

# Contents

- 1 Control Theory and Synchronization ..... 1**
  - 1.1 The Differential-Algebraic Point of View ..... 1
  - 1.2 Differential-Geometric Point of View ..... 7
  - 1.3 Synchronization of Chaotic Systems ..... 11
    - 1.3.1 Attractors..... 11
    - 1.3.2 Synchronization..... 12
    - 1.3.3 Some Examples of Synchronization ..... 13
    - 1.3.4 Types of Synchronization ..... 15
  - 1.4 Complete Synchronization ..... 15
  - 1.5 Phase Synchronization ..... 16
  - 1.6 Lag Synchronization ..... 17
  - 1.7 Generalized Synchronization ..... 17
  - 1.8 Some Classical Chaotic Systems ..... 18
    - 1.8.1 Lorenz System ..... 18
    - 1.8.2 Rössler System ..... 19
    - 1.8.3 Chua System ..... 20
    - 1.8.4 Colpitts System ..... 21
    - 1.8.5 Rikitake System ..... 22
    - 1.8.6 Duffing System ..... 22
    - 1.8.7 Van der Pol System ..... 23
  - 1.9 Fractional-Order Systems ..... 24
    - 1.9.1 Fractional-Order Operator Block in Simulink..... 25
  - 1.10 Why Fractional Order?..... 26
  - 1.11 Fractional Circuit ..... 27
  - References..... 30

<b>2</b>	<b>A Model-Free-Based Proportional Reduced-Order Observer Design for the Synchronization of Lorenz Systems</b>	<b>33</b>
2.1	Introduction	33
2.2	Synchronization of Lorenz System	35
2.2.1	Algebraic Observability Condition	35
2.2.2	Observer Synthesis	37
2.3	Numerical Results	41
2.4	Concluding Remarks	42
	References	42
<b>3</b>	<b>A Model-Free Sliding Observer to the Synchronization Problem Using Geometric Techniques</b>	<b>43</b>
3.1	Introduction	43
3.2	Observer Canonical Form of a Nonlinear System	44
3.3	Sliding-Mode Observer to the Synchronization Problem	45
3.4	Model-Based Observers to the Synchronization Problem	50
3.4.1	Bestle–Zeitz Observer for the Synchronization Problem	50
3.4.2	Thau Observer for Synchronization	53
3.5	Two Synchronization Problems	54
3.5.1	Lorenz System	54
3.5.2	Chua's Circuit	59
3.6	Concluding Remarks	61
	References	62
<b>4</b>	<b>Experimental Synchronization by Means of Observers</b>	<b>63</b>
4.1	Introduction	63
4.2	Exponential Polynomial Observer	64
4.2.1	Problem Statement	64
4.3	Asymptotic Reduced-Order Observer	68
4.4	High-Gain Observer	69
4.5	Synchronization by Means of Observers	69
4.5.1	Experimental Results	69
4.5.2	Synchronization of the Colpitts Oscillator Employing an Exponential Polynomial Observer	71
4.6	Synchronization with a High-Gain Observer	74
4.6.1	Synchronization of the Colpitts Oscillator by Means of an Asymptotic Reduced-Order Observer	75
4.6.2	Rössler System	76
4.6.3	Rikitake Oscillator	82
4.7	Bounded Error Observer Based Design of Synchronizing Chaotic Systems	93
4.8	Numerical Results	96
4.9	Conclusion	98
	References	98

<b>5</b>	<b>Synchronization of an Uncertain Rikitake System with Parametric Estimation</b>	101
5.1	Introduction	101
5.2	Problem Statement	102
5.2.1	Rikitake Model System	102
5.2.2	Some Algebraic Properties and Problem Formulation	103
5.3	Lyapunov-Based Formulation	104
5.4	Numerical Results	107
5.5	Concluding Remarks	109
	References	110
<b>6</b>	<b>Secure Communications and Synchronization via a Sliding-Mode Observer</b>	111
6.1	Introduction	111
6.2	Chaotic Communication Based on a Sliding-Mode Observer	112
6.3	Numerical Simulation	119
6.4	Conclusions	123
	References	123
<b>7</b>	<b>Synchronization and Antisynchronization of Chaotic Systems: A Differential and Algebraic Approach</b>	125
7.1	Introduction	125
7.2	Statement of the Problem	126
7.2.1	Antisynchronization	126
7.3	Synchronization and Antisynchronization of a Colpitts Oscillator	127
7.3.1	Observer Design	129
7.4	Numerical Results	131
7.5	Concluding Remarks	133
	References	133
<b>8</b>	<b>Synchronization of Chaotic Liouvillian Systems: An Application to Chua's Oscillator</b>	135
8.1	Introduction	135
8.2	Definitions	136
8.3	Problem Formulation and Main Result	137
8.3.1	Observer Convergence Analysis	139
8.4	Numerical and Experimental Results	141
8.4.1	Numerical Results	142
8.4.2	Experimental Results	146
8.5	Concluding Remarks	149
	References	150

<b>9</b>	<b>Synchronization of Partially Unknown Nonlinear Fractional-Order Systems</b>	153
9.1	Introduction	153
9.2	On Fractional Derivatives	154
9.2.1	Mittag-Leffler-Type Function	154
9.3	Main Result	155
9.4	Numerical Example	158
9.5	Conclusions	162
	References	162
<b>10</b>	<b>Generalized Synchronization via the Differential Primitive Element</b>	163
10.1	Introduction	163
10.2	Statement of the Problem and Main Results	164
10.3	Numerical Example	168
10.3.1	Stability Analysis	170
10.3.2	Simulation Results	170
10.4	Concluding Remarks	173
	References	173
<b>11</b>	<b>Generalized Synchronization for a Class of Nondifferentially Flat and Liouvillian Chaotic Systems</b>	175
11.1	Introduction	175
11.2	Definitions	176
11.3	Problem Statement and Methodology to Generalized Synchronization (GS)	179
11.4	Generalized Synchronization of Rössler and Chua Systems	181
11.5	Concluding Remarks	185
	References	186
<b>12</b>	<b>Generalized Multisynchronization by Means of a Family of Dynamical Feedbacks</b>	187
12.1	Introduction	187
12.2	Problem Formulation and Main Results	188
12.3	Generalized Synchronization of Multiple Decoupled Systems	195
12.4	Concluding Remarks	202
	References	202
<b>13</b>	<b>Fractional Generalized Synchronization in Nonlinear Fractional-Order Systems via Dynamical Feedback</b>	203
13.1	Introduction	203
13.2	Main Result	204
13.3	Fractional Generalized Synchronization Between Chua and Rössler Systems	209
13.4	Concluding Remarks	217
	References	217

<b>14 An Observer for a Class of Incommensurate Fractional-Order Systems</b> .....	219
14.1 Introduction.....	219
14.2 Basic Concepts .....	220
14.2.1 Mittag-Leffler-Type Functions .....	220
14.3 Problem Statement and Main Result .....	221
14.4 Numerical Results .....	224
14.5 Conclusions.....	235
References.....	235
<b>Appendix</b> .....	237
<b>Index</b> .....	239

Synchronization of Integral and Fractional Order Chaotic Systems

A Differential Algebraic and Differential Geometric

Approach With Selected Applications in Real-Time

Martínez-Guerra, R.; Pérez-Pinacho, C.A.; Gómez-Cortés, G.C.

2015, XXIV, 242 p. 115 illus., 80 illus. in color.,

Hardcover

ISBN: 978-3-319-15283-7