

# Hybrid LES/RANS of Internal Flows: A Case for More Advanced RANS

K. Hanjalić, D. Borello, G. Delibra and F. Rispoli

**Abstract** The Hybrid LES/RANS is emerging as the most viable modelling option for CFD of real-scale problems, at least in the aerospace design. Entrusting LES to resolve the intrinsic unsteadiness and three-dimensionality in the flow bulk reduces the modelling empiricism to a relatively small wall-adjacent RANS region, arguably justifying the use of very simple models. We argue, however, that for internal flows in complex passages, and involving heat and mass transfer, the role of the near-wall RANS should not be underestimated. The issue is discussed by two examples of flows in turbomachinery: a pinned internal-cooling passage in a turbine blade and tip leakage and wake in a compressor cascade with stagnant and moving casing. The examples illustrate the need for a topology-free wall-integration RANS model that accounts for versatile effects of multiple bounding walls. A HLR using an elliptic relaxation ( $v^2/k - f$ ) RANS model coupled with a dynamic LES showed to perform well in the cases considered.

## 1 Introduction

“The use of CFD in the aerospace design process is severely limited by the inability to accurately and reliably predict turbulent flows with significant regions of separation. Advances in Reynolds-averaged Navier-Stokes (RANS) modeling alone are unlikely to overcome this deficiency, while the use of Large-eddy simulation (LES) methods will remain impractical for various important applications for the foreseeable future, barring any radical advances in algorithmic technology. Hybrid RANS-LES and

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wall-modeled LES offer the best prospects for overcoming this obstacle although significant modeling issues remain to be addressed here as well...” ([15], CFD Vision 2030).

The HLRM conference series has been without any doubt on the right track. The appeal of using a RANS model to overcome the formidable demands for near-wall resolution for LES of complex high-Re-number wall-bounded flows, initiated by the pioneering DES approach of Spalart et al. [18], has triggered a number of ideas and concepts. While it is practically impossible (and, in principle, unimportant) to classify all proposals into distinct categories, in order to assess their physical rationales, potentials and snares, it is useful to distinguish the key presumptions and implications underlining each concept.<sup>1</sup> Arguably, most methods can be grouped into three categories:

- *zonal* RANS/LES that employ separate models in different flow regions, a RANS model close to solid walls (or in the complete attached wall layer), usually modified to match the conditions at the predefined or dynamic interface, and the conventional LES remote from walls;
- *blended* models in which the two methods are blended by a set of continuous (empirical) functions applied to the local turbulent stress tensor or the effective turbulent viscosity in terms of the local characteristic grid size and turbulence length scale;
- *seamless* schemes in which a single model (usually a modified, “sensitized” RANS) is employed throughout the entire flow, which only asymptotically approaches the conventional RANS and LES (or DNS) in the limits of the wall-distance approaching zero or infinity, respectively; the sensitizing of the RANS model is usually accomplished by a “grid-detecting” parameter (computer needs to “feel” the grid and to adjust the model) in term of the typical grid size and the turbulence length scale (as in blended schemes).

Other schemes have also been proposed that could not fit precisely into the above classification. An insightful and a thorough comparison of the rationales of the various concepts, and especially the identification of the common denominators and major conceptual differences, is still lacking. However, extensive scrutiny of various methods in a variety of flows undertaken over the past decade showed varied success, but no indisputable “winner” has emerged yet. Instead, the testing have opened a number of questions, which, irrespective of the models and matching criteria used, to a large degree still remain as actual as ever.

Admittedly, the DES concept (and its variants DDES, IDDES), using the one-equation RANS model of Spalart-Allmaras (SA), is an exception: its robustness and economy have at present no serious competitor when it comes to external aerodynamics at real-scale Re numbers, e.g. a complete airplanes [9], where the only (verifiable) integral parameters (lift and drag) matter. However, its heuristic simplicity and non-transparency, and in particular the use of the wall distance to represent

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<sup>1</sup> A comprehensive overview of various approaches and somewhat unorthodox classification can be found in Fröhlich and Terzi [10].

the turbulence length scale in the complete RANS area makes its applicability to geometrically complex internal flows questionable.

This paper attempts to make a case for more advanced RANS in the framework of HLR. It is argued that one of the key prerequisites, especially for internal flows of complex geometric configurations that may also include heat and mass transfer, particle deposition, phase change, near-wall chemical reactions, combustion and flames, as well as other surface-phenomena, is the choice of the adequate RANS model. The arguments are substantiated by discussing two cases relevant to turbomachinery: internal-cooling of gas- turbine blades and tip leakage in a compressor cascade.

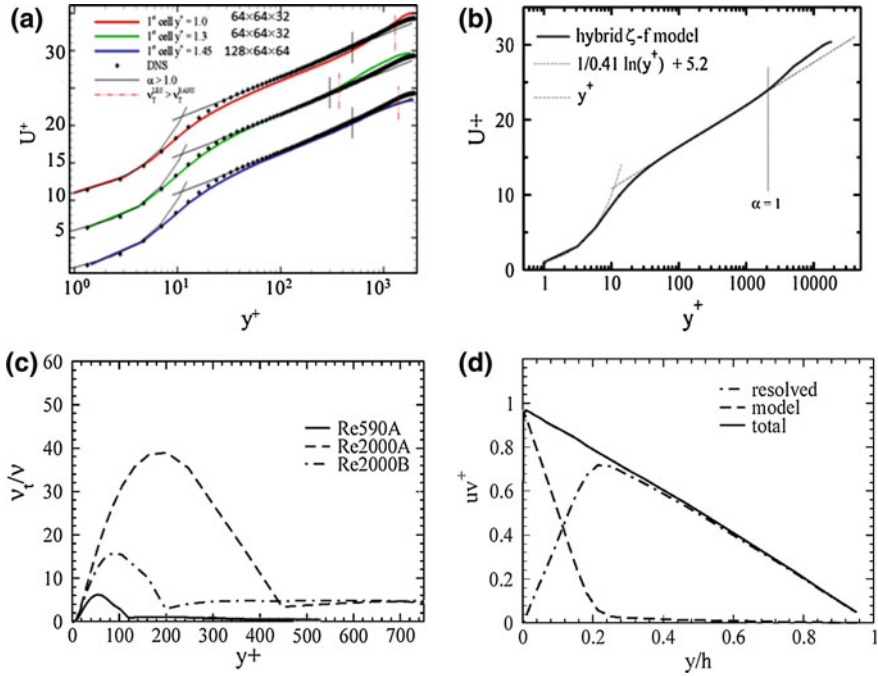
## 2 Importance of RANS in HLR of Complex Internal Flows

