5x_3 + aM_{sr}x_2 \\ m(x_3x_2 - x_4x_1) - cx_5 - \dfrac{1}{J}x_6 \\ 0 \end{bmatrix}.$$

The observation space $\mathcal{O}_{IMS,3,\omega_s=0}(x)$ of functions containing the components of h and closed under Lie derivation, is given by

$$\mathcal{O}_{IMS,3,\omega_s=0}(x) = \{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}^2h_1, L_{\mathbb{F}}^2h_2\}.$$

From Definition 2.11, and to verify the observability rank condition, the rank of matrix

$$d\mathcal{O}_{IMS,3,\omega_s=0}(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}^2h_1 \\ dL_{\mathbb{F}}^2h_2 \end{bmatrix},$$

has to be full rank (see Theorems 2.3 and 2.5). This is equivalent verifying if the determinant

$$Det(d\mathcal{O}_{IMS,3,\omega_s=0}(x)) = Det \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & ba & bpx_5 & bpx_4 & 0 \\ 0 & -\gamma & -bpx_5 & ba & -bpx_3 & 0 \\ a_7 & b_1 & a_8 & a_9 & a_{10} & \dfrac{-bpx_4}{J} \\ b_2 & a_{11} & a_{12} & a_{13} & a_{14} & \dfrac{bpx_3}{J} \end{bmatrix}$$

is different from zero, where

$$\begin{aligned}
b_1 &= bp(mx_3x_4 + M_{sr}ax_5) \\
b_2 &= bp(mx_3x_4 - M_{sr}ax_5) \\
a_7 &= -bpmx_4^2 + \gamma^2 + bM_{sr}a^2 \\
a_8 &= bpmx_4x_2 - \gamma ba - ba^2 + bp^2x_5^2 \\
a_9 &= bp\dot{x}_5 + bpmx_4x_1 - \gamma bpx_5 - 2bpax_5 \\
a_{10} &= -bpcx_4 - bp\gamma x_4 - 2bpax_4 + bp^2x_5x_3 + bpaM_{sr}x_2 \\
a_{11} &= -bpmx_3^2 + \gamma^2 + bM_{sr}a^2 \\
a_{12} &= -bp\dot{x}_5 + bpmx_3x_2 + \gamma bpx_5 + 2bapx_5 \\
a_{13} &= bpmx_3x_1 + \gamma ba + bp^2x_5^2 - ba^2 \\
a_{14} &= bpcx_3 + \gamma bpx_3 + 2bpax_3 - bpM_{sr}ax_1 + 2bp^2x_4x_5.
\end{aligned}$$

This is equivalent analyzing

$$Det(d\mathcal{O}_{IMS,3,\omega_s=0}(x)) = \frac{b^4p^3a}{J}\overbrace{(x_3^2+x_4^2)}^{\phi_{rd}^2+\phi_{rq}^2}(\dot{x}_5+\frac{a}{bp}x_5+\frac{p}{ba}x_5^3)$$
$$+\frac{b^3pM_{sr}a^2}{m}\overbrace{m(x_3x_2-x_4x_1)}^{T_e}. \tag{2.46}$$

Remark 2.6 Notice that the analysis of the determinant $d\mathcal{O}_{IMS,3,\omega_s=0}(x)$ is not an easy task. However, we can see that the points $T_e = 0$ and $\phi_{rd}^2 + \phi_{rq}^2 = 0$, appears as an observability singularity of the system. These conditions are not of practical interest, because these conditions are satisfied only if the machine has a flux equal to zero and then the electromechanical torque is obviously zero. The motor does not play any role with respect to the load.

