Preface

This text is a translation of the German edition. It closely follows the original; some errors and misprints were corrected.

München and Frankfurt/Main
January 2015

Martin Brokate
Götz Kersting

Preface to the German Edition

Modern measure and integration theory is a prominent descendant of Cantor's set theory, and it played an important role for the formation of the latter. The roots of measure and integration theory thus are found in areas usually attributed to pure mathematics. Nevertheless, it has gained importance particularly for areas of mathematics strongly linked to applications—for functional analysis, partial differential equations, applied analysis and control theory, numerical mathematics, potential theory, ergodic theory, probability theory, and statistics. Measure and integration theory thus cannot be subsumed so easily under the paradigm pure *versus* applied mathematics (a paradigm which nowadays tends to become less and less persuasive anyway).

It is under this view that we have written our textbook. Indeed we have in mind readers who want to utilize the theory elsewhere and are interested in a concise exposition of the most important results. At the same time, we aim at presenting measure and integration theory as a coherent and transparent system of assertions on areas, volumes, and integrals. We think that this can be done in a compact manner so that it can be integrated into a standard bachelor's curriculum in mathematics.

From the standpoint of mathematics, the core of measure and integration theory has largely reached its final form. Nevertheless, we think that concerning its presentation, there is still room for accentuation. Our arrangement of the content does not follow the format chosen by other authors. Here are some explanations.

We do not start with the existence and uniqueness theorems for measures. We believe that such an approach better fits the needs of the students: Initially, the convergence results for integrals are important; the construction of measures—however nicely it works out following Carathéodory—may be postponed for the start. For this reason we treat these constructions only at the end of our textbook (which does not prevent a lecturer from reorganizing the material, of course). There we have opted for a presentation which directly leads to the goal, avoiding the usual discussions of set systems like algebras of sets, semi-rings, etc. At some other places, too, there are new features.

We do not intend to present the theory in all its ramifications. We concentrate on the core (as we understand it) and, beyond that, display results which provide links to other areas of mathematics. Regarding analysis, this pertains, e.g., to the smoothing of functions by convolution as well as Jacobi's transformation formula. Concerning geometric measure theory, we discuss the Hausdorff measure and dimension. For probability theory, among other things, we treat kernels and measures on infinite products following Kolmogorov. With the final two chapters, we try to exhibit some connections to functional analysis which we find useful for understanding measure and integration theory. To guide the reader, we have marked some sections with an asterisk ($*$); they may be skipped at first reading.

As a prerequisite, we assume knowledge of the contents of the first-year bachelor courses in mathematics (as they are typically given in our home country). From topology, without comment we only use elementary concepts (open, closed, compact, neighborhood, continuity) in the setting of metric spaces. Anything exceeding that, we discuss by some means or other. Historical notes are found in footnotes.

A concise text as the one we aimed at cannot substitute any comprehensive exposition. We therefore do not intend to replace established textbooks like Elstrodt's [2], much less classical texts like those of Halmos [4] or Bauer [1]. In the appendix we mention these and other introductions to the theory. From all of them, we have benefitted a lot; we take the liberty not to document this in detail, as should be permitted in a textbook. We gladly have incorporated suggestions for the text as well as corrections due to Christian Böinghoff and Henning Sulzbach. We thank Birkhäuser for the pleasant and smooth collaboration.

München and Frankfurt/Main — Martin Brokate
March 2010 — Götz Kersting

http://www.springer.com/978-3-319-15364-3

Measure and Integral
Brokate, M.; Kersting, G.
2015, VII, 172 p. 24 illus., Softcover
ISBN: 978-3-319-15364-3
A product of Birkhäuser Basel