Employing LES to resolve large eddies in the flow bulk, expected to reproduce well the intrinsic unsteadiness and the essential large-scale 3D turbulence dynamics, relaxes the burden on the RANS and reduces, in principle, its task to provide only the wall boundary conditions. The modelling empiricism is thus confined to the wall-adjacent, presumed to be relatively small, portion of the flow, arguably justifying the use of very simple one- or two-equation RANS models.

While in many external flows with identifiable wall-attached flow regions, such arguments can be acceptable, they may not hold for simulation of internal flows in complex passages encountered in many engineering flows and especially involving heat and mass transfer and other surface phenomena. One can think of turbomachinery (pinned, ribbed, grooved or dimpled internal blade cooling passages subjected to rotation, blade-tip—casing gaps, labyrinth seals), or IC engines (valves, cooling jackets), compact heat exchangers, electronic packages, and many other examples where complex geometric topology may impede the proper capturing of the near-wall phenomena.

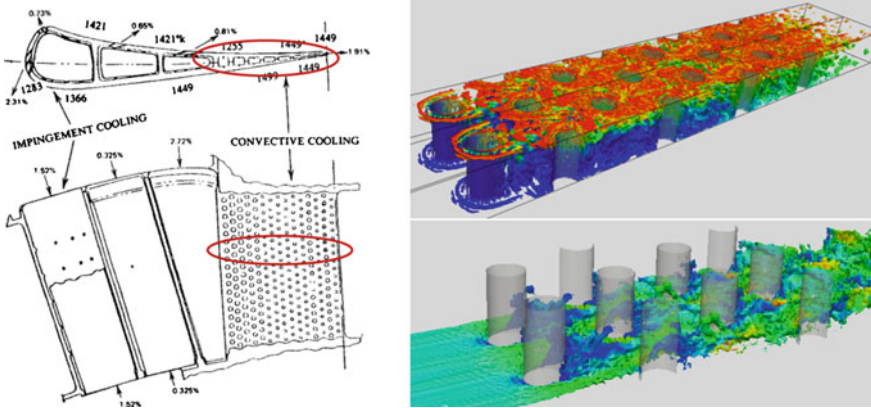
The Re numbers are rarely excessive as in full-scale external aerodynamics, but often moderate and, one may argue, suitable for LES. However, because of multiple bounding walls and a lack of full flow periodicity (even in periodic configurations), wall-resolving LES is still very grid-demanding. On the other hand, because of the full three-dimensionality, intrinsic unsteadiness and complex vortical structures, treating the complete domain with a URANS may not return the essential physics. Thus, a hybridization of LES in the regions away from walls with a RANS in wall-adjacent regions seems for this kind of flows the most feasible option. But as stated above, the RANS model should be thoughtfully chosen bearing in mind a number of flow specificities:

- corrugated configurations with corners, tips, grooves, protrusions, and accounting properly for the wall-blocking effect that may permeate over relatively large flow regions, require a model free from topological parameters;

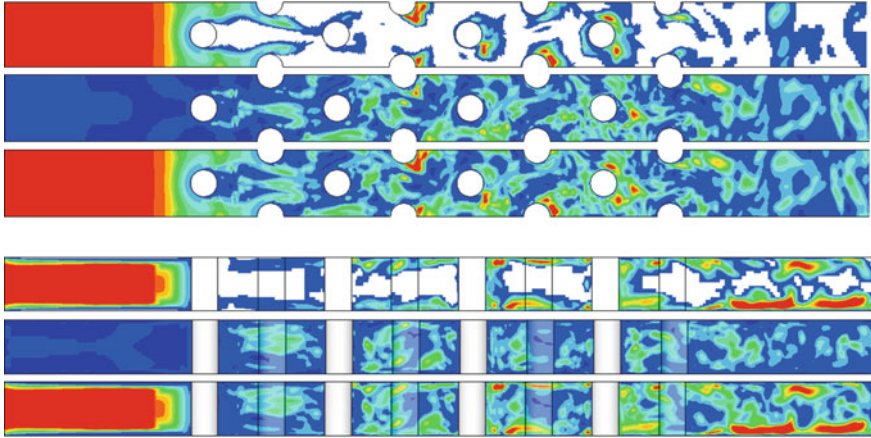


**Fig. 1** A HLRM of channel flow: effect of grid clustering and location of RANS and LES interface. **a** Velocity profiles for  $Re_\tau = 2,000$ , **b** for  $Re_\tau = 20,000$ ; **c** effective eddy viscosity for two  $Re$  numbers and grids; **d** turbulent shear stress

- multiple walls bounding the flows can exert strong influence beyond the thin boundary layers, which pose additional challenge especially if a relatively significant portion of the flow is entrusted to URANS (see Figs. 1 and 3);
- automatic gridding, common in handling complex geometries, may influence the location of the LES/RANS interface if it is based on the local cell size, changing/increasing the URANS region beyond that originally anticipated;
- complex topology will cause local separations, curvature, multiple vortex systems and their interactions, secondary flows, strong stress anisotropy, possible local laminarization (especially in rotating flows), all of which can, at least partially, occur within the URANS region;
- in most flows mentioned the confinement and topological periodicity does not allow long evolving time, the distortion scales are usually comparable with the intrinsic turbulence scales;
- the response of even a simple RANS to LES perturbation was found to be generally satisfactory [19] but it is reasonable to expect that more advanced anisotropy-capturing models will respond better than the more dissipative and diffusive linear one- or two-equation models.



