
Contents

Part I Mean-Variance Portfolio Analysis

1	Portfolio Selection: Introductory Comments	3
1.1	Asset Prices and Returns	3
1.2	Investor's Portfolio: Long and Short Positions	4
1.3	Return on a Portfolio	5
1.4	Mathematical Notation	8
2	Mean-Variance Portfolio Analysis: The Markowitz Model	11
2.1	Basic Notions	11
2.2	Optimization Problem: Formulation and Discussion	13
2.3	Assumptions	15
2.4	Efficient Portfolios and Efficient Frontier	16
3	Solution to the Markowitz Optimization Problem	19
3.1	Statement of the Main Result	19
3.2	Discussion	21
3.3	Proof of the Main Result	23
4	Properties of Efficient Portfolios	27
4.1	Mean and Variance of the Return on an Efficient Portfolio	27
4.2	Description of the Efficient Frontier	29
4.3	A Fund Separation Theorem	30
5	The Markowitz Model with a Risk-Free Asset	33
5.1	Data of the Model	33
5.2	Portfolio Optimization with a Risk-Free Asset	36
5.3	Solution to the Portfolio Selection Problem	38
6	Efficient Portfolios in a Market with a Risk-Free Asset	43
6.1	Expectations and Variances of Portfolio Returns	43
6.2	Efficient Frontier and the Capital Market Line	44
6.3	Tangency Portfolio	46
6.4	A Mutual Fund Theorem	50

7	Capital Asset Pricing Model (CAPM)	53
7.1	A General Result	53
7.2	An Equilibrium Approach to the CAPM	55
7.3	The Sharpe-Lintner-Mossin Formula	59
8	CAPM Continued	61
8.1	Security Market Line and the Pricing Formula	61
8.2	CAPM as a Factor Model	62
8.3	Applying Theory to Practice: Sharpe's and Jensen's Tests	64
9	Factor Models and the Ross-Huberman APT	69
9.1	Single- and Multi-Factor Models	69
9.2	Exact Factor Pricing	71
9.3	Ross-Huberman APT: Model Description	76
9.4	Formulation and Proof of the Main Result	78
10	Problems and Exercises I	83
 Part II Derivative Securities Pricing		
11	Dynamic Securities Market Model	105
11.1	Multi-Period Model of an Asset Market	105
11.2	Basic Securities and Derivative Securities	108
11.3	No-Arbitrage Pricing: Main Result	110
11.4	The No-Arbitrage Hypothesis and Net Present Value	112
12	Risk-Neutral Pricing	115
12.1	Risk-Neutral Measures	115
12.2	Fundamental Theorem of Asset Pricing	117
12.3	Asset Pricing in Complete Markets	119
13	The Cox–Ross–Rubinstein Binomial Model	125
13.1	The Structure of the Model	125
13.2	Completeness of the Model	127
13.3	Constructing a Risk-Neutral Measure	129
13.4	Examples	132
14	American Derivative Securities	137
14.1	The Notion of an American Derivative Security	137
14.2	Risk-Neutral Pricing of American Derivative Securities	139
14.3	The Pricing Algorithm	142
15	From Binomial Model to Black–Scholes Formula	145
15.1	Drift and Volatility	145
15.2	Modelling the Price Process	146
15.3	Binomial Approximation of the Price Process	147
15.4	Derivation of the Black–Scholes Formula	150
16	Problems and Exercises II	157

Part III Growth and Equilibrium

17 Capital Growth Theory	169
17.1 Growth-Optimal Investments	169
17.2 Strategies in Terms of Investment Proportions	171
17.3 Results for Simple Strategies	173
18 Capital Growth Theory: Continued	177
18.1 Log-Optimal Strategies	177
18.2 Growth-Optimal and Numeraire Strategies	179
18.3 Growth-Optimality for General Strategies	180
18.4 Volatility-Induced Growth	183
19 General Equilibrium Analysis of Financial Markets	187
19.1 Walrasian Equilibrium	187
19.2 On the Existence of Equilibrium	190
19.3 Rational Expectations and Equilibrium Pricing	192
19.4 Arbitrage and Equilibrium	194
20 Behavioral Equilibrium and Evolutionary Dynamics	197
20.1 A Behavioral Evolutionary Perspective	197
20.2 Survival Strategies	201
20.3 Links to the Classical Theory	203
21 Problems and Exercises III	205

Mathematical Appendices

A Facts from Linear Algebra	215
B Convexity and Optimization	219
Sources	223

Mathematical Financial Economics

A Basic Introduction

Evstigneev, I.; Hens, T.; Schenk-Hoppé, K.R.

2015, IX, 224 p. 21 illus., 3 illus. in color., Hardcover

ISBN: 978-3-319-16570-7