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Hangman

At a trial a prisoner is sentenced to death by the judge. The verdict reads “You will be executed next week, but the day on which you will be executed will be a surprise to you.” The prisoner reasons as follows. “I cannot be executed on Friday, because in that case I would not be surprised. But given that Friday is eliminated, then I cannot be executed on Thursday either, because that would then no longer be a surprise. And so on. Therefore the execution will not take place.” And so, his execution, that happened to be on Wednesday, came as a surprise.

So, after all, the judge was right. What error does the prisoner make in his reasoning?

The prisoner’s argument is very convincing. At first sight it seems as if it cannot be refuted at all. Still, the conclusion cannot be right. The prisoner rules out that the hanging will be on Thursday, and that it will be on Wednesday, and so on, but in fact the hanging is on Wednesday. Is it not easy to make clear where the error is. And therefore it is indeed called a paradox. To find the error in the prisoner’s reasoning, we first have to define what a “secret” is. Because initially the day of the hanging is a *secret*.

2.1 How to Guard a Secret?

The best way to guard a secret (like who you are in love with) is not ever to tell it to anyone. That is easier said than done. If your head is filled with the secret, it can happen to fall out of your mouth before you know it. And then it is no longer a secret. Someone might ask you why you are staring out of the window all the time, focusing on the horizon. You can then of course say that this is because you are guarding a secret. But that makes it less secret. If you really want to guard a secret, you had better not ever talk about it, because if you do, then you risk that the secret will be discovered.



It is also a bad idea to talk to yourself about your own secret. A classic of that kind (renewedly popular from the TV series *Once Upon a Time*) is the character Rumpelstiltskin in the Grimm Brothers fairy tale by the same name (1814). The queen promised her first-born child to Rumpelstiltskin. There is an escape clause: If the queen guesses correctly Rumpelstiltskin's name, then she can keep her child. She can guess three times. The first two guesses are incorrect. The tension rises. The queen's messenger now tells her that he saw in the forest, from a hidden place behind some bushes, a funny guy who was dancing while singing, loudly:

Heute back ich, morgen brau ich,
übermorgen hol ich der Königin ihr Kind;
ach, wie gut dass niemand weiß,
dass ich Rumpelstilzchen heiß!

Fortunately, the messenger understood German and could translate this into

Today I'll bake; tomorrow I'll brew,
Then I'll fetch the queen's new child,
It is good that no one knows,
Rumpelstiltskin is my name.

It was indeed Rumpelstiltskin who was singing this song, and so the queen finds out his name, and the third time her guess is correct: "Your name is Rumpelstiltskin." If only he had kept his mouth shut, it would have remained a secret.

The funny thing is, that the last two sentences, in a more convenient phrasing "Nobody knows that my name is Rumpelstiltskin," become false because Rumpelstiltskin is singing it. After this, it is no longer the case that nobody knows that his name is Rumpelstiltskin. The messenger now knows. This phenomenon is quite special. Apparently, it is possible to say something ("Nobody knows that I am in love with Stephanie") but because I am saying it, it becomes false. (In no time everyone, including Stephanie, knows that I am in love with her.) Usually when we say something, it remains true after we say it. But in exceptional cases, this is apparently false.

What is the relationship between hangings and fairy tales? The day of the hanging is a secret guarded by the judge, and the prisoner can only guess what the exact day is. The judge does not tell which day it is. What does it mean that the judge says that the day of the hanging will be a surprise? A surprise is something unexpected, it is something happening that you did not see coming. In the reasoning of the prisoner, "surprise" is entirely interpreted in terms of *knowledge*. The hanging is a surprise, because the prisoner does not *know* the day of the hanging in advance. A secret is no secret anymore if you are telling it

to someone, just as for the secret of Rumpelstiltskin. Similarly, a surprise is not a surprise anymore if you announce it. If you want to surprise someone with a big bunch of roses, then you should not let it appear from your behavior. If you say, “I am going to surprise Stephanie tomorrow with a big bunch of roses,” then the surprise is lost when she hears about it. If Rumpelstiltskin says, “Nobody knows that my name is Rumpelstiltskin,” then someone may get to know it.