**Fig. 2** *Left* a sketch of gas-turbine blade colling. *Right* LES of internal pinned passage ( $Re = 10,000$ ); *top* vortical structures ( $\nabla^2 p = 9$ ) coloured by temperature (*top* wall heated); *bottom* thermal plumes ( $T = 294\text{ K}$ ) coloured by velocity magnitude (*bottom* wall heated)



**Fig. 3** Viscosity contours in the midplane ( $z = 1.0$ ) and in the cross-plane ( $y = 1.25$ ) cutting through pins 1, 3, 5 and 7. Pins 2, 4, 6 and 8 (laterally staggered) are also shown (*shaded*). *Top* rows  $v_t^{RANS}$ ; *center*  $v_t^{LES}$ ; *bottom* effective viscosity  $v_t$

All these (and other) issues and phenomena place a large responsibility on the RANS model, so that its choice in the HLR context matters. More physics is required and the model should be capable of reproducing versatile effects especially in response to the outer LES. A fringe benefit is that a more capable RANS model allows to extend the RANS region over a larger portion of the flow, thus reducing the demand on the grid size and the computing time. The models should also be more transparent to make it possible for the user to ascertain a priori if the model used contains sufficient

physics or not. Robustness is the important requirement, but it is not the only one, and for many applications, accuracy should matter more.

The above requirements hint at the second-moment (Re-stress) closures, but applying a full differential model in the HLR context makes not much sense as the stress transport should be accounted for by the LES in the outer region and feed information through the interface. Moreover, such an approach would be very costly with marginal if any benefits, not to mention uncertainties in ensuring the adequate matching for all variables (e.g. all stress components) at the RANS/LES interface. A more viable option could be an algebraic stress model in conjunction with a wall-integration two-equation model (or even only one-equation model as e.g. in Schmidt and Breuer [16]). However, accommodating an algebraic stress model to complex near-wall configuration is not trivial, and may hamper the computational robustness and economy.

Another, perhaps still more attractive option is the elliptic-relaxation (ER) eddy-viscosity concept of Durbin [8], or one of several more robust model variants subsequently proposed. Although the ER-EVM involves solution of four transport equations, the experience shows that the computational penalty is modest since the equations are very simple and well coupled, ensuring fast convergence. The ER approach has been developed specifically to handle complex near wall flows without empirical damping functions and its elliptic relaxation accounting for inviscid wall blocking has a sound physical rationale. The approach seems thus well suited for HLR, especially for internal flows of complex geometric topology at high Re numbers where the RANS region may occupy more than just the attached portion of the flow. The arguments are considered valid irrespective of whether a zonal, blended, seamless or other hybridization concept is adopted.

In what follows we present briefly a HLR method that employs the ER concept for the URANS region, specifically, a robust linear elliptic relaxation  $\zeta (= v^2/k) - f$  model hybridized with the LES. The performance of the method is illustrated by two examples from turbomachinery: flow and heat transfer in a wall-bounded matrix of staggered cylindrical pins (mimicking internal cooling passage of a gas-turbine blade), and tip leakage and trailing vortices in a linear compressor cascade with stationary and moving casing. The examples illustrate the need for a topology free wall-integration (WIN) RANS model, capable of accounting for versatile effects of the bounding walls. The same approach has earlier been used in the simulation of impinging jet flow and heat transfer [11], flow around short wall-bounded cylinder found in plate-fin-and-tube heat exchangers [2], and, of course, in a plane channel over a range of Re numbers.

### 3 A HLR with Elliptic Relaxation EVM

The  $\zeta$ - $f$  model, chosen for the URANS region, follows Durbin's  $v^2$ - $f$  model, but solves a transport equation for the  $v^2/k$  ratio, which requires less stiff wall boundary conditions and proved generally to be more robust than the parent model. As a full

RANS model it proved to be successful and fast converging in a variety of flows (e.g. [12]). The rationale behind the HLR version [6, 11, 13] is to intervene in the sink term of the  $k$ -equation with a grid detecting parameter “ $\alpha$ ” by which the RANS eddy viscosity is damped to the subgrid-scale value of LES at the matching interface. A detailed description of the method can be found in Delibra et al. [6]. The full set of equations is provided in Appendix; here we list only the  $k$ -equation and the two limiters that control the switching from one to another model:

$$\frac{Dk}{Dt} = P_k - \alpha \varepsilon + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \quad (1)$$

$$\alpha = \max [1; L_{RANS}/L_{LES}] \quad \nu_t = \max(\nu_t^{RANS}, \nu_t^{LES}) \quad (2)$$

where  $k$  is the (RANS) modelled turbulent kinetic energy,  $\varepsilon$  its dissipation rate,  $L_{RANS} = k^{3/2}/\varepsilon$  and  $L_{LES} \equiv \Delta = C_\Delta (\Delta V)^{1/3}$ . Close to a wall  $\alpha = 1$  and the model acts as in the URANS formulation. Away from walls, where  $L_{RANS} > \Delta$ ,  $\alpha > 1$ ,  $k$  is damped, thus diminishing  $\nu_t^{RANS}$ . Eventually, when  $\nu_t^{RANS} < \nu_t^{LES}$ , the second constraint is activated and the conventional LES is resumed.

