Preface

Patterns have become a common theme in many fields of academic study. In programming, in particular, the book “Design Patterns” has become highly influential and it is now customary to discuss programs in terms of the patterns used. The programmers generally attribute the idea of a design pattern to architecture. The fundamental idea is that each field has a collection of ways of breaking down problems into component pieces. Understanding these methodologies explicitly then leads to greater comprehension, facilitates learning and simplifies problem solving. Rather than attempting a problem cold, one first sees whether known patterns work. Even if they all fail, understanding why they fail defines the problem.

In this book, my objective is to identify and teach many of the common patterns that arise in pure mathematics. I call these “Proof Patterns”. The main originality in the presentation is that examples are focussed about each pattern and drawn from different areas. This differs from the usual style of teaching pure mathematics where a topic is chosen and dissected; patterns are then drawn in as needed, and they are often not explicitly mentioned. After studying enough topics the learner picks up a variety of patterns, and the difficulty of studying a new area is often determined by the degree of unfamiliarity with its patterns.

This book is intended to do a variety of things. On one level, my objective is to teach the basic patterns. On another, it is intended as a taster for pure mathematics. The reader will gain a little knowledge on a variety of topics and hopefully learn a little about what pure mathematics is. On a third level, the intention of the book is to make a case for the explicit recognition of patterns when teaching pure mathematics. On a fourth level, it is simply an enjoyable romp through topics I love.

One powerful tool of pure mathematics which I intentionally avoid is that of abstraction. I believe that patterns and concepts are best learnt via the study of concrete objects wherever possible. Whilst one must go abstract eventually to obtain the full power and generality of results, a proof or pattern that has already been understood in a concrete setting is much easier to comprehend and apply.

A side effect of this avoidance is that the patterns are not formally defined since such definitions would require a great deal of abstraction.

The target reader of this book will already be familiar with the concept of proof but need not know much more. So whilst I assume very few results from pure mathematics, the reader who does not know what a proof is will struggle. There are several excellent texts such as Eccles, Velleman and Houston for such readers to study before reading here. In particular, I regard this book as a second book on proof, and my hope is that the reader will find that the approach here eases their study of many areas of pure mathematics.

I try to build up everything from the ground up as much as possible. I therefore try to avoid the "pull a big theorem out of the hat" style of mathematics presentation. The emphasis is much more on how to prove results rather than on trying to impress with theorems whose proofs are far beyond the book's scope. I do occasionally use concepts from analysis before these are formally defined such as a convergent sequence. Hopefully, the reader who has not studied analysis will be able to work with their intuitive notions of these objects.

Inevitably, as with many introductory books on proof, many examples are drawn from combinatorics and elementary number theory. This reflects the fact that these areas require fewer prerequisites than most and so patterns can be discussed in simple settings. However, I also draw on a variety of areas including group theory, linear algebra, computer science, analysis, topology, Euclidean geometry, and set theory to emphasize patterns' universality.

There is little if any originality in the mathematical results in this book: the objective was to provide a different presentation rather than new results. We look at the "Four-Colour problem" at various points. Our treatment is very much inspired by Robin Wilson's excellent book "Four colours suffice" and I recommend it to any reader whose interest has been piqued. A book with some similarities to this one but requiring a little more knowledge from the reader is "Proofs from the BOOK" by Aigner and Ziegler. The emphasis there is more on beauty in proof than on patterns and it is a good follow on for the reader who wants more. However, I do hope that any reader of this book will develop some appreciation for the beauty of mathematics.

Many of the patterns in this book have not been named before although they are in widespread use. I have therefore invented their names. I hope that these new names will prove popular. I apologise to those who dislike them.

For the reader who has forgotten or never knew mathematical terminology, I have included a glossary in Appendix A. This also includes definitions of the standard sets of numbers. For clarity, let me say right here that in this book 0 is a natural number. This is the way I was taught as an undergraduate and it is too firmly embedded in my psyche for me to use any other definition. The term *counting numbers* denoted $\mathbb{N}_1$ will be used for the natural numbers excluding zero.

This book is ultimately an expression of my philosophy of how to approach the teaching of mathematics. My views have been shaped by interactions with

innumerable former teachers, students and colleagues and I thank them all. I particularly thank Alan Beardon and Navin Ranasinghe for their detailed comments on a former version of the text. I also thank some anonymous referees for their constructive comments.

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