2.4.4.3 Case 3: $\dot{\phi}_{rd} = \dot{\phi}_{rq} = \omega_s = 0$

This case represents the operating condition when the fluxes are constant and the synchronous speed is equal to zero. Under these conditions, the induction motor (1.114) is described by the following state, input and measurable output as

$$x = \begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6 \end{bmatrix} = \begin{bmatrix} i_{sd}\\ i_{sq}\\ \phi_{rd}\\ \phi_{rq}\\ \Omega\\ T_l \end{bmatrix} \in \Re^6, \quad u = \begin{bmatrix} u_{sd}\\ u_{sq} \end{bmatrix}, \quad h(x) = \begin{bmatrix} h_1\\ h_2 \end{bmatrix} = \begin{bmatrix} x_1\\ x_2 \end{bmatrix} = \begin{bmatrix} i_{sd}\\ i_{sq} \end{bmatrix}$$

and

$$\mathbb{F}(x,u) = \begin{bmatrix} bax_3 + bpx_5x_4 - \gamma x_1 + m_1 v_{sd}\\ bax_4 - bpx_5x_3 - \gamma x_2 + m_1 v_{sq}\\ 0\\ 0\\ m(x_3x_2 - x_4x_1) - cx_5 - \frac{1}{J}x_6\\ 0 \end{bmatrix}.$$

The observation space $\mathcal{O}_{IMS,4}(x)$ generated by h, and closed under Lie derivation along of field $\mathbb{F}$, is given by

$$\mathcal{O}_{IMS,4}(x) = \{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}^2h_1, L_{\mathbb{F}}^2h_2\}.$$

From Definition 2.11, and by verifying the observability rank condition, it follows that the matrix

$$d\mathcal{O}_{IMS,4}(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}^2h_1 \\ dL_{\mathbb{F}}^2h_2 \end{bmatrix}$$

must be of full rank (see Theorems 2.3 and 2.5). It follows that matrix

$$d\mathcal{O}_{IMS,4}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & ba & bpx_5 & bpx_4 & 0 \\ 0 & -\gamma & -bpx_5 & ba & -bpx_3 & 0 \\ a'_7 & bpmx_3x_4 & a'_8 & a'_9 & a'_{10} & \dfrac{-bpx_4}{J} \\ bpmx_3x_4 & a'_{11} & a'_{12} & a'_{13} & a'_{14} & \dfrac{bpx_3}{J} \end{bmatrix}$$

where

$$\begin{aligned} a'_7 &= -bpmx_4^2 + \gamma^2 \\ a'_8 &= bpmx_4x_2 - \gamma ba \\ a'_9 &= bp\dot{x}_5 + bpmx_4x_1 - \gamma bpx_5 \\ a'_{10} &= -bpcx_4 - bp\gamma x_4 \\ a'_{11} &= -bpmx_3^2 + \gamma^2 \\ a'_{12} &= -bp\dot{x}_5 + bpmx_3x_2 + \gamma bpx_5 \\ a'_{13} &= bpmx_3x_1 + \gamma ba \\ a'_{14} &= bpcx_3 + \gamma bpx_3 \end{aligned}$$

has its determinant $Det(d\mathcal{O}_{IMS,4}(x))$ given by

$$Det(d\mathcal{O}_{IMS,4}(x)) = \frac{b^4p^3a}{J} \overbrace{(x_3^2 + x_4^2)}^{\phi_{rd}^2 + \phi_{rq}^2} \dot{x}_5.$$

must be different to zero. Notice that the determinant $Det(d\mathcal{O}_{IMS,4}(x))$ is equal to zero for

$$\phi_{rd}^2 + \phi_{rq}^2 = 0$$

or

$$\dot{\Omega} = \dot{x}_5 = 0.$$

Remark 2.7

- Case $\phi_{rd}^2 + \phi_{rq}^2 = 0$ has no practical interest because the IM cannot operate without flux. The case $\dot{x}_5 = 0$ implies that the rotor speed is constant. Then we can conclude that the determinant is zero if the speed is constant. Thus the observability of the IM cannot be establish under constant speed, with zero stator pulsation ω_s, the components of the rotor flux ϕ_{rd} and ϕ_{rq} are constants.
- Case 3 is important as the field-oriented control (a classical control strategy) imposes the flux ϕ_{rd} to be constant (i.e., $\dot{\phi}_{rd} = 0$) and the flux ϕ_{rq} to be equal zero. Then the observability of the IM is no longer satisfied when the speed is constant (steady state) and the stator pulsation ω_s is zero.
- From *Case 1* and *Case 3*, we can conclude that it is not possible to verify the observability of the induction motor by using only the stator current measurement and their derivatives up to order 2.