The use of the above defined  $L_{RANS}$ , while more physical, may pose some stability problems and in the two examples here presented we used  $L_{RANS} = \kappa x_n$ , where  $x_n$  is the local distance from the nearest wall, in which case  $C_\Delta = 1.3$ . It is noted that  $L_{RANS}$  serves *only* to define the control parameter  $\alpha$ , thus entering the turbulence model only in a narrow buffer region where  $\alpha > 1$  (usually only few grid nodes) between the URANS and the full LES.

Figure 1 illustrates the performance of the method in a plane channel flow over a range of Re numbers and for different grid density and distribution. The model switching locations ( $\alpha > 1$  and  $\nu_t^{RANS} = \nu_t^{LES}$ ) denoted in Fig. 1a show that a relatively large portion of the flow is handled by the URANS, whereas the buffer zone between the two limiters varies depending on the grid clustering, but without noticeable effect on the mean velocity distribution. It is noted that no artificial forcing was used to smoothen the profiles in the buffer region.

## 4 Examples 1: Pinned Matrix Bounded by Heated Walls

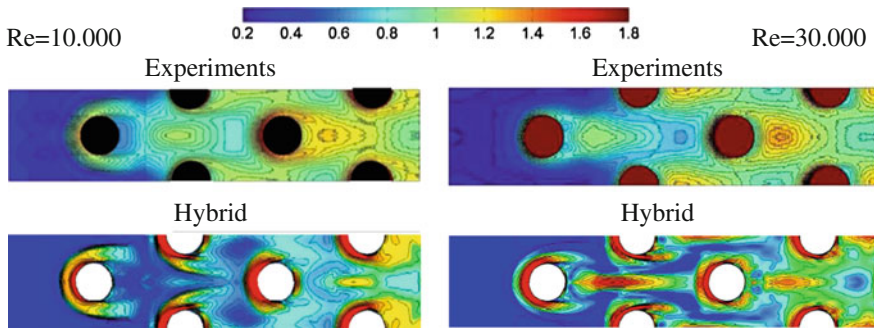
Figure 2 shows a sketch of the typical gas-turbine blade cooling system in which the interior of the trailing part is fitted by a matrix of staggered cylindrical pins. The pins act primarily as promoters of vortex shedding and turbulence to enhance blade cooling by interior cold air flow. A simplified setup with parallel, differentially heated walls mimicking the experiment of [1] for two Re numbers ( $10^4$  and  $3 \times 10^4$ ) was simulated by URANS, LES and HLR aimed at testing the optimum computational strategy. Details have been described in Delibra et al. [5–7]. The complexity of the vortical and plume structures that govern heat transfer and its enhancement is



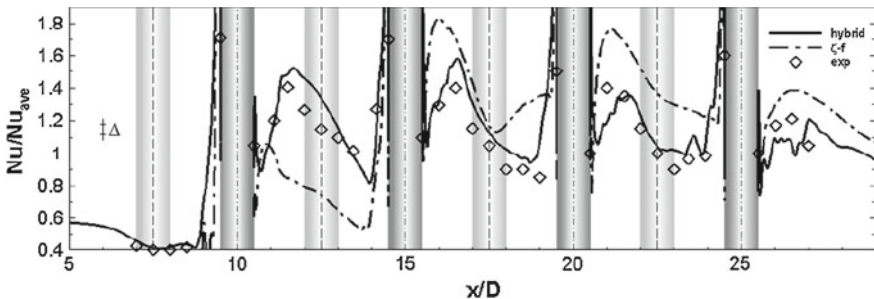
illustrated by wall-resolved LES (available only for  $Re = 10^4$ ) in Fig. 2 right. We discuss briefly some results obtained by HLR with moderate grids:  $1.3 \times 10^6$  and  $4.4 \times 10^6$  respectively for  $Re = 10^4$  and  $3 \times 10^4$ , compared with  $5 \times 10^6$  and  $15 \times 10^6$  for LES (the latter grid proving insufficient). Regardless of the models applied, a much coarser grid used in the HLR cannot reproduce the wide spectrum of scales shown in Fig. 2 right, especially if URANS covers substantial portion of the flow, as illustrated in Fig. 3. But the key question is how important are the unresolved small scales for predicting the wall heat transfer.

Figures 4 and 5 show the time averaged distribution of the Nusselt number on the heated wall, compared with the experiments (Fig. 4) and also with URANS using the same  $\zeta$ - $f$  model in the whole domain. The HLR showed obviously superior results, and also (for  $Re = 10^4$ ) in close agreement with the LES [6, 7].

Although in both the HLR and URANS the same (RANS) model is responsible for the wall heat transfer, the key difference is in the intensity of the outer forcing that influences the separation dynamics of vortex shedding especially behind the first pin row. As shown in Fig. 6, the velocity time records at a monitoring point just behind the pin (inset) show the HLR amplitude (well in agreement with the