2.2 A Bridge Too Far

When the judge says that the day of the hanging will be a surprise, he risks spoiling the surprise. If he had not said anything about the hanging, not even that it was going to be next week, it would not have mattered; surely the prisoner would then have been surprised by the hanging.

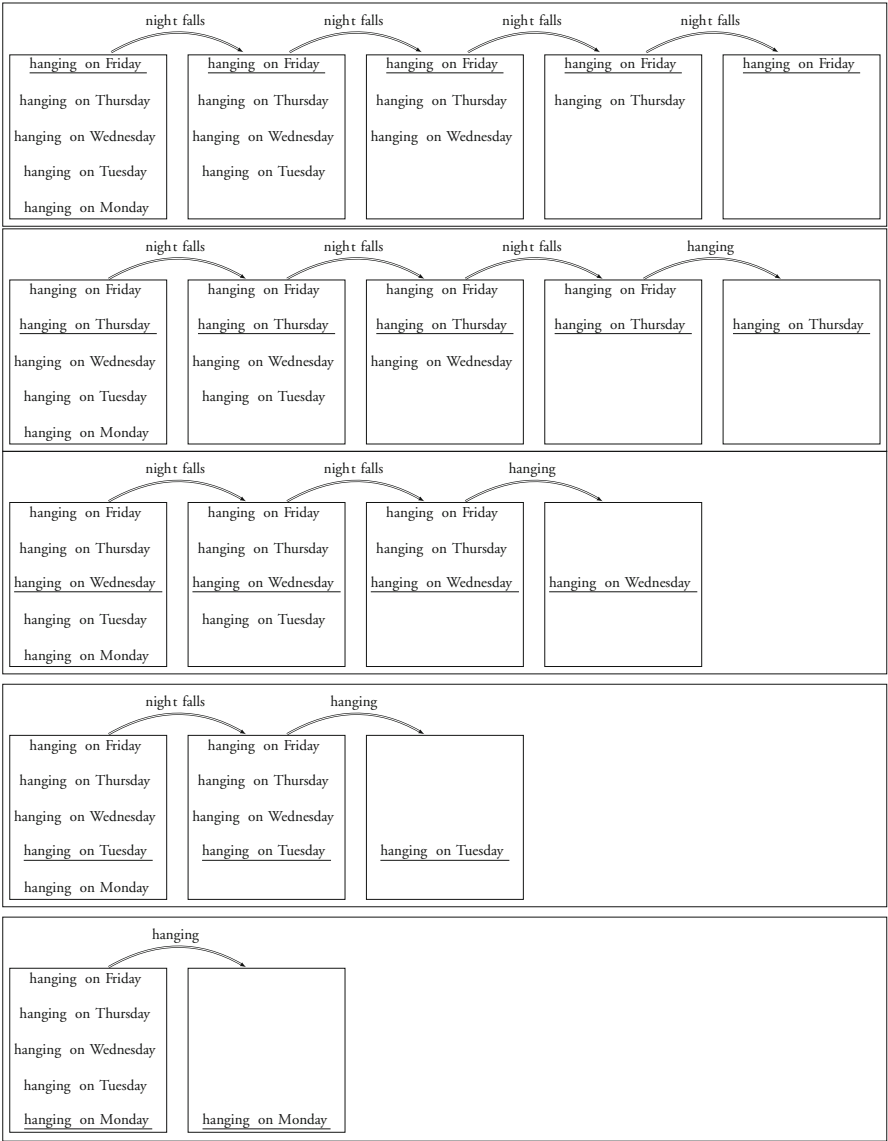
The error that the prisoner seems to make in his reasoning is that he does not realize that the judge may have spoilt the surprise by announcing it. Before the judge is saying that the day will be a surprise, the prisoner considers it possible that the hanging will take place on one of Monday, Tuesday, Wednesday, Thursday, or Friday. Now suppose nothing else has been announced about the day of the hanging. The prisoner would then know on Thursday night that the hanging will be on Friday. The hanging would then not be a surprise. On all other days, it would be a surprise. This, the judge also knows. But by saying that to the prisoner, he spoils the surprise. His announcement rules out that the hanging will be on Friday. Therefore, if the prisoner had not yet been hanged by Wednesday night, he could by that time have concluded that the hanging must be on Thursday. So now Thursday is special, instead of Friday.

