

## Chapter 2

# Theoretical Study in CWNs

In this chapter, the latest information theory based theoretical results in CWNs are introduced in detail. Compared to existing works on physical layer spectrum sensing and signal processing techniques, the cognitive information concept has been proposed to characterize the information sequence from radio environment awareness results. And the uncertainty of radio environment is depicted by novel definitions of geographic entropy and temporal entropy. Furthermore, the effectiveness of cognitive techniques which can eliminate the uncertainty of radio environment are verified by the proposed cognitive information metrics.

### 2.1 Theoretical Challenges in CWNs

Driven by technology innovations and various service requirements, new wireless network technologies with high data rate and wide spectrum band are developing rapidly. However, radio spectrum resources are facing spectrum scarcity and spectrum underutilization problems. Therefore, cognitive radio (CR) [1] has been proposed as one of the most promising technologies for the efficient spectrum utilization in recent years. With the flexible and comprehensive usage of available spectrum resources in [2] and [3], CR enables the optimal and efficient radio resource utilization. As Haykin in [1] pointed out, CR is an intelligent wireless communication system aware of its surrounding environment. Exploiting the information of radio environment with the environment awareness technologies, CR is able to learn and reconfigure itself to adapt to its environment. In this chapter, the cognitive information of radio environment by using information theoretic techniques is studied and the compression of cognitive information is proposed in CWNs.

Since the emergence of CR, the exploration and exploitation of the spectrum opportunities are extensively studied in the literatures [4]. Besides, advanced techniques such as machine learning is also applied in the network with cognitive

radio equipments to obtain the states information of primary users (PUs), which is essential for the dynamic spectrum access (DSA). However, the existing works mainly focus on the signal processing in spectrum sensing, while the characteristics and compression of cognitive information are ignored in the research of radio environment awareness technologies such as spectrum sensing. Therefore, the cognitive information concept has been proposed in this chapter to characterize the information sequence in the radio environment awareness results by using some novel information theoretic concepts, namely, cognitive information, geographic entropy and temporal entropy to reveal the features of cognitive information.

## 2.2 Information Theory Applied in Cognitive Wireless Networks

The information theories have been applied in spectrum sensing by researchers recently. The most representative literature is shown in [5] and [6], where the authors use the information theoretic criterion to detect the existence of PUs. Wang et al. in [5] applied the Akaike Information Criterion (AIC) and the minimum description length (MDL) criterion in the blind spectrum sensing. Liu et al. in [6] discovered the fact that the entropy of noise is smaller than the entropy of noise plus signal, thus the entropy based criterion can be used in spectrum sensing. Notice that the information theoretic techniques in previous literatures are mainly in the area of signal processing. The study of sensing-throughput tradeoff [7] can be regarded as the study of cognitive information. However, all these literatures do not explicitly address the behavior and characteristics of cognitive information and the information compression issues. As far as we know, the study of cognitive information is still very limited.

## 2.3 Geographic and Temporal Distribution Entropy Theory

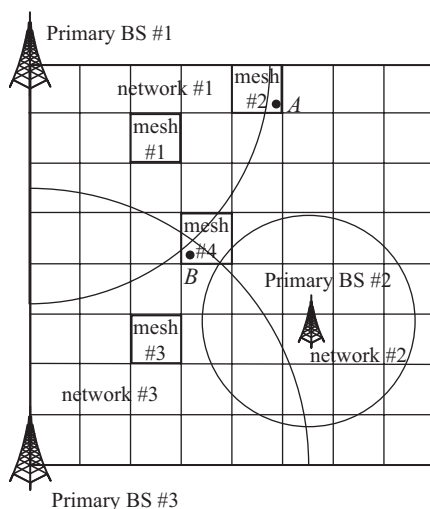
Facing the challenges of time-variant wireless channel, complex network radio resource management and a variety of user behaviors in wireless network environments, wireless networks need to implement the multi-domain environment cognition technologies, such as the spectrum sensing to discover vacant radio resources in the wireless domain, database based environment cognition technology to discover the vacant resources in the network domain and user domain. In order to provide a universal measurement for the cognitive information, information theoretic method is designed and proposed in this chapter. And cognitive information is defined as the uncertainty of the network environment that cognition technology can remove via the environment awareness techniques, such as spectrum sensing. Therefore, cognitive information is defined as the mutual information.

