Chapter 2
Lifetime Warranty: Policies and Cost Models

Abstract Lifetime warranties are relatively new concept. These types of warranties are becoming more and more popular as these provide assurance for a longer reliable service life, protection of customers against poor quality and the potential high cost of failure occurring during the long uncertain life of products. Formulation of attractive policies and cost models for these warranties is important to the manufacturer/dealer for estimating future costs to build it into the sales price. Modelling failures during the warranty period and the costs for such policies are complex since the useful life of products are uncertain and are not defined well and it is often difficult to tell about life measures for the longer period of coverage due to usage pattern/maintenance activities undertaken and uncertainties of costs over the period. This chapter focuses on defining lifetime, developing lifetime warranty policies and models for predicting failures and estimating costs for lifetime warranty policies. In this article, stochastic models for different lifetime warranty policies have been developed and analysed for the cost of these warranty policies when offered with sale of new product. The developed models are analysed by considering products with time dependent failure mode.

Keywords Lifetime warranty · Lifetime definition · Policies · Cost models

2.1 Introduction

One of the most attractive and widely used long term warranty policies is lifetime warranty. Lifetime warranty is relatively new concept and is becoming more popular as this type of warranty provides assurance for a longer reliable service life, protection of customers against poor quality and the potential high cost of failure occurring during the long uncertain life of products as well as a better customer peace of mind (Rahman and Chattopadhyay 2006). Currently, a large number of products are being sold with lifetime warranty policies. However, Lifetime is not defined clearly in many cases as the term lifetime is itself ambiguous. Different

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manufacturers use different definitions for lifetime of their products while offering a lifetime warranty such as:

'Sealevel Systems Inc. (http://www.sealevel.com/about.asp)' warrants their I/O products to conform and perform in accordance with published technical specifications to be free from defects due to materials and workmanship for the lifetime. "Lifetime" is defined as seven years after Sealevel discontinues manufacturing the product. The warranty period would, therefore, be technically around ten years from the date of purchase.

Celestron warrants their binocular to be free from the defects of materials and workmanship for its usable lifetime (http://www.celestron.com).

SOG Corporation Limited, a manufacturer of tools and blades provides warranty for their product as long as the customer uses the products for his/her own purpose and the coverage terminates with resell or transfer of the product (http://www.sogknives.com).

These examples shows that the measurement of a product's useful life (lifetime) varies from product to product and depends on the manufacturer's own perception. There is a need to develop a framework for lifetime and new policies useful for industries and also to develop mathematical models for estimating costs of such warranties for understanding future costs to build it into the product price. Estimation and analysis of costs for these policies is complex because of factors such as uncertainty of product useful life, cost of component parts, labour, downtime and ever increasing introduction of new products into the market. This chapter examines the development of lifetime warranty policies and models for estimating costs associated with these policies. The developed models are analysed by considering products with time dependent failure mode which reveals that product failure is only a function of its age.

The outline of the article is as follows. Section 2.1 provides an introduction of lifetime warranties. In Sect. 2.2, the roles and concept of lifetime warranty is discussed. In Sect. 2.3, a framework for long-term warranty policies is developed and discussed. Taxonomy for lifetime warranty policies is outlined in Sect. 2.4. Mathematical models are developed for predicting failures and estimating costs for different lifetime warranty policies in Sect. 2.5. The contribution of this research and scope for future research is discussed in the final Sect. 2.6.

2.2 Lifetime Warranty: Role and Concept

Lifetime warranties are covered for the life defined in the warranty policy and in general, it can be defined as the manufacturer/dealer's commitment to provide free or cost sharing repair or replacement of the sold product in case of failure due to design, manufacturing defects or quality problems which occur during the lifetime (useful life) of the product or the customer's ownership period.

Lifetime is defined in different ways by various providers of lifetime warranties. For example, a warrantor of an auto muffler may intend "lifetime" to be for the life of the car. In this case, the muffler warranty would be transferable to subsequent owners of the car and would remain in effect throughout the car's useful life. However, in most of the cases, a lifetime warranty can cover as long as the original purchaser of the product owns it or, lifetime can be as long as the original product survives. This is probably the least common usage of the term.

Termination of such warranty may arise due to technological obsolescence, design modifications, or change of component. For example, the lifetime warranty of a picture tube will cease if the TV unit is found to be out of service due to the failure of other critical components because of wear out or accident (Wells 1985) or the whole system is declared obsolete due to technological transformation such as Black and White televisions being replaced by Colour Televisions (in 1980s) and by the LCD and Plasma Televisions (in 2000s).

Therefore the useful lifetime can be defined as the lifetime of the product in the market and can be assumed to be terminated in some finite, random time horizon. Outdated technology is not covered by lifetime warranty. Therefore, the life term can be defined in any of the following ways (Chattopadhyay and Rahman 2008):

- **Technical life/Physical life**—the period over which the product might be expected to last physically (up to the period when replacement or major rehabilitation is physically required).
- **Technological life**—the period until technological obsolescence dictating replacement due to the development of a technologically superior alternative.
- **Commercial life/Economic life**—the period over which the need for the product exists, the period until economic obsolescence dictates replacement with an economic alternative.
- **Ownership life/Social and legal life**—the period until customer desire or legal ownership is retained or replacement change of ownership occurs.

However, a reasonable assumption is that it does not occur before the statutory warranty period and it does not exceed the Technical or Physical life of the product. Changes of ownership data for products like cars can be available from transport department data registering the resale of cars. Other categories are not easily predictable and can be estimated based on the time interval of new models/technologies and cost disadvantage of operating a product for a longer period when new generation products are more competitive in terms of operating cost. The distribution of ownership change can be modelled by using the resale age of new products.

The terms (e.g., coverage) can vary from item to item and can depend on the customer's and manufacturer's risk preferences. Taxonomy of lifetime warranty policies is presented in the following section.