To analyze the observability property of the induction motor from the measurements (the stator currents) and their derivatives up to order 2 in the case where the machine speed is constant ($\dot{\Omega} = 0$), the component of the flux are constant ($\dot{\phi}_{rd} = \dot{\phi}_{rq} = 0$) and the stator pulsation is zero ($\omega_s = 0$), is described in the following subsection.

2.4.4.4 Case 4: $\dot{\phi}_{rd} = \dot{\phi}_{rq} = \omega_s = 0$ and $\dot{\Omega} = 0$

For $\dot{\phi}_{rd} = \dot{\phi}_{rq} = \omega_s = 0$ and $\dot{\Omega} = 0$, the model of induction motor (1.115) is described by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \\ \Omega \end{bmatrix} \in \Re^5, \quad u = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix}, \quad h(x) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}$$

and

$$\mathbb{F}(x,u) = \begin{bmatrix} bax_3 + bpx_5x_4 - \gamma x_1 + m_1 v_{sd} \\ bax_4 - bpx_5x_3 - \gamma x_2 + m_1 v_{sq} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The observation space $\mathcal{O}_{IMS,5}(x)$ generated by the components of h and closed under Lie derivation along the field $\mathbb{F}$ is given by

$$\mathcal{O}_{IMS,5}(x) = \{h_1, h_2, L_{\mathbb{F}}h_1, L_{\mathbb{F}}h_2, L_{\mathbb{F}}^2h_1, L_f^2h_2, L_{\mathbb{F}}^3h_1, L_{\mathbb{F}}^3h_2, L_{\mathbb{F}}^4h_1, L_{\mathbb{F}}^4h_2\}.$$

The observability rank condition can be verified if matrix

$$
d\mathcal{O}_{IMS,5}(x) = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_{\mathbb{F}}h_1 \\ dL_{\mathbb{F}}h_2 \\ dL_{\mathbb{F}}^2 h_1 \\ dL_{\mathbb{F}}^2 h_2 \\ dL_{\mathbb{F}}^3 h_1 \\ dL_{\mathbb{F}}^3 h_2 \\ dL_{\mathbb{F}}^4 h_1 \\ dL_{\mathbb{F}}^4 h_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & ba & bpx_5 & bpx_4 & 0 \\ 0 & -\gamma & -bpx_5 & ba & -bpx_3 & 0 \\ \gamma^2 & 0 & -\gamma ba & -\gamma bpx_5 & -\gamma bpx_4 & 0 \\ 0 & \gamma^2 & \gamma bpx_5 & -\gamma ba & \gamma bpx_3 & 0 \\ -\gamma^3 & 0 & \gamma^2 ba & \gamma^2 bpx_5 & \gamma^2 bpx_4 & 0 \\ 0 & -\gamma^3 & -\gamma^2 bpx_5 & \gamma^2 ba & -\gamma^2 bpx_3 & 0 \\ \gamma^4 & 0 & -\gamma^3 ba & -\gamma^3 bpx_5 & -\gamma^3 bpx_4 & 0 \\ 0 & \gamma^4 & \gamma^3 bpx_5 & -\gamma^3 ba & \gamma^3 bpx_3 & 0 \end{bmatrix}
$$

is of full rank (see Theorems 2.3 and 2.5).

Thus the observability of the IM can be established under the following operation conditions of the machine: the (d, q)-components of rotor flux ϕ_{rd} and ϕ_{rq} are *constant*, *zero stator pulsation*, and *constant speed* even using the derivatives of high order of the measurements.

