Preface

Several years ago, I was intensively studying semilinear biharmonic elliptic equations, a topic quite far away from suspension bridges. In 2009, I was invited at a conference in Bertinoro (Italy) and I had the occasion to listen to a talk by Joe McKenna. I had already met Joe several times, including during a beautiful visit to the Corcovado in Rio de Janeiro, and I had also heard some of his talks. But it was on that occasion that I realized that a fourth order equation that he used to describe traveling waves in suspension bridges was identical to an equation that I obtained, after some change of variables, when studying radial entire solutions of semilinear biharmonic equations at critical growth, see [122]. That day I understood that some beautiful pieces of mathematics were certainly hidden into suspension bridges models.

Suddenly, I started getting interested in suspension bridges, in their history, in their mysteries. I spent some time in digging in the engineering literature and I found very interesting debates with divergent opinions leading quite naturally to open problems, of great appeal also for mathematicians. I soon discovered that problems related to structures, and therefore to nonlinear elasticity, are awfully complicated and very little information may be derived from a correct mathematical model. My personal challenge became to find reasonably simple mathematical models able to describe some of the phenomena visible in actual bridges and also able to give reliable responses to designers. This challenge had started some years earlier with the work by McKenna, followed by several other colleagues.

In this book I collected some of the historical material that I found in literature as well as the material that I produced in recent years, thanks to a wide and nice team of collaborators. Hopefully more mathematicians will find some interest in the models and in the open problems presented here, much work is still to be done, many improvements are needed on the models discussed in this monograph. And hopefully engineers will take advantage of several mathematical tools available for the study of nonlinear phenomena and, even better, use them for their future plans.

The main purpose of this book is to observe the static and dynamic behavior of suspension bridges and to try to fit them in suitable mathematical models. Several models suggested in literature are too poor both to describe with sufficient accuracy

the behavior of actual bridges and to give reliable responses. Classical mechanics tells us that **the models should be nonlinear with enough degrees of freedom** but, due to their difficulty, many tools of nonlinear analysis have been developed only in recent years. As a simple but extremely meaningful example, consider the classical Hill [141] equation

$$\ddot{y}(t) + a(t)y(t) = 0 \tag{1}$$

where a is a periodic function. For more than one century, mathematicians have sought refined criteria for the stability of the trivial solution $y \equiv 0$ of (1) but only in recent time, see e.g. [213], the stability of the trivial solution has been studied for nonlinear versions of the Hill equation. I come back to this problem with more details in Chap. 3.

Throughout the book I will discuss classical linear models and revisit them by placing suitable nonlinearities into the equations. Then I analyze the qualitative behavior of the solutions while it is not a primary scope of this book to reach exact quantitative responses. The following step should be to improve further the models and to put the correct values of all the parameters involved, in order to obtain also precise quantitative information and practical suggestions for future plans: some parameters may be determined theoretically whereas other parameters need to be determined experimentally. This step is usually called structural analysis and consists in studying the design and, for given structural geometry, materials and sizes. But this is beyond the scope of this book.

In order to shorten the exposition and to concentrate on the main core, I decided to drop all the proofs and I merely give precise references where to find most of them. Throughout the book I distinguish between Theorems (rigorous statements with all the assumptions) and Propositions (informal statements with qualitative assumptions). The parts of the text written in a smaller type are quotations taken from literature. The parts of the text written in **bold face** and entered in a minibox are statements by myself. One further remark: I give some biographical information only about the authors of historical and old contributions, the whole list of biographies being in the index named "Historical Biographies".

In the next few pages I describe in some detail the purposes and the contents of this book.

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