2.3 Taxonomy for Lifetime Warranty Policies

Taxonomy for the different types of lifetime warranty policies is shown in Fig. 2.1. Lifetime warranty policies can be divided into three main groups. These are: Group A, Free Rectification Lifetime Warranty (FRLTW) policies, Group B, Cost Sharing Lifetime Warranty (CSLTW) policies, and Group C, Trade in Lifetime Warranty (TLTW) policies. FRLTW and CSLTW are again divided into two sub-groups based

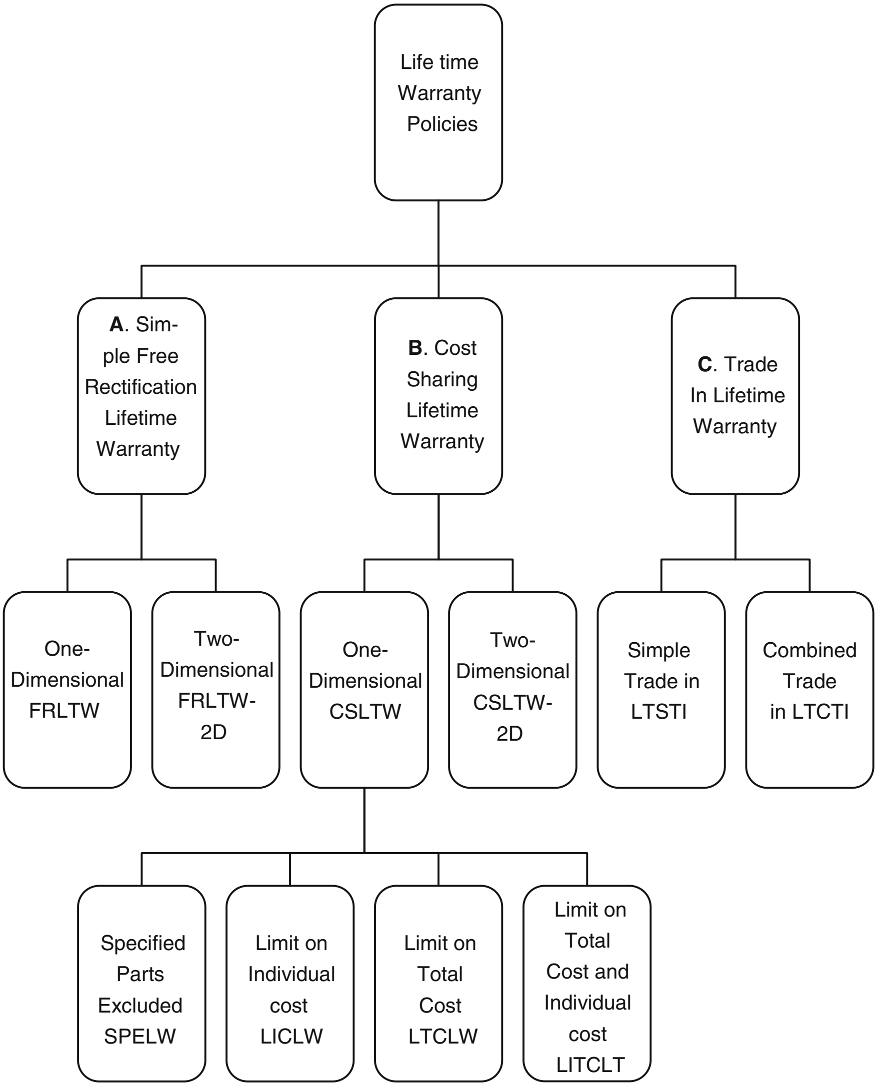

Fig. 2.1 Taxonomy for lifetime warranty policies

on the dimensions of the coverage i.e. a one dimensional policy covers only the age of the product whereas for two-dimensional policy it considers more than one dimension such as the age and usage of the sold product. Any failures within the lifetime age or lifetime usage are rectified according to the terms of the warranty policy. Here, this article confines reader's attention to one-dimensional policies and items sold individually. Group C 'Trade in' policies are divided into two groups. These are 'Simple Trade in' policies over the lifetime and 'Combined Trade-in' policies.

2.3.1 Lifetime Warranty Policies

The main complexity in this area is the uncertainties with useful life (lifetime) and subsequently the coverage periods. Another complexity is the uncertainty of servicing costs over longer uncertain periods of time. A brief description of all these policies is as follows:

Free Rectification Lifetime Warranty (FRLTW) Policies
Under these policies the manufacturer/dealers takes the responsibility to rectify any legal claim, free of cost to the customer throughout the lifetime of the product

Policy 1: *Lifetime Warranty on Age (One-Dimensional Case) with no Cost to Customer (FRLTW)*

Under this policy the manufacture/dealer takes the responsibility to rectify all defects and failures of the sold product due to design or manufacturing problems over the useful life or the defined lifetime of the product. Rectification can be a replacement, repair or in some cases refund. Unlike normal warranty, the coverage period for a lifetime warranty is uncertain and randomly variable.

Policy 2: *Lifetime Warranty on Both Age and Usage (Two-Dimensional Case) with no Cost to Customer (FRLTW-2D)*

Under this policy the manufacture/dealer rectifies all defects and failures of the sold product due to design or manufacturing problems over the lifelong age and lifelong usage of the product whichever comes first. Rectification can be a replacement or repair or in some cases a refund can be made. Here the coverage terminates at any age or usage due to the ownership change, technological obsolescence, technical or commercial reason.

Cost Sharing Lifetime Warranty (CSLTW) Policies
Under this policy, the customer and the manufacturer/dealer share the repair cost over the uncertain coverage period. The basis for the sharing can vary as indicated below. In line with [9], we propose four One-Dimensional Cost Sharing Lifetime Warranty (CSLTW) policies. These are: Specific Parts Exclusion (SPELTW), Limit

on Individual Cost (LICLTW), Limits on Total Cost (LTCLTW), and Limit on Individual and Total Cost Lifetime Warranty (LITLTW). These policies are described briefly as follows:

Policy 3: *Specified Parts Excluded Lifetime Warranty (SPELTW)*

Under this policy, the components of the product are grouped into two disjoint sets, Set-I (for inclusion) and Set-E (for exclusion). Here, the manufacture/dealer rectifies failed components belonging to Set-I at no cost to the buyer over the defined lifetime of the product. The costs of rectifying the failed components belonging to Set-E are borne by the customer. (Note: The rectification of failed components belonging to set E can be carried out either by the dealer or a third party).