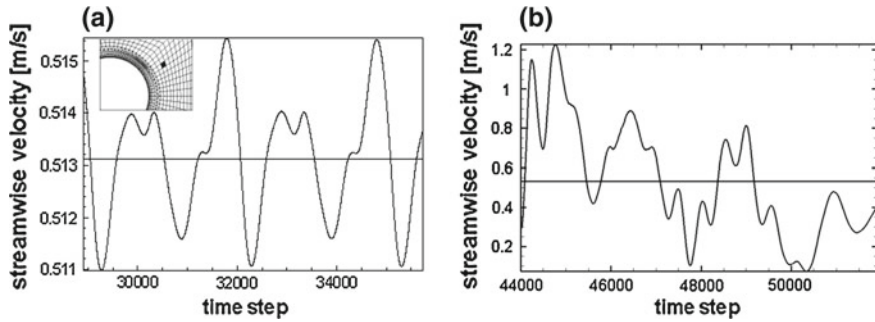


**Fig. 4** Time averaged Nusselt number on the heated wall (the first 4 pins) (normalised with the area-averaged  $Nu_{av}$ )



**Fig. 5** Nusselt number  $Nu/Nu_{av}$  on the heated endwall along the line cutting pins 2, 4, 6 and 8 for  $Re = 10^4$ . From Delibra et al. [6]. Symbols experiments, Ames et al. [1]





**Fig. 6** Velocity fluctuations in the near-wake of the 1st pin (*inset*) in the midplane ( $z = D$ ). **a** URANS, linear  $\zeta$ -f,  $St = 0.24$ , mean velocity 0.513 m/s, maximum amplitude: 0.004 m/s. **b** Hybrid  $\zeta$ -f-LES,  $St = 0.24$ , mean velocity 0.532 m/s, Maximum amplitude: 0.6 m/s

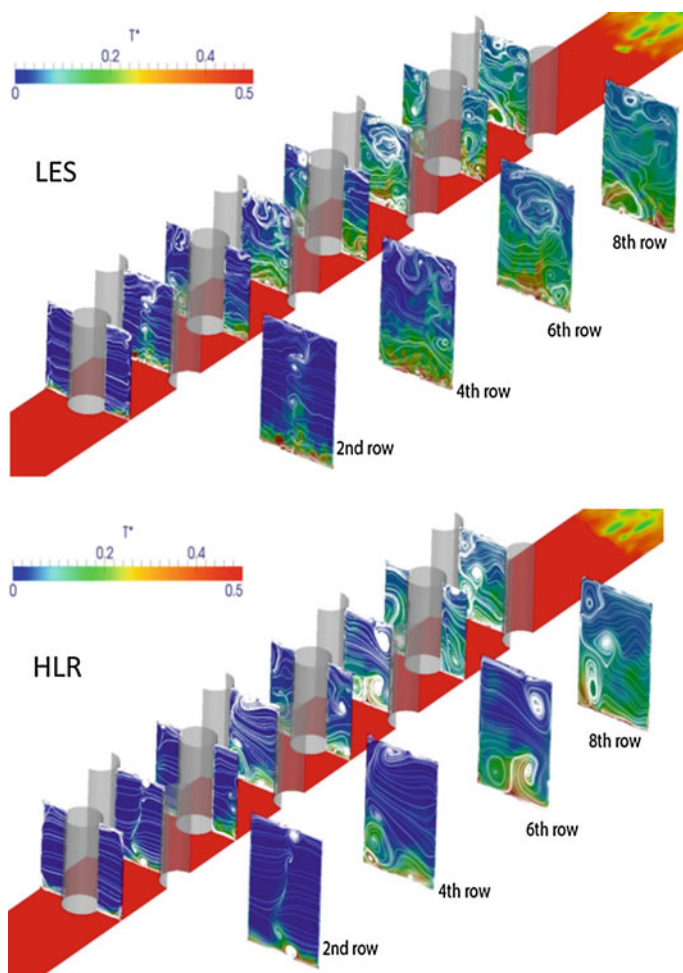
experiments, [6, 7]) to be two orders of magnitude larger than in the URANS, while both methods reproduce equally well the Strouhal number. Perhaps a more advanced RANS (nonlinear EVM or RSM) could do better but the here applied linear  $\zeta$ -f EVM is too simple to capture the instabilities and complex 3D vortical structures around and behind the pins, as illustrated in Fig. 7.

## 5 Examples 2: Tip Leakage and Trailing Vortices in Compressor Cascade with Stagnant and Moving Casing

The efforts to reduce the unproductive, loss-bearing but unavoidable blade-tip leakage and secondary flows rely at present much on the CFD. A sketch of a simplified low-speed compressor cascade with a casing, Fig. 8, illustrates the critical phenomena that pose challenge to modelling [20]. We discuss briefly some salient flow features and challenges for CFD and show some results for stagnant and moving casing using the HLR and URANS. More details can be found in Borello et al. [3, 4].

Fast distortion dominated by pressure and inviscid effects in the tip leakage makes the results insensitive to turbulence modelling (Fig. 10), though the model plays a role in capturing the boundary layer properties on the moving casing. However, the formation and development of the tip vortex and its interaction with the blade wake (Fig. 9) puts a high demand on the model.

