Preface

In the analysis and synthesis of contemporary systems, engineers and scientists are frequently confronted with increasingly complex models that may simultaneously include components whose states evolve along continuous time (continuous dynamics) and discrete instants (discrete dynamics); components whose descriptions may exhibit hysteresis nonlinearities, time lags or transportation delays, lumped parameters, spatially distributed parameters, uncertainties in the parameters, and the like; and components that cannot be described by the usual classical equations (ordinary differential equations, difference equations, functional differential equations, partial differential equations, and Volterra integrodifferential equations), as in the case of discrete-event systems, logic commands, Petri nets, and the like. The qualitative analysis of systems of this type may require results for finite-dimensional systems as well as infinite-dimensional systems, continuous-time systems as well as discrete-time systems, continuous continuous-time systems as well as discontinuous continuous-time systems (DDS), and hybrid systems involving a mixture of continuous and discrete dynamics.

Other than the first edition of this book, there are no texts on stability theory that are suitable to serve as a single source for the analysis of system models of the type described above. Most existing engineering texts on stability theory address finite-dimensional systems described by ordinary differential equations, and discrete-time systems are frequently treated as analogous afterthoughts or are relegated to books on sampled-data control systems. On the other hand, books on the stability theory of infinite-dimensional dynamical systems usually focus on specific classes of systems (determined, e.g., by functional differential equations, partial differential equations, and so forth). Finally, the literature on the stability theory of discontinuous dynamical systems (DDS) is presently scattered throughout journals and conference proceedings. Consequently, to become reasonably proficient in the stability analysis of contemporary dynamical systems of the type described above may require considerable investment of time. As was the case for the first edition, the present updated version of this book aims to fill this void. To accomplish this, the book addresses again four general areas: the representation and modeling of a variety of dynamical systems of the type described above, the presentation of the Lyapunov and Lagrange stability theory for dynamical systems defined on general metric spaces, the specialization of this stability theory to finite-dimensional dynamical systems, and the specialization of this stability theory to infinite-dimensional dynamical systems. Throughout the book, the applicability of the developed theory is demonstrated by means of numerous specific examples and applications to important classes of systems.

In his groundbreaking result concerning the global asymptotic stability of an equilibrium, Lyapunov makes use of positive definite, radially unbounded, and decrescent scalar-valued functions of the system state and time (called *Lyapunov functions*) which, when evaluated along the motions of a finite-dimensional dynamical system (determined by ordinary differential equations), decrease *monotonically* with increasing time and approach zero as time approaches infinity. In this book, such functions are called *monotonic Lyapunov functions*.

Lyapunov's famous result for global asymptotic stability of an equilibrium yields sufficient conditions. Subsequently, necessary conditions for uniform asymptotic stability in the large were established as well. However, for general dynamical systems, there are no results which constitute necessary *and* sufficient conditions for the uniform asymptotic stability of an equilibrium. This points to limitations inherent in the Lyapunov results which comprise the *Direct Method of Lyapunov* (also called the *Second Method of Lyapunov*).

In recent works (concerning the qualitative analysis of discontinuous dynamical systems, including switched systems and hybrid systems), stability and boundedness results were discovered which involve the existence of Lyapunov-like functions which, when evaluated along the motions of a dynamical system, still need to approach zero as time approaches infinity; however, these functions no longer need to decrease monotonically with increasing time along the system motions. In this book, such functions are called *non-monotonic Lyapunov functions*.

As in the case of the *classical Lyapunov stability and boundedness results* (involving monotonic Lyapunov functions), the stability and boundedness results involving non-monotonic Lyapunov functions constitute sufficient conditions or necessary conditions. For general dynamical systems, no results which constitute necessary *and* sufficient conditions are known. However, it turns out that, in general, the principal stability and boundedness results involving monotonic Lyapunov functions will always reduce to corresponding results involving non-monotonic Lyapunov functions. Furthermore, for many cases, the classical Lyapunov results involving monotonic Lyapunov functions turn out to be more conservative than the corresponding results involving non-monotonic Lyapunov functions.