Policy 4: *Limit on Individual Cost Lifetime Warranty (LICLTW)*

Under this policy, if the cost of a rectification on each occasion is below the limit c_I, then it is borne completely by the manufacturer/dealer and the customer pays nothing. If the cost of a rectification exceeds c_I, the buyer pays all the costs in excess of c_I (i.e. Cost of rectification—c_I). This continues until the termination of lifetime.

Policy 5: *Limit on Total Cost Lifetime Warranty (LTCLTW)*

Under this policy the manufacturer/dealer's obligation ceases when the total repair cost over the lifetime exceeds c_T. As a result the warranty ceases at an uncertain lifetime L or earlier if the total repair cost, at any time during the lifetime, exceeds a prefixed cut off cost c_T. Here, the warranty coverage is uncertain not only for uncertainty in exceeding total cost limit but also for the uncertainty of lifetime.

Policy 6: *Limit on Individual and Total Cost Lifetime Warranty (LITLTW)*

Under this policy, the cost to the manufacturer/dealer has an upper limit (c_I) for each rectification and the warranty ceases when the total cost to the dealer (subsequent to the sale) exceeds a cut off cost c_T or the termination of the product life due to the defined reasons, whichever occurs first. The customer pays the difference between rectification cost and the dealer's cost if the individual rectification cost exceeds cost limit c_I.

Trade in with Lifetime Policies

The two main types of Trade in with lifetime warranty policies are as follows

Policy 7: *Simple Trade in Lifetime Warranty (STLTW)*

Under this policy, the customer has an option to get a replacement at a reduced cost of trade-in for the used one. In this type of warranty the old used product is repurchased by the manufacturer/dealers. The repurchased price would be a proportion of the original purchased price depending on the age of the old/used product i.e. the repurchased price (trade in price) $P_t = P_0 \frac{a}{E(L)}$, where P_0 and a are the

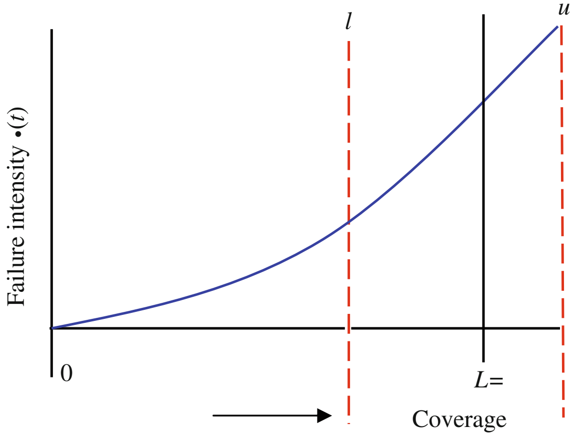

Fig. 2.2 Failure intensity over the lifetime warranty coverage period

original purchased price and age and condition of the product at the time of trade in. $E(L)$ is the expected lifetime of the product. And this can in real life be negotiable.

Policy 8: *Combined Trade in with Lifetime Warranty (CTLTW) policy*

Under this policy the failed or defective product is rectified free of cost to the customer up to a certain time w and if the product fails any time beyond w over the rest of the lifetime (L) the failed product is repurchased by the manufacturer/dealer at a reduced price (see Fig. 2.2). Clearly, this policy coverage time is divided into two terms. These are: (1) up to w, the warranty condition is similar to that of the normal free rectification warranty and (2) from the age w to defined lifetime, L the warranty terms and condition follows the simple lifetime trade-in policy.

2.4 Modelling Cost for Lifetime Warranty Policies

Most real life products are treated as a system comprising of several components and failures are modelled at the system or sub-system level.

It is assumed the product cumulative failure distribution $F(t)$ with density function $f(t) = dF(t)/dt$ and the failure intensity function (Λ)

$$\Lambda(t) = \frac{f(t)}{(1 - F(t))} \tag{2.1}$$

2.4.1 Assumptions

- Item failures are statistically independent.
- Item failure, in a probabilistic sense, is only a function of its age.

- The time to carry out a rectification action by repair or replacement is negligible compared to the mean time between failures and this time is ignored.
- An item failure results in an immediate claim and all claims are valid.

2.4.2 Notations

We use the following notations for the purpose of Chap. 4

α	Increasing rate of cost due to inflation and other factors
β	Shape parameter
δ	Discount rate (annuity)
η	Product characteristic life parameter
λ	Inverse characteristic parameter
ρ	Parameter for the truncated exponential distribution used in the life distribution of products
μ	Mean of the failure distribution
λ	Failure intensity
$\lambda(t)$	Intensity function for system failure
L	Product lifetime
N_i	Year i, where, $i = 1, 2, 3,\ldots,L$
a	Lifetime coverage
$H(a)$	Distribution function of the lifetime (useful life)
$h(a)$	Density function associated with $H(a)$
l	Lower limit of the defined lifetime
u	Upper limit of the defined lifetime
$\bar{c}$	Expected cost of each rectification over the lifetime (system level)
γ	Parameter for cost distribution

2.4.3 Modelling Lifetime

The next step is to model the uncertain useful life of the product. The upper limit of the coverage period is uncertain since the termination of life is random and unknown (see Fig. 2.2). Conditioned on the coverage $L = a$, one can capture this uncertainty by binding lower limit of lifetime by l and upper limit by u. It means warranty of the product can be terminated at any time between l and u which is unlike fixed term warranty offer. One can model this as a random variable with a distribution function $H(a)$ with $H(l) = P(a \leq l) = 0$ and $H(u) = P(a \leq u) = 1$

$h(a)$ is the probability density function of coverage period a associated with $H(a)$ and

$$h(a) = \frac{dH(a)}{da} \tag{2.2}$$

One form of $H(a)$, is truncated exponential distribution (see Chattopadhyay and Murthy 2000).

$$H(a) \text{ is } \frac{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho a}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} \text{ which gives a } h(a) = \frac{\rho \mathrm{e}^{-\rho a}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} \tag{2.3}$$

The mean value of useful life of the sold product can be expressed by

$$\mu_L = E(a) = \frac{\left(l\mathrm{e}^{-\rho l} - u\mathrm{e}^{-\rho u}\right) + \left(\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}\right)/\rho}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} \tag{2.4}$$

ρ is parameter for the truncated exponential distribution used in the life distribution of products. In real life distribution of lifetime coverage might not be possible to model using a particular distribution and can be modelled using a probability mass function.