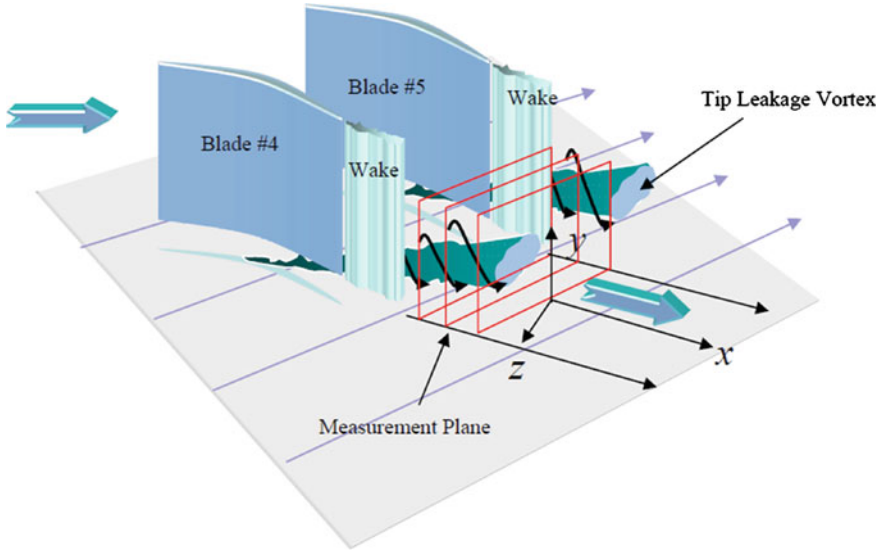
Figure 10 illustrates the insensitivity of computation within the tip clearance to the model used: the URANS and HLR give similar results, close to the experiments for the  $U$  and  $V$  component, though less satisfactory for  $W$ , presumably due to inadequate meshing or imperfect mimicking of the clearance configuration). The choice of the model affects, however, the formation and development of the tip vortex, the HLR capturing a broader spectrum of the scales even compared with the non-linear URANS, especially for moving casing, Fig. 11.



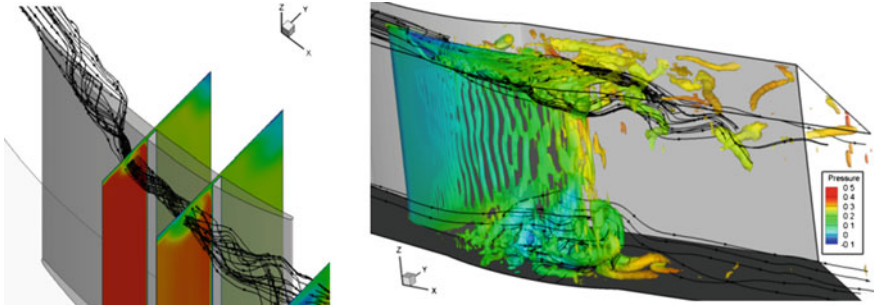
**Fig. 7** Instantaneous streamlines and temperature field (*colours*) in the cut-planes show close structural similarity between LES and HLR

The superiority of the HLR over the URANS is further illustrated in Fig. 12 showing a view of vortical structures and streamlines for the case of moving casing. The hybrid simulation shows stronger, longer-lived, elongated structures in the trails of both the tip and hub vortices, expected to affect the wake. Both methods capture a horseshoe vortex at the hub, but surprisingly, the URANS returns a large and stronger recirculation immediately downstream the blade.

The results for the wake, however, showed to be more model-sensitive as illustrated in Figs. 13 and 14. The experimental data available in the planes indicated by red-quadrangles in Fig. 8, make it also possible to provide some quantitative test of the performances of the models considered. Figure 13, depicting the wall-normal

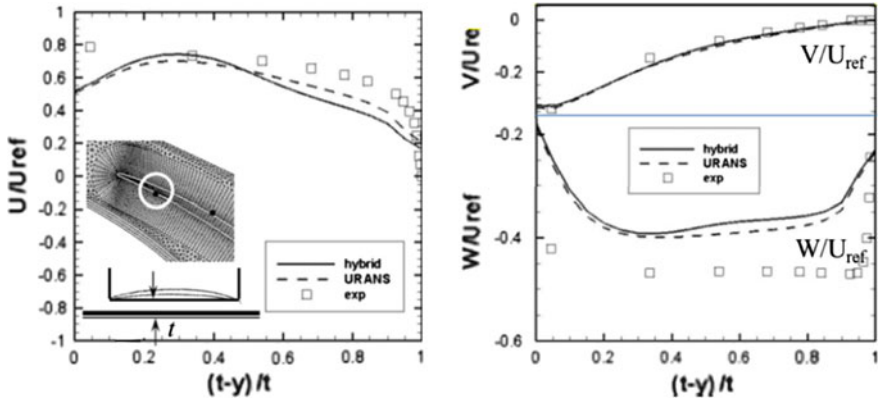


**Fig. 8** A sketch of the cascade and tip leakage vortices; measuring planes indicated by *red* quadrangles behind the cascade. From Wang and Devenport [20]

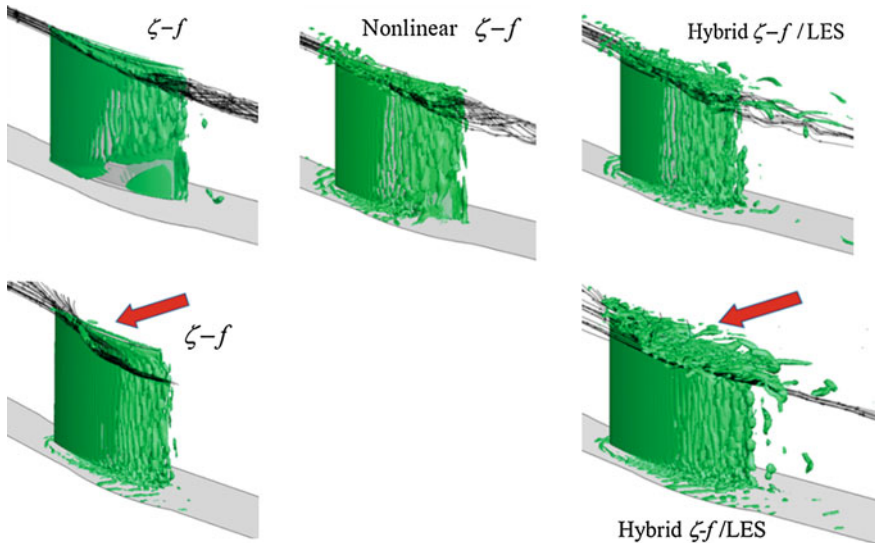


**Fig. 9** Instantaneous HLR streamlines over the blade tip (*left*) and vortical structures on the blade, over the tip and the hub identified by pressure-coloured isosurfaces of  $\nabla^2 p = 60$  (*right*), over a linear compressor cascade with moving casing [4]

velocity field in the cross-plane at the first measuring plane ( $x/c_a = 1.37$ ), shows that only the HLR captures properly the locations as well as the velocity intensity in the two counter-rotating tip-leakage vortices. The linear and non-linear  $\xi$ - $f$  RANS models, when applied in the whole domain (URANS computations), show very similar patterns in between, as well as the vortices strengths and their distance as in the experiment, but shifted laterally towards the suction side, indicating at some wake and vortices deflection.