2.4.5 Unobservability Line

From (2.40), the stator pulsation (2.39) can be expressed as follows:

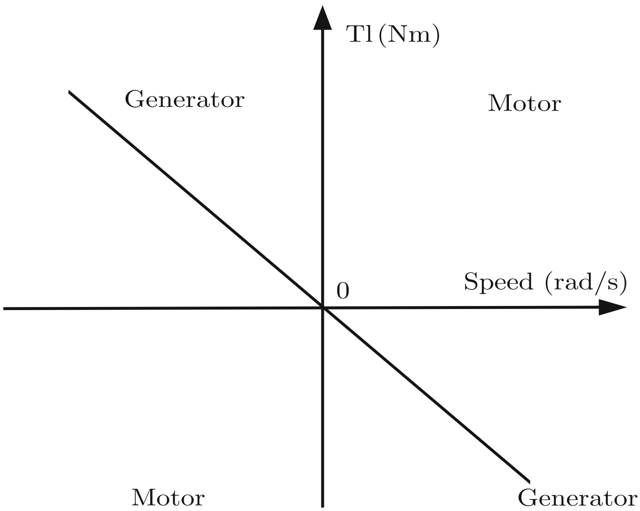

Fig. 2.1 Unobservability line in the plane (T_l, Ω)

$$\omega_s = p\Omega + \frac{R_r T_e}{p\phi_{rd}^2}. \tag{2.47}$$

For $\omega_s = 0$ and ϕ_{rd} constant, we obtain that the electromagnetic torque is given by

$$T_e = -K\Omega \tag{2.48}$$

where $K = \dfrac{P^2\phi_{rd}^2}{R_r}$. If the machine speed is constant ($\dot{\Omega} = 0$), the dynamical equation (1.105) becomes

$$T_e = f_v\Omega + T_l. \tag{2.49}$$

From (2.48) and (2.49) a line can be drawn in the load torque-mechanical speed plane (T_l, Ω) (see Fig. 2.1):

$$T_l = -M\Omega \tag{2.50}$$

with $M = \dfrac{P^2\phi_{rd}^2}{R_r} + f_v$.

This unobservability line is located in the second and fourth quadrants of the plane (T_l, Ω), when the machine operates in generator mode (the load torque and the mechanical speed are of the opposite sign) as shown in Fig. 2.1. This line is used to check industrial drives in order to characterize their sensorless behavior at slow speed.

2.5 Normal Forms for Observer Design

As seen in the above sections, there are different structures used to represent a nonlinear system, in particular to represent the AC machines. Normal forms are obtained based on the information available from measurement and from the observation objectives. Furthermore, there are a large number of observers which have been developed for linear and nonlinear systems. Several efforts have been made to construct an observer for a general class of nonlinear system. Extensions of the linear case have been proposed which are adaptations of linear observers to nonlinear systems.

They have been derived using different techniques or methodologies:

Extended observers: general Luenberger observer, Kalman filter, state affine systems observer, linear plus an output injection, high gain observer, adaptive observers mainly. These observers have an estimation error that converges exponentially or asymptotically to zero.

Sliding mode observers: classical sliding mode, super-twisting, high-order sliding mode, adaptive sliding mode for instance. One the most important characteristics of these observers is their finite-time convergence to zero and their robustness under uncertainties.

However, when considering nonlinear systems, the construction of an observer is not easy (see [4, 30, 42]). We can distinguish two classes of systems: those of which are observable for any input and those that have singular inputs.

For those which are observable for any input, i.e., *uniformly observable*, the first results have been obtained in the case when the nonlinear system can be transformed, by means of a diffeomorphism, into a linear system plus an output–input injection.

Consider the class of nonlinear system described by a state-space representation of the form (2.18). System (2.18) can be transformed into one of the following state-space representations:

(1) *Linear system plus an output–input injection*

$$\begin{cases} \dot{\xi} = A\xi + \phi(u, y) \\ y = C\xi \end{cases} \tag{2.51}$$

which is observable for any input, if and only if the pair (C, A) is observable.