2.4.4 Modelling Repair Cost

The cost of each repair C is, in general, a random variable because an item failure is due to the failure of one or more of its components and cost of repair or replacement varies with components. This cost can be modelled as the same over the period of life (an assumption and valid for simple product consists of few components) and can be represented by a distribution function $G(c)$, i.e.

$$G(c) = P\{C \le c\} \tag{2.5}$$

Then the expected cost of each rectification action, $E(C) = \bar{c}$, is given by

$$E(C) = \bar{c} = \int_0^\infty cg(c)dc \tag{2.6}$$

If $G(c)$ is considered to be an exponential distribution with exponential parameter γ so that

$$G(c) = 1 - \mathrm{e}^{-\gamma c} \tag{2.7}$$

Then the expected cost of each rectification with cost parameter γ is given by

$$\bar{c} = [1/\gamma] \tag{2.8}$$

Moreover the cost can vary subsequently in the longer uncertain period of contract and a negotiation clause can be included. For a very long product life, a discount factor can be added to the cost model.

2.4.5 Modelling Costs for Free Rectification Lifetime Warranty (FRLTW)

For products with longer lifetimes such as a car, the total warranty cost is uncertain over longer period of time due to the uncertainties of lifetime and the costs of servicing claims. If the lifetime is more than one year, the future cost will be affected by increased labour cost, inflation and devaluation of money over time.

Let the life of the product be L years. This L is itself uncertain due to the ownership change, technological obsolescence, technical and commercial reasons.

Let the expected number of failures and the expected cost of warranty for each occasion within the year i be $E(N_i)$ and $E(C_i)$ respectively. Costs of premiums are assumed to occur at the beginning of every year.

The present value of the total expected cost for lifetime warranty can be modelled as

$$E(C) = \sum_{i=1}^{L} E(N_i)E(C_i)P(N_i) \times \left(\frac{1+\alpha}{1+\delta}\right)^{i-1} \tag{2.9}$$

where, $P(N_i)$ = Probability that lifetime terminates at year i. $P(N_i)$ is a function of ownership change, technological, obsolescence, technical and commercial life of products.

α denotes the increase of cost due to inflation, labour costs and others and δ represent the discount rate of money since the expected cost of warranty would be affected by the inflation, increased labour and other costs, and the discounting of money for a product with long lifetime.

This model is a complex one and analytical solution is intractable. Therefore, a simulation approach can be used to solve this model.

For the purpose of model simplification, it is assumed that the costs are not affected by α and δ. This assumption is realistic for products with shorter lifetimes i.e. product with lifetime close to one year.

The total cost of warranty during the lifetime can then be expressed as

$$E(C) = E(C)E(a) \tag{2.10}$$

where, $E(a)$ is the expected number of failures over the lifetime.

Under this policy manufacture/dealer are obliged to rectify all defects and failures of the sold product due to design or manufacturing problems over the defined lifetime.

Here, it is also assumed the product as a system composed of a number of components. Rectification can be a repair or replacement. Product failures are modelled as occurring according to a point process with an intensity function $\lambda(t)$ where t represents the age of the product (this assumption is true for most of the real life products). $\lambda(t)$ is an increasing function of t indicating that the number of failures, in a statistical sense, increases with age. As a result, $N(t)$, the number of failures over the warranty period is a random variable with

$$P\{N(t) = n\} = \frac{\left[\int_0^a \Lambda(t)dt\right]^n \mathrm{e}^{-\int_0^a \Lambda(t)dt}}{n!} \tag{2.11}$$

Since, the useful life of the product can be terminated at any time due to the ownership change, technological obsolescence, technical or commercial reason. And the lifetime is be bounded by a lower limit (l) and upper limit (u) at statutory base.

So, the expected number of failures during the lifetime can be expressed as

$$E[N(l,u)] = \int_l^u \{\int_0^a \Lambda(t)dt\} h(a)da \tag{2.12}$$

Therefore, the total cost for Model FRLTW-1D due to warranty claims over the lifetime of the product can be given by

$$E[C(l,u)] = E(C)E[N(l,u)] = \bar{c}\left[\int_l^u \{\int_0^a \Lambda(t)dt\} h(a)da\right]$$

or

$$E(C(l,u)) = \frac{1}{\gamma}\left[\int_l^u \{\int_0^a \Lambda(t)dt\} h(a)da\right] \tag{2.13}$$

2.4.6 Modelling Costs for Cost Sharing Lifetime Warranty (CSLTW)

2.4.6.1 Specified Parts Exclusive Lifetime Warranty

The components of the item are grouped into two disjointed sets; I and E. Failures of components belonging to Set I are covered by lifetime free replacement warranty and those belonging to Set E are not covered under warranty. Let $N_I(l, u)$ and $N_E(l, u)$ denote the number of failures of the warranty covered and non warranty covered components over the lifetime warranty period. One can model the failures during warranty using two different approaches. It is assumed that the total costs were not affected by the inflation or the discounting of money.

Approach 1

Here the failures are modelled by considering point process with intensity function $\Lambda(t)$. However, with each point (corresponding to a failure) there is a mark which indicates whether the item failure is covered under warranty or not. This mark is modelled as a binary random variable Y with $Y = 1$ indicating that the failure is covered and $Y = 0$ indicating that the failure is not covered under warranty.

$$\text{Let } P\{Y = 0\} = p \text{ and } P\{Y = 1\} = 1 - p = q$$

In this case, the expected number of failures of the components of the sold product that are covered with free replacement warranty over the lifetime is given by

$$E[N_I(l,u)] = \int_l^u \left\{ q \int_0^a \Lambda(t)dt \right\} h(a)da \tag{2.14}$$

where, l and u are the lower and upper limits of the useful life distribution and a is the lifetime coverage of the product.