In cognitive wireless networks, the radio environment, also named by the wireless domain, is monitored by spectrum sensing equipments. If the radio environment is complex, the intensity of information must be high enough to eliminate the uncertainty of the radio environment. In this section, the uncertainty of radio environment is defined and studied. Besides, the notions of geographic entropy and temporal entropy are proposed to measure the uncertainty of radio environment in space and time dimensions respectively. Then, the relation between the intensity of cognitive information and the uncertainty of the radio environment is studied thereafter. In order to quantify the complexity of wireless networks coverage in different locations, the geographic entropy based Radio Environment Information (REI) and Radio Parameter Error (RPE) representation approaches are proposed respectively.

### 2.3.1 REI Representation

Cognitive radio allows the optimization of radio resource utilization by exploiting vacant spectrum resources, such as the spectrum holes, which exist in the space and time dimensions. Considering the complex radio environment information caused by the difference of wireless network distributions, the radio environment information (REI) is defined as a parameter representing the available radio resources at a specific location which can be utilized by CWNs. And the characteristics of the REI are investigated in the spatial dimension. As illustrated in Fig. 2.1, there are three primary base stations (BSs), when a cognitive radio is in the coverage of primary BSs, the resources of primary BSs are not available for this cognitive radio. However, cognitive radios can exploit the spatial spectrum holes when it is outside of the coverage of primary BSs.

**Fig. 2.1** Scenario of multiple primary networks



To represent the REI,  $R(k, x, y)$  is defined as the binary representation of network  $k$  at location  $(x, y)$  in Eq. (2.1). The radio parameter at a specific location is denoted by the sum of the binary representations for all networks in Eq. (2.2) based on the assumptions in [8, 9], where  $T$  is the number of networks. The entire region is divided into uniform small squares, which is denoted as “mesh”. Moreover, different meshes represent various coverage conditions of multiple radio access networks. The radio parameter of mesh  $i$  is denoted in Eq. (2.3), where  $p_{ij}$  is the fraction of the area in mesh  $i$  with radio parameter  $j$ ,  $\sum_{j=1}^N p_{ij} = 1$ ,  $N = 2^T$  is the number of radio parameters.

In Fig. 2.1, there are  $T = 3$  primary networks and the number of radio parameters is  $N = 2^T = 8$ . Therefore, the radio parameter in mesh 2 is 1 according to Eq. (2.3).

$$R(k, x, y) = \begin{cases} 1 & \text{if network } k \text{ is detected at } (x, y) \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

$$I(x, y) = \sum_{k=1}^T R(k, x, y) \times 2^{k-1} \quad (2.2)$$

$$P = \arg \max_j p_{ij} \quad (2.3)$$

### 2.3.2 Geographic Entropy

Based on the assumption that each mesh represents the most commonly available network information in its specific location, the radio parameter error (RPE) is defined in Eq. (2.4) and the RPE of the entire region is defined in Eq. (2.5), where  $M$  is the number of mesh and  $\alpha_i$  is the proportion area of mesh  $i$  comparing to the entire

region which satisfies  $\sum_{i=1}^M \alpha_i = 1$ . The Geographic Entropy (GE) of a mesh is defined

to quantify the uncertainty of the radio environment information, which is caused by the difference of wireless network distributions. In Fig. 2.1, the REI in mesh 1 is much more certain than that in mesh 4 in terms of the distribution of different networks. The REI in mesh 4 is more composite and complex, causing a much bigger uncertainty. Similar to the Shannon entropy [10], the geographic entropy of mesh  $i$  is denoted by  $H_i$  in Eq. (2.6). And the geographic entropy of the entire region is defined by  $H$  in Eq. (2.7). For a uniform size mesh division approach,  $\alpha_i = \frac{1}{M}$  and  $H$  are shown in Eq. (2.8).

$$p_{e,i} = 1 - \max_j p_{ij} \quad (2.4)$$

$$p_e = \sum_{i=1}^M \alpha_i p_{e,i} \quad (2.5)$$

$$H_i = - \sum_{j=1}^N p_{ij} \log p_{ij} \quad (2.6)$$

$$H = \sum_{i=1}^M \alpha_i H_i = - \sum_{i=1}^M \alpha_i \sum_{n=1}^N p_{ij} \log p_{ij} \quad (2.7)$$

$$H = \frac{1}{M} \sum_{i=1}^M H_i \quad (2.8)$$

Therefore, by using the geographic entropy concept, the uncertainty of the REI and the corresponding RPE values are quantified, which can be applied to analyze the relation between the number of mesh  $M$  and RPE. It can be applied in the design of new algorithms for the efficient environment information awareness.

