Chapter 2
Fundamental Concepts in Multibody Dynamics

Abstract In this chapter, the fundamental ingredients related to formulation of the equations of motion for multibody systems are described. In particular, aspects such as degrees of freedom, types of coordinates, basic kinematics joints and types of analysis in multibody systems are briefly characterized. Illustrative examples of application are also presented to better clarify the fundamental issues for spatial rigid multibody systems, which are of crucial importance in the formulation development of mathematical models of mechanical systems, as well as its computational implementation.

Keywords Degrees of freedom · Types of coordinates · Kinematic joints

Prior to establish the equations of motion that govern the dynamic behavior of multibody systems, it is first necessary to select the manner how to describe them. The description variables must be able to characterize, at any instant of time, the configuration of the system, that is, the position of all the material points that compose the bodies. The description variables, also called generalized coordinates, must uniquely define the position of the system components at any instant of time during the multibody system analysis. The expression generalized coordinates is employed to include both linear and angular coordinates (Huston 1990).

The minimum number of variables necessary to fully describe the configuration of a system is denominated as degrees of freedom (DoF) of the system, or simply mobility (Müller 2009). When the configuration of a multibody system is completely defined by the orientation of one of its bodies, the system is said to have one degree of freedom. The number of degrees of freedom can also be defined as the number of independent generalized coordinates required to uniquely describe the configuration of a system. It is evident that the knowledge of the number of degrees of freedom is of prime importance in the processes of modeling and analysis of multibody systems. It is known that for the spatial case, each body has six degrees of freedom. Introducing a kinematic joint to a system, the total number of DoF will be reduced by the number of constraints imposed by the joint. It is clear that the number of constraints depends on the number and type of joint applied to the system, where the constraints must be independent from one another. The number

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of degrees of freedom of a multibody system can be evaluated as the difference between the system coordinates and the number of independent constraints. The mathematical expression that summarizes this idea is known as the Grüebler-Kutzback criterion and is written as (Shigley and Uicker 1995)

$$n_{DoF} = 6n_b - m \tag{2.1}$$

where n_b represents the number of bodies that compose the multibody system and m is the number of independent constraints. For example, the spatial four-bar mechanism illustrated in Fig. 2.1 has six spherical joint constraints and ten revolute joint constraints, yielding two degrees of freedom.

Determining the number of degrees of freedom of a multibody system is mostly the first step in analyzing mechanical system, which typically consist of several bodies interconnected by different types of joints and force elements. When the number of degrees of freedom is negative, it denotes an over constrained non solvable mechanism. Zero or null degrees of freedom represents a structure, that is, a nonmovable system. Finally, when a multibody system has a positive number of degrees of freedom, it indicates a resolvable mechanism.

It is not unanimous and it is not a simple task either to define a criterion to classify the different types of coordinates that can be used to describe the configuration of multibody systems. A general and broad embracing rule to group the generalized coordinates is to divide them into independent and dependent coordinates (Wehage

Fig. 2.1 Spatial four-bar linkage with two degrees of freedom

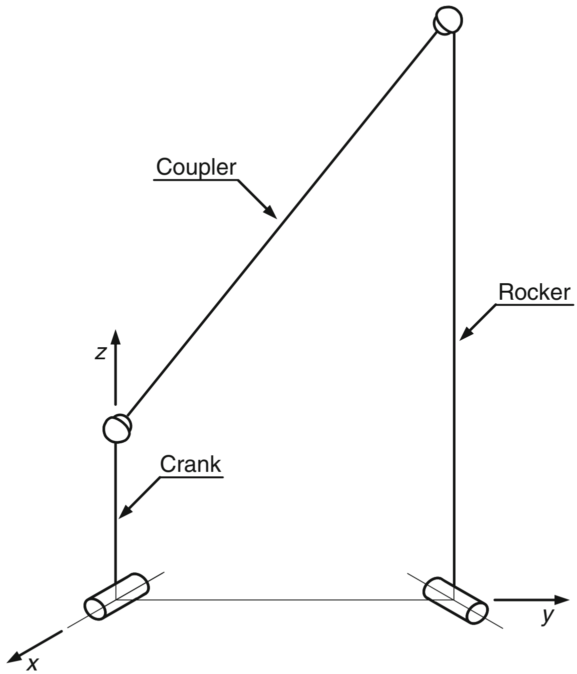

Fig. 2.2 Types of coordinates frequently used in multibody systems formulation

Types of coordinates
- Independent coordinates
- Dependent coordinates
  - Absolute coordinates
  - Relative coordinates
  - Natural coordinates

and Haug 1982). The independent coordinates are free to vary arbitrarily, while the dependent coordinates are required to satisfy the equations of constraints. Additionally, the dependent coordinates are classified as absolute coordinates (Orlandea et al. 1977), relative coordinates (Chace 1967) and natural coordinates (Jalón and Bayo 1994). Figure 2.2 summarizes the different types of coordinates most frequently used to describe the configuration of multibody systems.