Then, the expected warranty cost to the manufacturer/dealer, $E[C_m(l,u)]$ associated with the covered components, is given by

$$E[C_m(l,u)] = \bar{c}q \int_l^u \left\{ \int_0^a \Lambda(t)dt \right\} h(a)da \tag{2.15}$$

Similarly, the expected number of failures of components that are not covered with free replacement under this warranty policy is given by

$$E[N_E(l,u)] = \int_l^u \left\{ p \int_0^a \Lambda(t)dt \right\} h(a)da \tag{2.16}$$

And the expected cost to the customer over the warranty period, $E[C_b(l,u)]$, is given by

$$E[C_b(l,u)] = \bar{c}p \int_l^u \left\{ \int_0^a \Lambda(t)dt \right\} h(a)da \tag{2.17}$$

Approach 2

Here, the component failures belonging to Set I are modelled by a failure intensity function $\Lambda_I(t)$ and those belonging to Set E are modelled by another failure intensity function $\Lambda_E(t)$. Both of these are increasing functions of time t. Then expected number of failures $E[N_I(l,u)]$ for warranty covered components and $E[N_E(l,u)]$ for uncovered components are distributed according to non-stationary Poisson processes with intensity functions $\Lambda_I(t)$ and $\Lambda_E(t)$ respectively. In this case, the expected manufacturer's warranty cost for component included and customer's repair costs for components excluded can be expressed as Eqs. (2.18) and (2.19) respectively.

Manufacturer's cost (warranty cost for included component, Set I):

$$\begin{aligned} E[N_I(l,u)] &= \int_l^u \left\{ \int_0^a \Lambda_I(t)dt \right\} h(a)da \\ E[C_m(l,u)] &= \bar{c}_I \int_l^u \left\{ \int_0^a \Lambda_I(t)dt \right\} h(a)da \end{aligned} \tag{2.18}$$

Customer's cost (repair cost of warranty excluded component, for Set E)

$$\begin{aligned} E[N_E(l,u)] &= \int_l^u \left\{ \int_0^a \Lambda_E(t)dt \right\} h(a)da \\ E[C_b(l,u)] &= \bar{c}_E \int_l^u \left\{ \int_0^a \Lambda_E(t)dt \right\} h(a)da \end{aligned} \tag{2.19}$$

2.4.6.2 Limit on Individual Cost Lifetime Warranty

Here the cost of individual claims to the manufacturer is limited to fixed cost c_I whereby the manufacturer carries out all rectification action free of cost to the customer if the cost of rectification is below a limit c_I. If the cost of rectification exceeds c_I, then the customer pays the difference between the cost of rectification and c_I. That means manufacturer/dealer pays all costs up to c_I and the customer pays an amount (C-c_I), where C is the total rectification costs of an individual claim.

Then for the jth failure,

$$M_j = \min\{C_j, c_I\} \tag{2.20}$$

$$B_j = \min\{0, (C_j - c_I)\} \tag{2.21}$$

where M_j and B_j represent the manufacturer's cost and customer's costs for jth failure respectively.

Let the individual cost of rectification, C_j be given by a distribution function $G(c)$ with density function $g(c)$.

Therefore, the expected cost of each rectification to the manufacturer/dealer is given by

$$\bar{c}_m = \int_0^{c_I} cg(c)dc + c_I\bar{G}(c_I) \tag{2.22}$$

and that to the customer is given by

$$\bar{c}_b = \int_{c_I}^{\infty} (C - c_I)g(c)dc \tag{2.23}$$

The expected number of failures is given by

$$E[N(l,u)] = \int_l^u \{\int_0^a \Lambda(t)dt\}h(a)da \tag{2.24}$$

The total expected warranty cost to the manufacturer/dealer, $E[C_m(l,u)]$, is given by

$$E[C_m(l,u)] = \bar{c}_m[\int_l^u \{\int_0^a \Lambda(t)dt\}h(a)da] \tag{2.25}$$

and the total expected cost to the customer over the lifetime, $E[C_b(L)]$, is given by

$$E[C_b(l,u)] = \bar{c}_b[\int_l^u \{\int_0^a \Lambda(t)dt\}h(a)da] \tag{2.26}$$

2.5 Analysis of the Models

2.5.1 Analysis of the Model FRLTW

Here, the rate of product failure is an increasing function of age (as assumed). Since the number of failed components at each failure is very small relative to the number of components in the item, the rectification action can be viewed as having a negligible impact on the failure rate of the product as a whole. In other words, the failure rate after a repair is nearly the same as that just before the failure. Such a repair action is called a minimal repair (Barlow and Hunter 1960). This type of failure can be modelled as the Non-Homogeneous Poisson's Process (NHPP). In this case $\Lambda(t)$ is the failure rate associated with the failure distribution for the product. A simple form for $\Lambda(t)$ can be derived as follows:

$$F(t) = 1 - \exp(-(\lambda t)^{\beta}) \tag{2.27}$$

and

$$f(t) = \frac{d[F(t)]}{dt} = \lambda\beta(\lambda t)^{\beta-1}\exp\left(-(\lambda t)^{\beta}\right) \tag{2.28}$$

From Eq. (2.1), $\Lambda(t)$ is given by:

$$\Lambda(t) = \frac{f(t)}{1-F(t)} = \frac{\lambda\beta(\lambda t)^{\beta-1}\exp(-(\lambda t)^{\beta})}{1-(1-\exp(-(\lambda t)^{\beta}))} = \lambda\beta(\lambda t)^{\beta-1} \tag{2.29}$$

with the parameters $\beta > 1$ and $\lambda > 0$. This is an increasing function of t. Note that this corresponds to the failure rate of a two-parameter Weibull distribution.