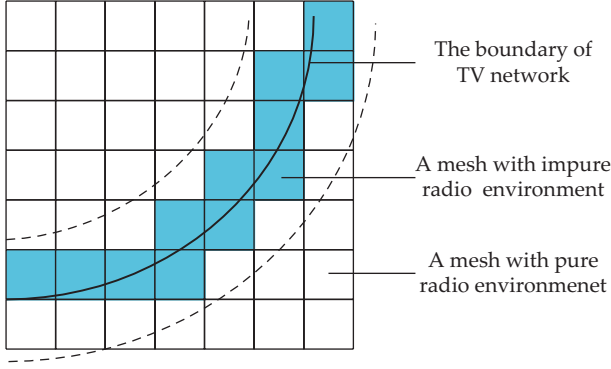
Moreover, the properties of geographic entropy and RPE are investigated, considering the relation between  $M$  and the RPE based on the proposed geographic entropy concept.

**Theorem 1** The GE of the entire region is  $O(\frac{1}{\sqrt{M}}) \rightarrow 0$ , where  $M$  is the number of meshes.

**Proof** The meshes with impure radio environment are distributed along the boundaries of primary networks. Denote the length of all these boundaries as  $\xi$ , the length of a mesh edge as  $\varepsilon$ , and the length of the entire region's edge as  $L$ . Then,  $M = (\frac{L}{\varepsilon})^2$  and the number of mesh with impure radio environment is upper bounded by  $K$  in Eq. (2.9).

$$K \leq \frac{2\xi\sqrt{2}\varepsilon}{\varepsilon^2} = \frac{2\sqrt{2}\xi}{\varepsilon} \quad (2.9)$$

As shown by the solid curve in Fig. 2.2, the result is obtained by considering the corresponding packing problem along the boundaries of detection region of TV primary networks. Moving each point on this curve in two normal directions with a distance of  $\sqrt{2}\varepsilon$ , two dash curves are achieved hereafter. The area between these



**Fig. 2.2** The boundary of detection region determines an upper bound on  $K$

two dash curves is  $2\xi\sqrt{2}\varepsilon$ . All the meshes with a pure radio environment are located between these two dash curves, so the upper bound on  $K$  is  $2\xi\sqrt{2}\varepsilon$  divided by the area of a mesh.

$$H \leq \frac{1}{M} K \log N \leq \frac{1}{M} \frac{2\sqrt{2}\xi}{\varepsilon} \log N = \frac{1}{\sqrt{M}} \frac{2\sqrt{2}\xi \log N}{\sqrt{S}} \quad (2.10)$$

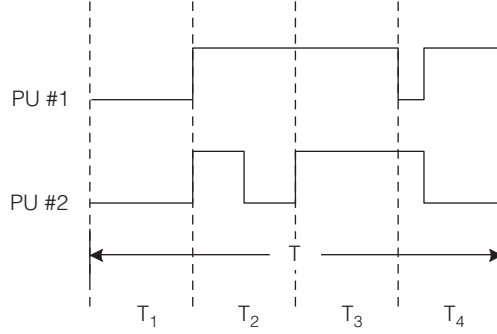
Since  $H_i \leq \log N$ , thus an upper bound of the geographic entropy is indicated in Eq. (2.10), where  $S$  is the area of the entire region. Thus  $O\left(\frac{1}{\sqrt{M}}\right) \rightarrow 0$  is an upper bound of  $H$ .

### 2.3.3 Temporal Entropy

Considering the proposed geographic entropy to quantify the uncertainty of various wireless networks distribution, the intensity of cognitive information distribution in the time dimension is defined by the cognitive information flow (CIF), which is a dynamic and ordered information sequence of the radio environment sensing results in CWNs. CIF contains the information of spectrum occupancies, namely, the states of spectrum holes. Furthermore, the information in CIF will flow to cognitive radio equipments or the database that stores the spectrum occupancy information.