In general, there are different manners of describing the configuration of a multibody system. In other words, there are many types of coordinates that can be helpful in the formulation of the equations of motion for multibody systems. The dilemma of selection of the type of coordinates to be used depends on the type of problem to be analyzed. In fact, the choice of the most appropriate set of coordinates is not indifferent, being a tradeoff between the advantages and drawbacks associated with each type of coordinates. A valuable comparison of the main types of coordinates are presented and discussed by Nikravesh (1988), Shabana (1989) and Jalón and Bayo (1994), where the pros and cons of each type of coordinates are highlighted. In particular, Shabana (1989) called attention for the selection of the most adequate type of coordinates to be used when modeling flexible multibody systems, which is a much more relevant task.

It is known that the degrees of freedom in a multibody system are directly related to the types of kinematic constraints considered, namely, those associated with kinematic joints. Furthermore, each type of joint allows for certain relative motions between adjacent bodies and constrains others. Figure 2.3 illustrates four of the most basic and frequently used kinematic joints when modeling multibody systems,

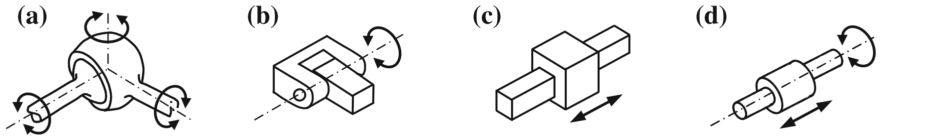

Fig. 2.3 Basic kinematic joints used in multibody systems: **a** spherical joint; **b** revolute joint; **c** translational joint; **d** cylindrical joint

in which their denomination, and relative degrees of freedom permitted are also represented (Reuleaux 1963).

Multibody systems methodologies include the main two phases: (*i*) development of mathematical models of multibody systems and (*ii*) implementation of computational procedures to perform the simulation, analysis and optimization of the global motion produced. Modeling or formulation is the process of generating and assembling the necessary equations of motion, when solved, would reveal the behavior of a multibody system. In a simple manner, there are two modeling approaches that can be considered, namely the point coordinates formulation and the body coordinates formulation. Broadly, it can be said that in the point coordinates formulation, the coordinates represent the joints and the constraints represent the bodies, whereas in the body coordinates formulation, the coordinates represent the bodies and the constraint represent the joints (Nikravesh 2008).

By describing the geometric configuration of a system with point coordinates formulation, the multibody system is represented as a multiple particle system. This collection of interconnected points usually stands for the joints in the system. Then, each point is assigned to a set of coordinates for which the kinematic constraints are constructed. Thus, the number of constraint equations only depends on the number and type of joint applied to the system. Using this formulation, the coordinates represent the joints and the applied constraints represent the bodies. Although this formulation could be realized in a computer program, the coordinates are not associated with the bodies. The analysis of a multibody system can be more convenient if the governing equations are solved for coordinates which correspond to the bodies directly.

The body coordinates formulation is a systematic approach to obtain the equations of motion for multibody systems based on the Newton-Euler equations. While other formulations describe the equations of motion in terms of generalized coordinates and generalized velocities, this formulation includes all coordinates and velocities of the involved bodies, which are expressed as the absolute coordinates and velocities. The resulting number of equations is large compared with other methods and, therefore, inappropriate for solving by hand. However, the equations are rather simple, although nonlinear, versatile and very suitable for the implementation in a computational program.

In a broad sense, the analysis of mechanical systems may be performed statically or dynamically. While statics denotes the study of stationary systems, i.e., time invariant systems, dynamics deals with the study of moving systems, i.e., systems whose behavior is time dependent. Furthermore, the branch of dynamics can be divided into two main disciplines, namely the kinematics and kinetics. In the kinematic analysis, the geometric aspects of motion are considered independently of forces that produce the motion. More precisely, kinematics deals with the study of the displacement, velocity and acceleration. In turn, kinetics is the study of the motion characteristics and the relation to the forces that produce the motion. Unlike the case of static and kinematic analysis, where only algebraic equations are utilized, in the kinetic analysis, the motion of a mechanical system is described by second-order differential equations (Nikravesh 1988).

It is very common to refer kinetic analysis as dynamic analysis because kinetic analysis must be based on the knowledge of the kinematic analysis of a system as well. Therefore, in this work, the term dynamic will be used instead of kinetic. In studying the dynamics of a mechanical system, there are two different types of analysis that can be performed, namely forward dynamics and inverse dynamics. In the forward dynamic analysis, the external forces acting on the bodies of a system are known and the resulting motion is obtained by solving the equations of motion. On the other hand, in the inverse dynamic analysis, a specific motion for a multibody system is sought and the objective is to determine the forces that are required to produce such a motion. In the context of the present work, methods of kinematics and forward dynamics are employed.

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