By substituting value of $\Lambda(t)$ from Eq. (2.29) into Eq. (2.13), the total expected cost can be expressed by

$$E[C(l,u)] = \bar{c}[\int_{l}^{u} \{\int_{0}^{a} \lambda\beta(\lambda t)^{\beta-1}dt\}h(a)da] \tag{2.30}$$

which then can be expressed by after integration

$$E[C(l,u)] = \bar{c}[\int_{l}^{u} (\lambda a)^{\beta}h(a)da] \tag{2.31}$$

Now substituting value of $h(a)$ from Eq. (2.3) in Eq. (2.31),

$$E[C(l,u)] = \bar{c}[\int_{l}^{u} (\lambda a)^{\beta}\frac{\rho \mathrm{e}^{-\rho L}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}}da] \tag{2.32}$$

Now Eq. (2.32) can be solved by using mathematical software such as MATLAB, MAPLE etc. Let $\beta = 2$ and $\lambda = 0.443$ per year. This implies that the mean time to first failure is close to 2 years. Let the expected cost of each repair be \$100 (with $\gamma = 0.01$) and let lifetime parameter be $\rho = 0.4$. The expected warranty costs, $E[C(l,u)]$ for different combinations of l and u are shown in Table 2.1.

Table 2.1 Warranty cost ($) to the manufacturer for lifetime free rectification to customer (FRLTW: one dimensional model)

u (upper limit)	*l* (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
2	30	43	59	–	–	–	275
4	85	109	136	167	201	237	310
4.5	100	127	157	191	229	268	345
5	115	144	178	215	256	299	380

Table 2.1 shows that the warranty cost to the manufacturer/dealer increases as the lifetime increases. Table 2.1 also shows that the warranty cost increases as the range of lifetime increases. This implies the longer the span of lifetime the more the number of failures and higher the cost to the manufacturer/dealer.

2.5.2 Analysis of the Cost Sharing Models CSLTW

2.5.2.1 Analysis of Specific Parts Exclusive Lifetime Warranty (SPELTW)

Manufacturer's and customer's costs for specific parts excluded policy Approach 1:

For the *j*th failure of *i*th component of an item/product, we can define Y_{ji} as follows:

$$Y_{ji} = \begin{cases} 1 & for, \quad i \in SetI \\ 0 & for, \quad i \in SetE \end{cases} \tag{2.33}$$

Let M_j and B_j denote the cost to the manufacturer and the customer associated with the *j*th failure under warranty. Since the costs are shared, we have for the *j*th failure,

$$M_j = Y_{ji} C_j \tag{2.34}$$

$$B_j = (1 - Y_{ji}) C_j \tag{2.35}$$

As a result, the total cost to the manufacturer over the warranty period, $C_M(a)$, is given by

$$C_m(a) = \sum_{j=1}^{N(a)} M_j \tag{2.36}$$

where M_j is given by (2.34). Similarly, the total cost to the customer over the warranty period (because of non-covered components), $C_b(a)$, is given by

$$C_b(a) = \sum_{j=1}^{N(a)} B_j \tag{2.37}$$

where B_j is given by (2.35).

Let p denote the probability that a product failure is due to the failure of components from Set I. Then $E[Y_{ji}] = q$. The expected number of failures covered under warranty is given by (2.14) and not covered under warranty is given by (2.16). Then, the expected warranty cost to the manufacturer, $E[C_m(l,u)]$ associated with the covered components, is given by

$$E[C_m(l,u)] = E[Y_{ji}C_j]E[N(L)] = \bar{c}q \int_l^u \left\{ \int_0^a \Lambda(t)dt \right\} h(a)da \tag{2.38}$$

When it follows the failure follows the NHPP, the expected cost can be expressed as

$$E[C_m(l,u)] = \bar{c}_I q \int_l^u \left\{ \int_0^a \lambda\beta(\lambda t)^{\beta-1} dt \right\} h(a)da$$

and finally,

$$E[C_m(l,u)] = \bar{c}_I q \int_l^u (\lambda a)^\beta \frac{\rho \mathrm{e}^{-\rho a}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} da \tag{2.39}$$

And the expected cost to the customer over the warranty period, $E[C_b(l,u)]$, is given by

$$E[C_b(l,u)] = \bar{c}_E p \int_l^u (\lambda a)^\beta \frac{\rho \mathrm{e}^{-\rho a}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} da \tag{2.40}$$

where, the cost of each rectification resulting from failures due to components belonging to Set E and Set I are C_E and C_I respectively. This is modelled by distribution functions $G_E(c)$ and $G_I(c)$ with mean $\bar{c}_E$ and $\bar{c}_I$ respectively. Expressions for the mean values can be followed by the Eq. (4.8) using the appropriate distribution function.

For both the Eqs. (2.39) and (2.40) the value of $h(a)$ is substituted from Eq. (4.3).

Let, the shape parameter of the product $\beta = 2$ and inverse characteristic parameter $\lambda = 0.443$ per year. Let the lifetime distribution parameter $\rho = 0.4$. This implies that the mean time to first failure, $\mu = 2$ years. Let expected cost of rectifications for warranty included and warranty excluded components be $\bar{c}_I = \$100$ and $\bar{c}_E = \$70$

Table 2.2 Warranty cost ($) to the manufacturer for lifetime SPELTW: Approach 1

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
2	18.4	26.3	35.9	–	–	–	–
4	51.4	65.5	82.1	100.6	120.9	142.4	165
4.5	60.4	76.2	94.5	115.0	137.4	161.2	1862
5	69.3	86.7	106.8	129.3	153.8	179.9	2074

Table 2.3 Warranty cost ($) to the customer for lifetime SPELTW: Approach 1

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
2	8.6	12.3	16.7	–	–	–	–
4	24	30.6	38.3	46.9	56.4	66.5	77.0
4.5	28.1	35.5	44.1	53.68	64.12	75.2	86.9
5	32.3	40.4	49.8	60.35	71.77	83.9	96.8

respectively and let the probability of warranted excluded component $p = 0.4$ (40 %), and probability of warranty covered component $q = 1 - p = 0.6$.

The expected warranty costs to the manufacturer and the customer for different combination of lower useful life (l) and upper useful life (u) limits are shown in Tables 2.2 and 2.3 respectively. Figure 2.3 can be developed from Tables 2.2 and 2.3, which shows the cost trend for different combinations of l and u. This Figure exhibits that warranty cost increase with the increase of lifetime.

