

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Motivation	1
1.2	The Virtual Work	2
1.3	Literature Survey	4
1.3.1	Foundations of Continuum Mechanics	4
1.3.2	Beam Theory	7
1.4	Aim and Scope	8
1.5	Outline	9
	References	10
 <b>Part I Geometric Continuum Mechanics</b>		
<b>2</b>	<b>Kinematics</b>	17
2.1	Body and Space	17
2.2	Spatial Virtual Displacement Field	20
2.3	Configuration Space	25
2.4	Affine Connection	29
	References	32
<b>3</b>	<b>Force Representations</b>	33
3.1	Principle of Virtual Work	33
3.2	Classical Nonlinear Continuum Mechanics	36
	References	41

## Part II Induced Beam Theories

<b>4 Preliminaries</b>	45
4.1 Fundamental Principles of a Continuous Body	45
4.2 Constrained Position Fields	48
4.3 Intrinsic and Induced Beam Theories	49
References	52
<b>5 Classical Nonlinear Beam Theories</b>	55
5.1 Kinematical Assumptions	55
5.2 Virtual Work Contributions	59
5.2.1 Virtual Work Contributions of Internal Forces	60
5.2.2 Virtual Work Contributions of Inertia Forces	61
5.2.3 Virtual Work Contributions of External Forces	63
5.2.4 The Boundary Value Problem	64
5.3 Nonlinear Timoshenko Beam Theory	65
5.4 Nonlinear Euler–Bernoulli Beam Theory	68
5.5 Nonlinear Kirchhoff Beam Theory	69
5.6 Literature Survey of Numerical Implementations	70
References	71
<b>6 Classical Linearized Beam Theories</b>	75
6.1 Linearized Beam Kinematics	75
6.2 The Boundary Value Problem of the Classical Linearized Beam Theory	78
6.3 Linearized Timoshenko Beam Theory	79
6.4 Linearized Euler–Bernoulli Beam Theory	80
6.5 Linearized Kirchhoff Beam Theory	80
Reference	81
<b>7 Classical Plane Linearized Beam Theories</b>	83
7.1 Constrained Position Fields in Linear Elasticity	84
7.2 The Plane Linearized Timoshenko Beam	85
7.2.1 Kinematics, Virtual Work and the Boundary Value Problem	85
7.2.2 Constraint Stresses of the Plane Timoshenko Beam	89
7.3 The Plane Linearized Euler–Bernoulli Beam	94
7.3.1 Kinematics, Virtual Work and the Boundary Value Problem	94
7.3.2 Constraint Stresses of the Plane Euler–Bernoulli Beam	96
7.4 The Plane Linearized Kirchhoff Beam	98
References	99

<b>8</b>	<b>Augmented Nonlinear Beam Theories</b>	101
8.1	The Nonlinear Cosserat Beam	101
8.1.1	Kinematical Assumptions	102
8.1.2	Virtual Work Contribution of Internal Forces	103
8.1.3	Virtual Work Contribution of Inertia Forces	104
8.1.4	Virtual Work Contribution of External Forces	105
8.1.5	The Boundary Value Problem	105
8.1.6	Constitutive Law and Restrictions on Internal Forces	106
8.2	The Nonlinear Saint–Venant Beam	107
8.2.1	Kinematical Assumptions	108
8.2.2	Virtual Work Contribution of Internal Forces	109
8.2.3	Virtual Work Contribution of Inertia Forces	111
8.2.4	Virtual Work of External Forces	112
8.2.5	The Boundary Value Problem	113
8.2.6	Constitutive Laws	114
	References	114
<b>9</b>	<b>Conclusions and Outlook</b>	117
9.1	Geometric Continuum Mechanics	117
9.2	Induced Beam Theories	119
	References	121
	<b>Appendix A: Multilinear Algebra</b>	123
	<b>Appendix B: Properties of the Cross Product</b>	141
	<b>Index</b>	143

Geometric Continuum Mechanics and Induced Beam  
Theories

Eugster, S.

2015, IX, 146 p. 12 illus., Hardcover

ISBN: 978-3-319-16494-6