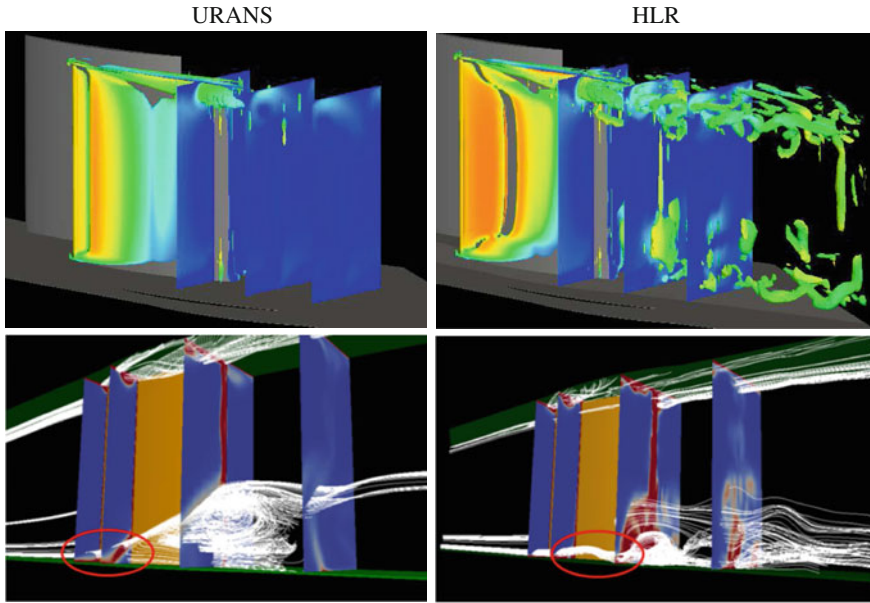


**Fig. 10** Profiles of the mean velocity component in the tip-gap with stationary casing at the indicated position (*inset*), normalised with the reference velocity. *Symbols* experiments, *full line* hybrid  $\zeta$ -f-LES; *dashed line* URANS  $\zeta$ -f

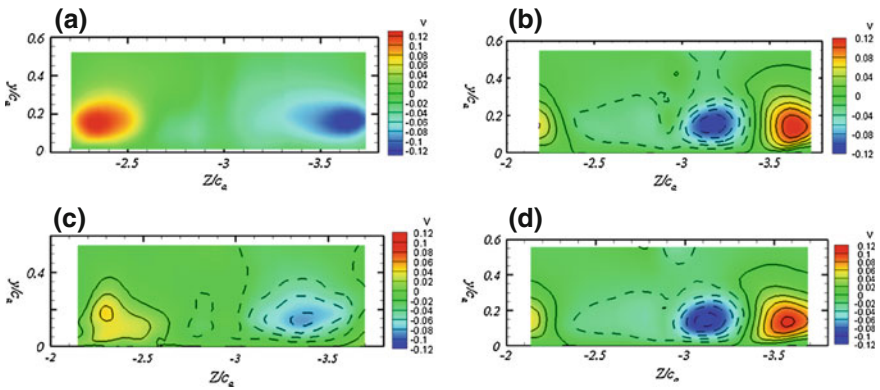


**Fig. 11** Vortical structures in and behind the tip leakage, identified by  $Q = 35$  isosurfaces for different models. *Top* stationary casing; *bottom* moving casing

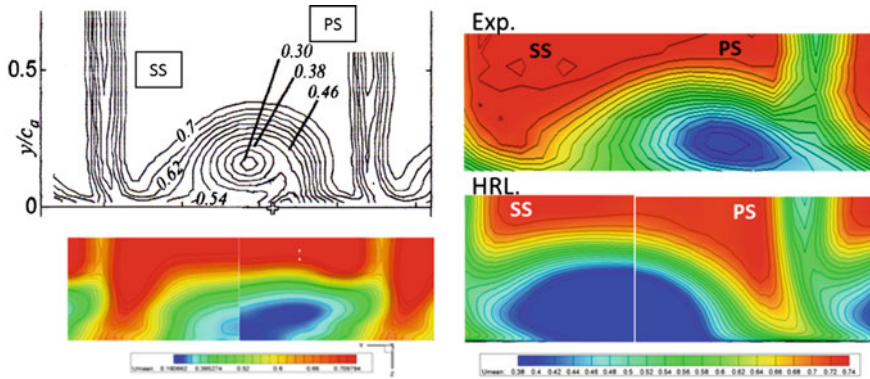
The axial velocity fields, presented in Fig. 14 for two cross-planes (at  $x/c_a = 1.37$  and 1.51), show similar agreement with the experiment, both in the intensity and distribution—though admittedly less satisfactory for  $x/c_a = 1.51$ . The velocity contours clearly display the blade wakes (vortical patterns) and the tip vortices between the blades, close to the casing wall.