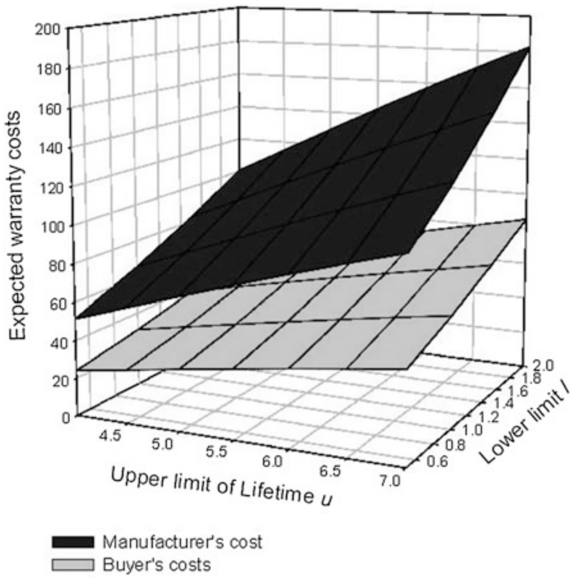

Fig. 2.3 Expected lifetime warranty cost ($) to the manufacturer and the buyer for SPELTW-Approach 1

Figure 2.3 shows that both the manufacturer and customer's cost increase with the increase of lifetime. In this example, with the same failure intensity, the customer's cost curve (lighter) is flatter than that of the manufacturer (darker) cost curve. This is dependent on the proportion of warranty covered components and the expected cost of rectification of the warranty covered components. The higher the proportion of warranty covered components and the expected rectification cost, the higher will be the manufacturer's expected warranty cost and the more steep will be the manufacturer's cost curve and vice versa. Note that under this policy, the manufacturer has to bear the costs of failures of all warranted components whereas the buyer has to bear the rectification costs of all non-warranted components.

Manufacturer's and customer's costs for specified parts excluded policy Approach 2:

Let the mean costs of rectification of each part belonging to the *Set E* and *Set I* be $\bar{c}_E$ and $\bar{c}_I$ respectively. Expressions for the mean values are given by Eq. (2.8) using the appropriate distribution function.

Here, intensity functions of Set E component and Set I components are different. The expected number of failures covered under warranty is given by the Eq. (2.18) and not covered under warranty is given by the Eq. (2.19).

If the failures of components belonged to Set I, follow the Non-homogeneous Poisson Process with shape parameters β_I and inverse characteristic parameter λ_I and, then the Eq. (2.18) can expressed as

$$E[C_m(l,u)] = \bar{c}_I \int_l^u (\lambda_I a)^\beta I \frac{\rho \mathrm{e}^{-\rho a}}{\mathrm{e}^{-\rho l} - \mathrm{e}^{-\rho u}} da \tag{2.41}$$

Similarly, the expected cost to the customer over the warranty period, $E[C_b(L)]$, is given by

$$E[C_b(l,u)] = \bar{c}_E \int_l^u (\lambda_E a)^\beta E \left[\frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}}\right] da \tag{2.42}$$

where, the failure of components follows the NHPP with parameters λ_E and β_E.

Let the parameter values for failures of components belonging to Set E be $\lambda_E = 0.241$ per month and $\beta_E = 2.31$ and let the expected cost of each rectification $\bar{c}_E = \$70$. The corresponding values for set I are $\lambda_I = 0.443$ per month, $\beta_I = 2$ and $\bar{c}_I = \$100$. The expected warranty costs to the manufacturer/dealer $E[C_M(l,u)]$ and to the customer $E[C_b(l,u)]$ for different combination of u and l are shown in Tables 2.4 and 2.5 respectively.

Tables 2.4 and 2.5 show that the warranty costs for both the manufacturer and the buyers increase with the increase of the statutory base of the warranty and also with the increase of the useful life of the product. It also indicates that the warranty costs are sensitive to the difference between the lower and upper limit of the life time.

Table 2.4 Lifetime warranty cost for specified parts excluded policy Approach 2 (manufacturer's cost)

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
2	28.4	29.4	52.4				
4	78.8	92.1	138.8	137.6	162.9	189	216
4.5	83.9	105.0	128.8	154.8	182.42	211	240
5	94.7	117.3	143.5	171.6	201.59	232	265

Table 2.5 Lifetime warranty cost for specified parts excluded policy Approach 2 (customer's cost)

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
2	4.5	6.7	9.5				
4	10.7	19.8	25.2	31.6	38.7	46.6	55.2
4.5	18.6	23.7	29.9	37.0	45.1	53.9	63.5
5	22.0	27.7	34.6	42.6	51.3	61.4	72.1

Figure 2.4 can be developed from the Tables 2.4 and 2.5 which shows that the total expected warranty costs for both the manufacturer and the buyers increase with the increase of the statutory base of the warranty and also with the increase of the useful life of the product.

Fig. 2.4 Expected lifetime warranty cost ($) to the manufacturer and buyer for SPELTW-Approach 2

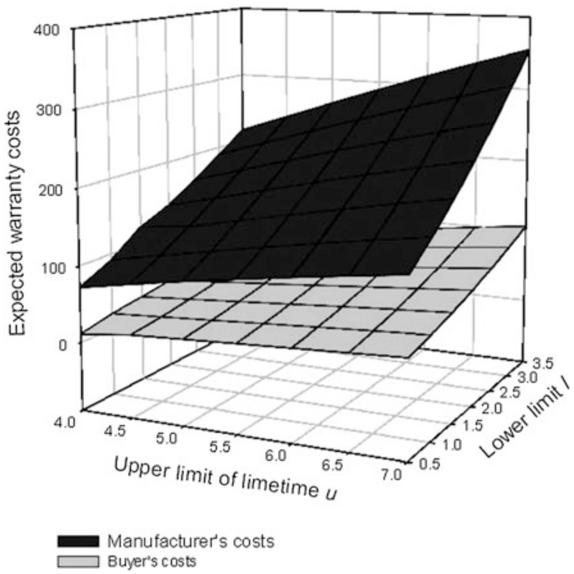

Figure 2.4 also shows that under the assumed failure rates and proportion of warranted component the customer's cost curve is flatter than the manufacturer's cost. This is because here the failure intensity of warranted components is assumed to be higher than that of the unwarranted components. This is also affected by the proportion of warranted and unwarranted components. The higher the proportion of the warranty covered components, the steeper will be the manufacturer cost curve and lower will be the customer cost curve. This implies that both the manufacturer's and customer's costs are influenced by the proportion warranted components and the failure intensities of the warranted and unwarranted components if the cost of rectification is same for both the manufacturer and the customer.

