

Preface

This is the third volume of the series of books of problems in C_p -theory entitled “A C_p -Theory Problem Book”, i.e., this book is a continuation of the two volumes subtitled *Topological and Function Spaces* and *Special Features of Function Spaces*. The series was conceived as an introduction to C_p -theory with the hope that each volume could also be used as a reference guide for specialists.

The first volume provides a self-contained introduction to general topology and C_p -theory and contains some highly non-trivial state-of-the-art results. For example, Section 1.4 presents Shapirovsky’s theorem on the existence of a point-countable π -base in any compact space of countable tightness and Section 1.5 brings the reader to the frontier of the modern knowledge about realcompactness in the context of function spaces.

The second volume covers a wide variety of topics in C_p -theory and general topology at the professional level, bringing the reader to the frontiers of modern research. It presents, among other things, a self-contained introduction to Advanced Set Theory and Descriptive Set Theory, providing a basis for working with most popular axioms independent of ZFC.

This present volume basically deals with compactness and its generalizations in the context of function spaces. It continues dealing with topology and C_p -theory at a professional level. The main objective is to develop from scratch the theory of compact spaces most used in Functional Analysis, i.e., Corson compacta, Eberlein compacta, and Gul’ko compacta.

In Section 1.1 of Chapter 1, we build up the necessary background presenting the basic results on spaces $C_p(X)$ when X has a compact-like property. In this section, the reader will find the classical theorem of Grothendieck, a very deep theorem of Reznichenko on ω -monolithity, under $\text{MA} + \neg\text{CH}$, of a compact space X if $C_p(X)$ is Lindelöf, as well as the results of Okunev and Tamano on non-productivity of the Lindelöf property in spaces $C_p(X)$.

The main material of this volume is placed in Sections 1.2–1.4 of Chapter 1. Here we undertake a reasonably complete and up-to-date development of the theory of Corson, Gul’ko, and Eberlein compacta. Section 1.5 develops the theory of splittable

spaces and gives far-reaching applications of extension operators in both C_p -theory and general topology.

We use all topological methods developed in the first two volumes, so we refer to their problems and solutions when necessary. Of course, the author did his best to keep *every* solution as independent as possible, so a short argument could be repeated several times in different places.

The author wants to emphasize that if a postgraduate student mastered the material of the first two volumes, it will be more than sufficient to understand every problem and solution of this book. However, for a concrete topic much less might be needed. Finally, let me outline some points which show the potential usefulness of the present work.

- the only background needed is some knowledge of set theory and real numbers; any reasonable course in calculus covers everything needed to understand this book;
- the student can learn all of general topology required without recurring to any textbook or papers; the amount of general topology is strictly minimal and is presented in such a way that the student works with the spaces $C_p(X)$ from the very beginning;
- what is said in the previous paragraph is true as well if a mathematician working outside of topology (in functional analysis, for example) wants to use results or methods of C_p -theory; he (or she) will find them easily in a concentrated form or with full proofs if there is such a need;
- the material we present here is up to date and brings the reader to the frontier of knowledge in a reasonable number of important areas of C_p -theory;
- this book seems to be the first self-contained introduction to C_p -theory. Although there is an excellent textbook written by Arhangel'skii (1992a), it heavily depends on the reader's good knowledge of general topology.

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