The state of radio environment depicts the state of PUs (busy or idle). And the state of PUs is defined as the binary representation of spectrum occupancy at time  $t$ .

$$I(t) = \begin{cases} 1 & \text{If spectrum band is occupied} \\ 0 & \text{Otherwise} \end{cases} \quad (2.11)$$



**Fig. 2.3** Temporal entropy of two primary users

For the PUs operating on the single spectrum band,  $I(t)$  is a “0–1” random variable (r.v.). In Fig. 2.3, the spectrum occupancy of two primary users is observed during a time interval  $T$ , which is divided into  $N$  time windows as  $T_1, T_2, \dots, T_N$  ( $N=4$ ). If  $T_i = T_j, \forall i \neq j$  and the SU detects the spectrum at the beginning of time window  $T_i, \forall i$ , then  $T/N$  is the spectrum sensing period. The state of PU during time window  $T_i$  is denoted as  $X_i$  (1 or 0), which is a random variable (r.v.) and is defined in Eq. (2.12), where  $p_{ij}$  is the fraction of time in time window  $T_i$  when the state of PU is  $j \in \{0,1\}$ .

$$X_i = \arg \max_j p_{ij} \quad (2.12)$$

To capture the dynamic state of PUs, we define the temporal entropy to characterize the uncertainty of the states of spectrum occupancy. The temporal entropy of the entire interval  $T$  is defined as follows.

$$H = \frac{1}{N} H(X_1, X_2, \dots, X_N) \quad (2.13)$$

This is the form of entropy rate, i.e., the average entropy of each symbol. Since different values of  $X_i$  are independent, the temporal entropy can be simplified as follows.

$$H = \frac{1}{N} \sum_{i=1}^N H(X_i) \stackrel{(a)}{=} \frac{1}{N} \sum_{i=1}^N H_i \quad (2.14)$$

Specifically,  $H_i$  is the entropy of spectrum occupancies in time window  $T_i$ , defined as

$$H_i = - \sum_{j=1}^M p_{ij} \log p_{ij} \quad (2.15)$$

where  $M$  is the number of states. For spectrum sensing in single spectrum band,  $M=2$ .  $p_{ij}$  is the fraction of time with state  $j$  in time window  $T_i$ , which satisfies  $\sum_{j=1}^M p_{ij} = 1, \forall i$ .

The error probability of time window  $T_i$  is defined in Eq. (2.16), where  $X_i$  is defined in Eq. (2.12).

$$p_{e,i} = 1 - \max_j p_{ij} = 1 - p_{iX_i} \quad (2.16)$$

Equation (2.16) reveals that even if the spectrum sensing is perfect (no miss detection or false alarm), SU may still make mistakes.

As illustrated in Fig. 2.3, for PU #1, at the beginning of  $T_p$ , SU declares that PU #1 is idle, so SU will transmit signals during  $T_p$ , and it will interfere PU #1 during most of the remaining time, leading to another case of “miss detection”. Similarly, for PU #2, at the beginning of  $T_p$ , SU declares that PU #2 is busy, so SU does not transmit signals during the remaining time, while PU #2 is idle for most of the time, resulting in another case of “false alarm”. Thus, even though SUs detect the spectrum perfectly, other forms of miss detection and false alarm are inevitable. The error probability is used to measure this kind of mistakes. The error probability of the entire time interval is denoted in Eq. (2.17).

$$p_e = \sum_{i=1}^N \frac{T_i}{T} p_{e,i} \stackrel{(b)}{=} \frac{1}{N} \sum_{i=1}^N p_{e,i} \quad (2.17)$$

Similarly, (b) holds when the time windows are equal. As to the mathematical properties of temporal entropy, the theorem is achieved below.

**Theorem 2** The temporal entropy is  $H = O\left(\frac{1}{N}\right)$ , where  $N$  is the number of time windows and  $\lim_{N \rightarrow \infty} H = 0$ .

## 2.4 Cognitive Information Metrics

The cognitive information is defined as a metric to evaluate the uncertainty of both internal and external environments. Such uncertainty can be eliminated by other systems or nodes using cognitive awareness techniques. In CWNs,  $X(i)$  is used to represent the internal and external environment states of system A at the time instance  $i$ . The cognitive awareness result of system A's state by system B using cognitive awareness techniques is denoted by  $Y(i)$ , then the output sequence can be expressed by  $Y(1), Y(2), \dots, Y(N)$ , where  $N$  is the number of time instances. The averaged mutual information is defined in Eq. (2.18). It is defined generally as the



uncertainty of system A's states which is removed by system B in  $N$  times using the cognitive information awareness techniques. However, as the correlation between the states of two time instances is unknown, it is difficult to find a closed-form solution for Eq. (2.18). Therefore, the simplified form of Eq. (2.18) with  $N=1$  is achieved and the cognitive information is defined below.

$$\begin{aligned} & \frac{1}{N} I(X(1), X(2), \dots, X(N); Y(1), Y(2), \dots, Y(N)) \\ &= \frac{1}{N} H(X(1), X(2), \dots, X(N)) - \frac{1}{N} H(X(1), X(2), \dots, X(N) | Y(1), Y(2), \dots, Y(N)) \end{aligned} \quad (2.18)$$

**Definition 1** Cognitive information is defined as the mutual information,

$$I(X; Y) = H(X) - H(X | Y) \quad (2.19)$$

$X$  is the state of system A and  $Y$  is the inference result of  $X$  by system B using cognitive awareness techniques. Both  $X$  and  $Y$  are random variables.  $H(X)$  is the entropy of system A's states and  $H(X | Y)$  is the conditional entropy.