**Fig. 12** URANS (*left*) and Hybrid LES/RANS (*right*) of flow in a linear compressor cascade with moving casing. *Top* Vortex structure ( $Q$ ), coloured by velocity; *bottom* instantaneous streamline pattern illustrating the horseshoe vortex (*encircled*), with focus on the hub vortex



**Fig. 13** Wall-normal velocity in the tip-vortex wake at  $x/c_a = 1.37$ . **a** Experiments Muthana and Devenport [14]. **b** Linear  $\zeta$ - $f$  RANS; **c** Hybrid linear  $\zeta$ - $f$ /LES; **d** Nonlinear  $\zeta$ - $f$  RANS



**Fig. 14** Axial velocity in wake of the tip vortex for stationary casing at  $x/c_a = 1.37$  (left) and for moving casing at  $x/c_a = 1.51$  in a linear compressor cascade. *Top row* experiments; *bottom row* Hybrid  $\zeta$ - $f$

## 6 Conclusions

The paper attempts to make a case for using more advanced and transparent RANS models in the Hybrid LES/RANS methods when targeting internal flows of industrial relevance. Most flows in questions are featured by complex geometrical configurations, full three-dimensionality and intrinsic unsteadiness even when the flow is steady in the bulk. The omnipresent bounding walls exert especially strong inviscid (blocking) effects that may permeate over large regions of the flow. If a substantial flow portion is entrusted to RANS, intentionally (for the sake of economy) or not (automatic gridding, difficulties in controlling the grid), the URANS region may include local separation, multiple vortex interactions, secondary flows, strong anisotropy, local transition and laminarization. All these and other phenomena, especially when heat and mass transfer are considered, require a RANS model free from topological parameters and capable of capturing the basic physics of the near-wall flow and ensuring sufficient receptivity to the LES forcing at the interface. Some illustrations are provided with a zonal approaches using an elliptic relaxation,  $\zeta (= v^2/k) - f$ , model to provide the near-wall conditions for the dynamic Smagorinsky LES. The arguments are substantiated by two examples from turbomachinery flows and heat transfer. It is noted that the here reported RANS can be used irrespective of whether a zonal, blended, seamless or other hybridization concept is adopted.

We further argue that reducing empirical contents and making the model more physically transparent and more understandable should be the general requirements, at least for treating internal flows of complex topology. Robustness and economy matter, but should not be decisive since, for credible prediction of unknown situations, physics matters more. Advanced and physically sounder models are inevitably numerically more challenging, require more computational time and effort, but often only by a margin. Admittedly, curing the robustness problem requires more skill,



but serious design and optimization tasks should not be entrusted to non-specialist anyhow, except perhaps, for preliminary considerations and rough estimates.

As for an outlook, in the authors' opinion the CFD community has long been entrenched by too many constraints of the conventional rigid Navier-Stokes eddy-viscosity Ansatz and segregated solvers, which have in general hampered the use of more advanced models. The stability issues should be resolved by innovative algorithms tailored for specific modelled equation sets not necessarily coupled by eddy viscosity (e.g. second-moment closures) rather than insisting on simpler turbulence models. Arguing in favour of most simple heuristic turbulence models because they “work” and that all what matters is simplicity, robustness and economy, may be paralleled by an argument that one should stick to a simple first-order upwind convection scheme because it always “works”. As noted by Spalart [17], “The more capable the RANS component is, the lower costs of the hybrid computations will be. Therefore, the switch to LES in some regions does not remove the incentive to further the RANS technology”.

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## Appendix: The HLR $\zeta$ - $f$ Model

$$\begin{aligned} \frac{Dk}{Dt} &= \mathcal{D}_k + P_k - \alpha \varepsilon; & \frac{D\varepsilon}{Dt} &= \mathcal{D}_\varepsilon + \frac{C_{\varepsilon 1} P_k - C_{\varepsilon 2} \varepsilon}{\tau} \\ \frac{D\zeta}{Dt} &= \mathcal{D}_\zeta + f - \frac{\zeta}{k} P_k; & L^2 \nabla^2 f - f &= \frac{1}{\tau} \left( C_1 + C_2 \frac{P_k}{\varepsilon} \right) \left( \zeta - \frac{2}{3} \right) \\ \alpha &= \max \left( 1, \frac{L_{RANS}}{\Delta} \right); & L_{RANS} &= k^{3/2} / \varepsilon; \\ \Delta &= C_\Delta (\Delta V)^{1/3}; & \mathcal{D}_\phi &= \frac{\partial}{\partial x_j} \left[ \left( v + \frac{v_t}{\sigma_\phi} \right) \frac{\partial \phi}{\partial x_j} \right]; \\ v_t^{RANS} &= c_\mu \tau \zeta k; & v_t^{LES} &= (c_s \Delta)^2 \bar{S}; & v_t &= \max(v_t^{RANS}, v_t^{LES}) \\ \tau &= \max \left[ \min \left( \frac{k}{\varepsilon}, \frac{0.6}{\zeta c_\mu \sqrt{6S^2}} \right), c_\tau \left( \frac{v}{\varepsilon} \right)^{1/2} \right]; \\ L &= c_L \max \left[ \min \left( \frac{k^{3/2}}{\varepsilon}, \frac{k^{1/2}}{\zeta c_\mu \sqrt{6S^2}} \right), c_\eta \left( \frac{v^3}{\varepsilon} \right)^{1/4} \right] \end{aligned}$$

$c_\mu$	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$C_1$	$C_2$	$\sigma_k$	$\sigma_\varepsilon$	$\sigma_\zeta$	$c_\tau$	$c_\eta$	$c_L$	$c_\Delta$
0.22	$1.4(1 + 0.012/\zeta)$	1.9	0.4	0.65	1.0	1.3	1.2	6	85	0.36	1.5

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