The first edition of this book contains many stability and boundedness results involving non-monotonic Lyapunov functions. While these results were primarily discovered in the qualitative analysis of *discontinuous dynamical systems* (including switched and hybrid systems), it was recognized that these results are applicable to *continuous dynamical systems* as well. Nevertheless, the terms "monotonic Lyapunov function" and "non-monotonic Lyapunov function" do not appear in the first edition of this book. There are two reasons for this. First, at the time of the publication of this book, the roles of the monotonic and the non-monotonic behavior of Lyapunov functions along system motions were perhaps not fully appreciated. Secondly, the development of the stability and boundedness theory involving non-monotonic Lyapunov functions was incomplete. For example, when the first edition of this book appeared, no invariance stability and boundedness results involving non-monotonic Lyapunov functions had been discovered, the stability and boundedness results for discrete-time dynamical systems involving non-monotonic Lyapunov functions had not been established, results involving multiple non-monotonic Lyapunov functions had not been developed yet, and so forth.

Once the aforementioned obstacles had been removed, a presentation of all the principal Lyapunov stability and boundedness results involving monotonic and non-monotonic Lyapunov functions was made possible. To accomplish this, altogether eleven new sections have been added to the first edition of this book. Furthermore, the entire text is replete with explanations of the roles of monotonic and non-monotonic Lyapunov functions for the various results being presented.

In developing the subject on hand, we first establish the Lyapunov and Lagrange stability results for general dynamical systems defined on metric spaces. Next, we present corresponding results for finite-dimensional dynamical systems and infinite-dimensional dynamical systems. Our presentation is very efficient because the stability and boundedness results of finite-dimensional and infinite-dimensional dynamical systems are, in many cases, direct consequences of the corresponding stability and boundedness results of general dynamical systems defined on metric spaces.

In our presentation of the various stability and boundedness results, we use a prescribed road map. First, for the case of continuous-time dynamical systems, we establish results involving non-monotonic Lyapunov functions. These results are applicable to discontinuous as well as continuous dynamical systems. Next, we present the corresponding classical Lyapunov stability and boundedness results for continuous-time dynamical systems. We prove these results by showing that whenever the hypotheses of a given classical result involving monotonic Lyapunov functions are satisfied, then the hypotheses of the corresponding result involving non-monotonic Lyapunov functions are also satisfied. We establish the stability and boundedness results for discrete-time dynamical systems involving non-monotonic and monotonic Lyapunov functions in an identical manner. Alternatively, since every discrete-time dynamical system can be associated with a discontinuous continuous-time dynamical system with identical stability and boundedness properties, we also make use of the above stability and boundedness results for continuous-time systems involving non-monotonic Lyapunov functions to establish the various Lyapunov stability and boundedness results for discrete-time systems. In addition to being very efficient in establishing the various assertions on hand, the above method of proof enables us also to conclude that the various stability and boundedness results involving monotonic Lyapunov functions will always reduce to corresponding results involving non-monotonic Lyapunov functions. Furthermore, the relationship between discontinuous continuous-time systems and discrete-time systems discussed above enables us to establish a unifying stability theory for continuous-time systems and discrete-time systems.

Given that the various classical Lyapunov stability and boundedness results always reduce to corresponding results involving non-monotonic Lyapunov functions, it is natural to ask whether or not the converse to these statements is true. To this end, we identify specific dynamical systems to show that for several important stability types, converses to the above statements are not true. For example, for a specific class of dynamical systems, we show that there *does not exist* a Lyapunov function which satisfies the hypotheses of the classical Lyapunov result for uniform asymptotic stability, while for the same dynamical system, there *does exist* a Lyapunov function which satisfies the hypotheses of the result for uniform asymptotic stability involving non-monotonic Lyapunov functions. Using this method, we prove that for several stability types, the classical Lyapunov results involving monotonic Lyapunov functions are more conservative than the corresponding results involving non-monotonic Lyapunov functions.

This book is suitable for a formal graduate course in stability theory of dynamical systems or for self-study by researchers and practitioners with an interest in systems theory in the following areas: all engineering disciplines, computer science, physics, chemistry, life sciences, and economics. It is assumed that the reader of this book has some background in linear algebra, analysis, and ordinary differential equations.

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Notre Dame, IN, USA — Anthony N. Michel
St. Cloud, MN, USA — Ling Hou
Chicago, IL, USA — Derong Liu
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