

# Preface

In the last centuries continuum mechanics developed from a theory treating very specific problems to a general theory suitable for many applications. Continuum mechanics started with the description of one-dimensional continua where Euler's elastica is maybe its most famous problem. With the seminal work of Cauchy on the existence of the stress tensor in a three-dimensional continuum, the foundations of modern continuum mechanics have been laid down. After a century with the paradigm of infinitesimal deformations and linear elastic material laws, the second half of the twentieth century has been dominated by finite strain theories with large deformations and nonlinear material laws. Especially with the emergence of the computer and its fast rising power, to date, it is possible to treat more complex mechanical behavior than ever before. Nevertheless, an axiomatic consideration of continuum mechanics together with an appropriate mathematical framework is still a major challenge. The foundations of mechanics deal with the identification of the fundamental objects and the postulation of its principles. Due to the high level of abstraction, the mathematical discipline of intrinsic differential geometry seems to be best suited for the description of continuum mechanics. Step-by-step, additional mathematical structure can be introduced and motivated by the underlying physics. Without specifications of constitutive laws, geometric continuum mechanics is on the one hand coordinate independent and on the other hand a priori metric independent. Since a geometric continuum mechanics generalizes the well-established objects of the classical theories, every single object has to be rethought and evaluated if it is fundamental or not.

This book is intended to make the reverse direction of the historical development. It starts with an attempt of geometric continuum mechanics where body and physical space are assumed to be smooth manifolds. Combining the mechanical principles of Paul Germain from the 1970s with an intrinsic differential geometric description of continuum mechanics of Reuven Segev of the 1980s, the principle of virtual work emerges as the fundamental principle of continuum mechanics. In the second part of the book, the classical model of the physical space, the three-dimensional Euclidean space, is assumed and induced beam theories are treated as an application of continuum mechanics. Then it is possible to consider a beam as a

continuous body with a constrained position field guaranteed by a perfect constraint stress field. Defining a constrained position field and applying the restricted kinematics to the principle of virtual work of a continuous body, the constraint stresses are eliminated due to the principle of d'Alembert–Lagrange and the weak variational form of an appropriate beam theory is induced directly. This induced approach to beam theory relates the point of view of beams as generalized one-dimensional continua to the theory of continuous bodies. In this work all classical beam theories, in which the cross sections remain rigid and plain, are presented. Additionally, augmented beam theories allowing for cross section deformations are derived using the very same procedure. All theories are suitable for large displacements and large rotations. The obtained weak variational forms of the appropriate beam theories serve then as the basis for the numerical implementation by finite elements.

The work presented in this book has been carried out during my time as research assistant at the Center of Mechanics at the ETH Zurich and appeared as doctoral thesis with the title “On the Foundations of Continuum Mechanics and its Application to Beam Theories”. I was accompanied by many people whom I would like to thank for their kind support of my work. I am very thankful to my supervisor Prof. Dr.-Ing. Dr.-Ing. habil. Christoph Glocker for supporting and guiding my research. I have got the opportunity to delve into the very foundations of mechanics and by the way to improve my mathematical background enormously. His distinct idea of mechanics, based on the principle of virtual work as its fundamental principle, has always been a clear guideline to my work. Special thanks go to Prof. Dr. ir. habil. Remco Leine who has taught me the art of academic writing, has been an ideal of how to present research results, and has always been a critical voice in my research. I am looking forward to an intensive time as “Akademischer Rat” working together with him at the Institute for Nonlinear Mechanics at the University of Stuttgart. Many thanks go to Dr.-Ing. O. Papes who hooked me as a student to continuum mechanics and who infiltrated my mind by the concept of a geometric description of continuum mechanics. Finally, I would like to thank my family and friends for their support and continuous encouragement.

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