(2) *Triangular form*

The generalization of the above class of nonlinear systems is of the form

$$\begin{cases} \dot{\xi} = A\xi + \phi(u, \xi) \\ y = C\xi \end{cases} \tag{2.52}$$

where the term $\phi(u, \xi)$ is in the triangular form, i.e.,

$$\phi(u, \xi) = (\phi(u, \xi_1), \phi(u, \xi_1, \xi_2), \ldots, \phi(u, \xi_1, \ldots, \xi_n))^T,$$

which has been introduced in [28].

Notice that these classes of nonlinear systems are observable for any input, so the observer design is possible.

An interesting class of systems which will be studied in the book is:

(3) *State affine system plus an input–output injection*

This class of systems is represented as

$$\begin{cases} \dot{\xi} = A(u)\xi + \phi(u, y) \\ y = C\xi \end{cases} \tag{2.53}$$

where the components of the matrix A depends on the input u.

Notice that this system has inputs rendering the system unobservable. These inputs are called *bad inputs* [87]. Despite this fact, stronger notions like persistency is used to design observers, i.e., there exists an observer working for the class of persistent inputs.

(4) *State affine system plus a nonlinear term*

A general class of state affine systems is given by the class of systems of the form

$$\begin{cases} \dot{\xi} = A(u, y, s)\xi + \phi(u, \xi) \\ y = C\xi \end{cases} \tag{2.54}$$

for which it is possible to design an observer. Notice that the matrix $A(u, y, s)$ depends on the input u, the output y, and a known signal s [83].

Regarding these above classes of systems, several authors are interested to characterize them, where necessary and sufficient conditions are given to transform a general nonlinear system into state affine systems plus an output–input injection or plus a nonlinear term.

(5) *Interconnected state affine system plus nonlinear terms*
Finally, we can find systems that can be partitioned in a set of interconnected subsystem, represented in subsystems of the following form

$$\begin{cases} \dot{\xi}_1 = A_1(u)\xi_1 + \phi_1(u, \xi_2, \ldots, \xi_r) \\ \dot{\xi}_2 = A_2(u)\xi_2 + \phi_1(u, \xi_1, \ldots, \xi_r) \\ \ldots \\ \dot{\xi}_r = A_r(u)\xi_r + \phi_r(u, \xi_2, \ldots, \xi_{r-1}) \\ y_1 = C\xi_1 \\ y_2 = C\xi_2 \\ \ldots \\ y_r = C\xi_r. \end{cases} \tag{2.55}$$

In Chap. 3, two main classes of observers for nonlinear systems will be considered to reconstruct the components of the state vector which are not measurable.

(1) *Extended observers*: high gain observer, observer for state affine system, and nonlinear interconnected observers.
(2) *Sliding mode observers*: for nonlinear systems: Super-twisting and high order sliding mode observers.

2.6 Conclusions

One of the most important structural properties of dynamical systems has been studied in this chapter: the observability of nonlinear systems. As it has been seen in this chapter, the nonlinear observability property can depend on the input (explicitly or implicitly), and some definitions have been introduced to classify the inputs (universal and persistent inputs). Then, the observability of the AC machines has been analyzed, and the conditions under which the PMSM and the IM are observable have been determined along with their physical interpretation. This will be useful in the subsequent chapters to guarantee the convergence of the designed observers.

2.7 Bibliographical Notes

The observability study of nonlinear systems and next, the design of an observer is generally not a trivial task. Concerning the observability of nonlinear systems, the main definitions, used in this book, can be found in [42, 46, 65]. A classical observability criterion can be defined by using an observation space closed under Lie derivation as introduced in [46]. The key role of the input for the observability of the nonlinear systems is described in [3].

From these definitions, some authors have studied the observability of the AC machines. Nevertheless, the studies on the synchronous motor observability are rather uncommon, even if some results can be found in [20, 40, 92, 93]. Similarly, for the induction motor observability analysis, some results are available in [11, 32, 44]. In [32], sufficient conditions under which the induction motor loses the observability property have been presented.

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