**Remark 1** The entropy  $H(X)$  is useful in the source coding of the system A's states, which is essential to reduce the cognitive information exchanging among different systems or nodes using cognitive information awareness techniques.

**Remark 2** The cognitive information  $I(X; Y)$  is defined as the information that one system "transmits" to another system or node, namely, the uncertainty of the states of system A that can be removed by system B using cognitive awareness techniques.

Considering the imperfect cognitive awareness in CWNs ( $H(X | Y) \neq 0$ ), the error probability exists and can be defined by Eq. (2.20).

$$p_e = \Pr\{X \neq Y\}, \quad (2.20)$$

Generally, the states of one system or node are complex and changing dynamically, including the radio, network, user and policy multi-domains. For simplicity, it is assumed that the node usually has two states, the idle state and the busy state. For example, the spectrum occupancy state is taken into account and the expression of the error probability is depicted in Eq. (2.21).

$$p_e = p_0 p_f + p_1 p_m, \quad (2.21)$$

$p_0$  is the probability that the node is in the idle state "0" and  $p_1$  is the probability that the node is in the busy state "1",  $p_f$  is the probability of false alarm and  $p_m$  is the probability of miss detection. Thus, the cognitive information is depicted in Eq. (2.22).

$$\begin{aligned}
I(X; Y) &= H(X) - H(X | Y) = H(Y) - H(Y | X) \\
&= - \sum_{i \in \{0,1\}} q_i \log q_i + \sum_{x \in \{0,1\}} \sum_{y \in \{0,1\}} p(x) p(y | x) \log p(y | x),
\end{aligned} \tag{2.22}$$

And the expressions of  $q_0, q_1$  denote the probabilities of the idle state “0” and busy state “1” of the detection results, as  $q_0 = p_0(1 - p_f) + p_1 p_m$  and  $q_1 = p_0 p_f + p_1(1 - p_m)$ .

In this section, the mathematical features of cognitive information are described in detail by using the cognitive information as a metric for energy detection, cooperative spectrum sensing and network performance evaluation.

### 2.4.1 Cognitive Information Metrics in Energy Detection

When the system A’s signal  $x(t)$  is transmitted through wireless channel with channel gain  $h(t)$ , the received signal from another system or node B using cognitive awareness techniques is  $y(t)$ . It follows a binary hypothesis:  $H_0$  (when signal  $x(t)$  is absent) and  $H_1$  (when signal  $x(t)$  exists).

$$y(t) = \begin{cases} w(t) & : H_0 \\ x(t)h(t) + w(t) & : H_1 \end{cases}, \tag{2.23}$$

And  $w(t)$  is the additive white Gaussian noise (AWGN), which is assumed to be a circularly symmetric complex Gaussian random variable with the mean zero and one-sided power spectral density  $\sigma_w^2$ , namely,  $w(t) \sim \mathcal{N}(0, \sigma_w^2)$ .

The decision statistic is  $Y = \frac{1}{N} \sum_{i=1}^N |y(i)|^2$ . With the assumption that  $x(n)$  is complex Phase-shift Keying (PSK) modulated and  $w(n)$  is circularly symmetric complex Gaussian (CSCG) noise, the probabilities of miss detection  $p_m = \Pr\{H_0 | H_1\}$  and false alarm  $p_f = \Pr\{H_1 | H_0\}$  are shown in Eqs. (2.24) and (2.25) in [11].

$$p_m(\varepsilon, N) = Q\left(\left(\gamma + 1 - \frac{\varepsilon}{\sigma_w^2}\right) \sqrt{\frac{N}{2\gamma + 1}}\right), \tag{2.24}$$

$$p_f(\varepsilon, N) = Q\left(\left(\frac{\varepsilon}{\sigma_w^2} - 1\right) \sqrt{N}\right), \tag{2.25}$$

$N$  is the number of samples,  $\gamma$  is the signal to noise ratio (SNR) at the signal receiver node B,  $\sigma_w^2$  is the power spectral density of AWGN and  $\varepsilon$  is the decision



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