Preface

Time-delay systems have been intensively studied in various disciplines. On one hand, delay phenomena exist in many dynamical systems encountered in engineering, physics, chemistry, biology, and economics. For instance, examples can be found in population dynamics [63], biological systems [76], as well as engineering systems [92]. On the other hand, sometimes, complex dynamics can be well approximated by a time-delay system (see [36] and the references therein) and a high-order linear model can be approximately "reduced" to a low-order one with delays (see [47]). In other words, delays may induce some simplifications in modeling a dynamical system (less parameters to be taken into account) although the corresponding time-delay system is infinite dimensional.

It is well known that the delay considerably affects the system stability and related performances. One may tend to have some intuition that the delay always has a *negative* effect (i.e., increasing the value of delay in a system must deteriorate the system dynamics and even brings instability). This intuition does not always hold. As pointed out in the control literature, the delay may have a *positive* effect on the system dynamics. For instance, several examples in this monograph illustrate that increasing the delay properly may stabilize some unstable systems.

In this book, we will mainly focus on the analysis of the delay's effect on the stability and our objective is to find the whole stability domain with respect to the delay parameter in the case of linear systems with commensurate delays. Both retarded and neutral systems will be addressed. This problem, referred to as the *complete stability problem* for time-delay systems, has attracted a lot of attention since the 1950s, but it has not received full characterization. For an overview of the stability study of time-delay systems, one may refer to [102].

Actually, the complete stability problem is generally much more complicated than we can expect, due to the intricate spectral characteristics. *First*, a time-delay system has infinitely many characteristic roots. For this reason, time-delay systems represent a class of infinite-dimensional systems. In this context, it is important to point out that, given a delay, the unstable roots (if any) are always in finite number for a retarded time-delay system as well as a neutral time-delay system whose neutral operator is stable. By existing mathematical tools, it is impossible to

accurately detect all the infinitely many characteristic roots. *Second*, a critical imaginary root of a time-delay system has infinitely many critical delays. Thus, a thorough asymptotic behavior analysis for a general time-delay system is very difficult to achieve.

In our opinion, the current bottleneck mainly lies in the involved singularities associated with the spectra (the case without singularities can be studied by the existing methods). As a matter of fact, the complexity of the singular case (as illustrated by various examples proposed in this book) was underestimated until recently.

In order to systematically address the singularities and eventually solve the complete stability problem, a new methodology will be proposed in this book. Roughly speaking, a singular point of a time-delay system can be reformulated (from a new analytical curve perspective) such that its asymptotic behavior can be studied from the corresponding singular point of the frequency-sweeping curves associated to this time-delay system. Since such an approach covers the regular case, it is quite general. The methodology proposed in this book is called a *new frequency-sweeping framework*. It is worth mentioning that the origin of the classical frequency-sweeping method for studying the stability of time-delay systems goes back to Tsypkin in 1946 (see [114]). Further insights into such approaches and techniques will be addressed throughout this volume.

Outline of the Book

First, a new analytic curve perspective will be introduced, making the line of this book distinct from the existing ones in the literature. From this new perspective, the asymptotic behavior of the critical imaginary roots with respect to the critical delays can be systematically investigated. One of the most important results is that the asymptotic behavior of a critical imaginary root can be accurately described and studied by means of the Puiseux series. Next, we will propose to prove the *general invariance property*, to overcome the peculiarity that a critical imaginary root has infinitely many critical delays. In order to determine whether the general invariance property holds, we will improve the classical frequency-sweeping method by adopting the analytic curve perspective. We will show that the asymptotic behavior of the frequency-sweeping curves can be reflected by the dual Puiseux series. With the help of such dual Puiseux series, the frequency-sweeping curves will play an important role in studying the complete stability problem. The general invariance property will be confirmed by studying carefully the (equivalence) relation between the Puiseux series and the dual Puiseux series. As a consequence, the complete stability problem can be fully solved with the following important results.

- An explicit expression of the number of the unstable roots with respect to the delay parameter will be found. By using such a function, the analysis and design of time-delay systems may be significantly simplified.
- The ultimate stability problem (the system stability property when the delay approaches infinity) can be thoroughly studied. Moreover, all time-delay systems, according to their ultimate stability properties, may be classified into three types: Type 1: A time-delay system has infinitely many unstable roots as delay tends to infinity, Type 2: A time-delay system has a fixed number of unstable roots for all delay values (delay-independently hyperbolic system), and Type 3: A time-delay system has a fixed number of unstable roots except at the critical delays.
- A simple frequency-sweeping criterion will be presented. Using this graphical test, the asymptotic behavior analysis for the critical imaginary roots at all the positive critical delays can be fulfilled by simply observing the frequency-sweeping curves (without any calculation).

In most part of the book, the time-delay systems under consideration are of retarded type with commensurate delays. The proposed methodology will be extended to the time-delay systems of neutral type with commensurate delays, by paying attention to the additional features of the corresponding neutral operators.

The book is mainly based on the contributions of the authors in the last five years, namely [66–74]. Further extensions could be made in the future, based on the methodology proposed in this book.

Further Extensions

In our opinion, the ideas proposed here may be applied to some other problems not covered in this book. In the sequel, we list two possible directions:

From τ-decomposition to D-decomposition: If one differentiates the system parameters in two categories: delays and others, appropriate methods have been developed to handle the stability problems in frequency-domain. More precisely, the τ-decomposition [64] corresponds to the case when the delay is "free" and the other parameters are "fixed". Similarly, the *D*-decomposition [89] corresponds to the counterpart case: "free" parameters for all system parameters, except the delay which is assumed to be "fixed". This book addresses the τ-decomposition problem. Despite the differences between the two classes of problems, the methodology proposed here may bring some useful insights into the *D*-decomposition problem as it also relies on the asymptotic behavior analysis of the critical characteristic roots.

From single delay parameter to multiple delay parameters: As mentioned earlier, the time-delay systems considered in this book are supposed to have commensurate delays. This means that the problem involves in fact only one

(delay) parameter and we will see that the results to be presented are mathematically elegant. However, if the delays are not commensurate, the problem has multiple (delay) parameters and will become much more complicated (the problem is generally nondeterministic polynomial (NP)-hard [113]). Extending the results of this book to the problem with multiple incommensurate delays may not be straightforward as, to the best of the authors' knowledge, we still lack an effective mathematical tool for multiple-parameter asymptotic behavior analysis.

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