2.5.2.2 Analysis of Limit on Individual Cost (LICLTW) Model

Under this policy, the total expected warranty cost to the manufacturer and customer over the lifetime, are given by the Eqs. (2.25) and (2.26) respectively. When failures of the product follow the Non-homogeneous Poisson process with inverse characteristic life parameters λ and shape parameter β, the manufacturer's total expected warranty cost over the lifetime can written as

$$E[C_m(l,u)] = \left\{ \int_0^{c_I} cg(c)dc + c_I\bar{G}(c_I) \right\} \int_l^u (\lambda a)^\beta \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} da \qquad (2.43)$$

and the total expected cost to the customer over the lifetime, $E[C_b(l,u)]$, can be expressed as

$$E[C_b(l,u)] = \left\{ \int_{c_I}^{\infty} (C - c_I)g(c)dc \right\} \left\{ \int_l^u (\lambda a)^\beta \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} da \right\} \qquad (2.44)$$

In solving Eqs. (2.43) and (2.44) numerically, it is assumed the rectification costs for the product is exponentially distributed over the lifetime with parameter γ. And $\gamma = 0.01$.

Using the product failure data with shape parameter $\beta = 2$ and inverse characteristic life parameter $\lambda = 0.443$ per year. Let the manufacturer's set individual limit cost c_I = \$125 and it is also assumed that the cost of each rectification is not exceeding \$200 which implies that the customer has to pay all the costs of rectification from 125 to \$200. The expected warranty costs, $E[C_m(l,u)]$ $E[C_b(l,u)]$ for different combinations of l and u are shown in Tables 2.6 and 2.7.

Figure 2.5 can be generated from the Tables 2.6 and 2.7.

The plots in the Fig. 2.5 show that both the manufacturer's and buyer's/customer's expected warranty cost are increasing function of useful life of the products and the cost also increases with the increase of span limits. It also implies that under

Table 2.6 Manufacturer's/dealers costs for policy LICLTW

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
4	45.9	58.6	73.4	90.0	108	127	147
4.5	54.0	68.1	84.5	102	122	144	166
5	62.0	77.5	95.9	115	137	160	185
5.5	69.9	86.8	106	128	152	177	204

Table 2.7 Customer's costs for policy LICLTW

u (upper limit)	l (lower limit of useful life in years)						
	0.5	1	1.5	2	2.5	3	3.5
4	9.7	12.3	15.5	19.0	22.8	27	31
4.5	11.4	14.4	17.8	21.7	25.9	304	35
5	13.1	16.4	20.2	24.4	29.0	34	39
5.5	14.7	18.3	22.4	27.1	32.1	37	43

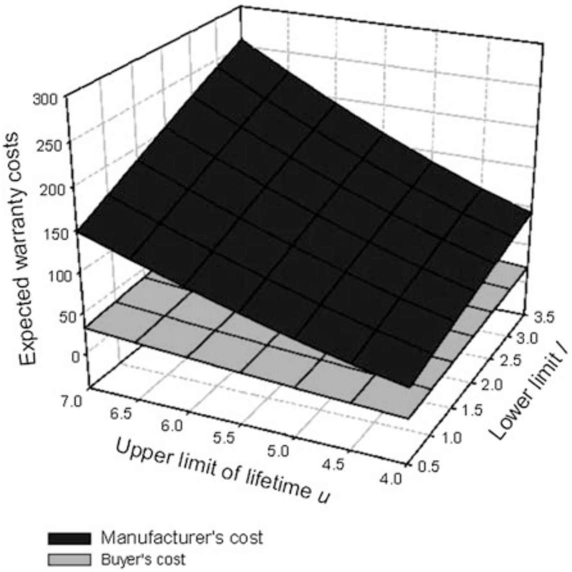

Fig. 2.5 Expected lifetime warranty cost ($) to the manufacturer and buyer for LICLTW

the conditions and assumptions of this example the manufacturer's expected warranty costs are higher than that of the buyer's. This is because the mean cost of rectification of this particular product is considered $100 (as expected cost is estimated, $\bar{c} = \frac{1}{\gamma}$) while manufacturer's cost limit was set $125. Most of the costs are less than the manufacturer's limit cost ($125). This implies that the both the manufacturer's and the customer's costs are sensitive to mean cost of rectification and the manufacturer's limit on cost per rectification. Therefore the manufacturer should fix the limit on each cost by carefully estimating the expected cost of rectification.

2.6 Conclusion

Lifetime warranties are a relatively new concept. Different people use the measure of lifetime in different ways which results in complexities in modeling the costs of the lifetime warranty policies. Despite the complexities in determining the life of a product and the difficulties involved in making decisions on servicing of a warranty claim it is becoming more attractive to the practitioners.

In this article, life measures applicable to lifetime warranty policies are defined. In this article, taxonomy for various lifetime warranty policies have been proposed and developed. Cost models for different lifetime warranty policies have been developed and analysed with a numerical examples.

There is huge scope for future research in this area. This includes modeling failures and costs for different types of trade in lifetime warranty policies, modeling failures and costs for two Dimensional policies, expected life cycle costs (LCC) of operation over the life time and modelling complex systems with repairable and non repairable components and provision of second-hand components in servicing warranties. Effect of various servicing strategies such as overhauling or preventive maintenance on the long term warranty costs, multi dimensional service contracts and consumer choice have not been studied adequately. These areas can also be considered in the future.

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