However, the prisoner takes the argument further—and too far: He assumes that even after the judge’s announcement, the day of the hanging remains a surprise. And, therefore, he thinks he can rule out not only Friday but also Thursday, and Wednesday, and Tuesday, and Monday. But that is carrying it too far. Only Friday can be ruled out.

In fact, the hanging is on Wednesday. So, if the prisoner would not get more information, that would still be a surprise.

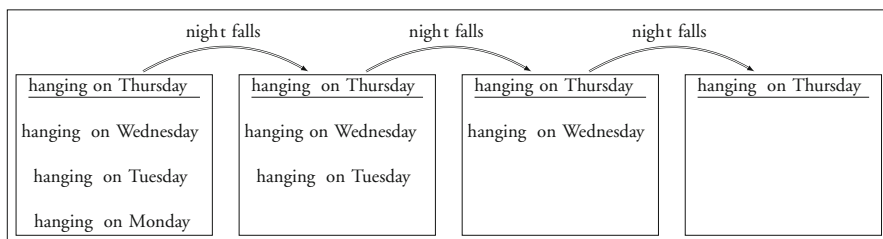
Let us illustrate this by constructing models. We assume that initially the prisoner only knows that there will be a hanging some day next week (a working day: Monday to Friday). So, this is before the judge announces that the day of the hanging will be a surprise. In that case, how will the prisoner’s information change with the passing of that coming week? Below we can see this depicted for the different days that the hanging can take place. Two events may reduce the uncertainty for the prisoner: Nightfall will rule out that the hanging is on the current day and thus reduces the uncertainty, but the hanging itself will confirm that it is on the current day and thus also reduces the uncertainty.

Hang on, what does it mean for a prisoner who has been hanged and who is dead, to know on Friday that he has been hanged on Thursday? Dead prisoners do not know anything. True enough, but this is an artifact of our setting of the riddle! In another version, the riddle concerns a surprise *exam* given by a schoolmaster to his pupils. Then, on Friday you will still know that the exam has been on Thursday. We can also imagine ourselves, as problem solvers, to be the agents observing the scenario and whose knowledge is being modeled. The problem solver will still know on Friday that the prisoner has been hanged on Thursday.



What is remarkable in these different scenarios is that there is only one occasion where the actual hanging does remove the uncertainty about the day of the hanging, for the prisoner. Namely, when the hanging is on Friday. Because (only) on that occasion the prisoner can determine the night before the hanging that the hanging will take place on Friday. So, for the prisoner there is only one day where the hanging will not be a surprise: Friday. If the judge announces that the hanging will be a surprise, this then rules out that the hanging is on Friday.

After the judge's announcement it is not necessarily so that the hanging will be a surprise. But there is now another scenario in the picture above where the hanging will not be a surprise, namely where Friday has been eliminated and where the hanging will be on Thursday. The prisoner does not know this in advance but knows that the hanging will be on Thursday when Wednesday night falls.



2.3 Versions

Puzzle 6 Suppose that the judge has answered the question “On which day?” by “That will not be a surprise.” On which day will the hanging then take place?

A different wording of the riddle is not about a judge surprising a prisoner with the day of a hanging, but about a schoolmaster or teacher surprising a class with the day of an examination: “You will get an exam next week, but the day of the exam will be a surprise for you.” Then, of course, the class only learns that the examination will not be on Friday. It is therefore also known as the surprise exam paradox. We now discuss a further version of that.

Puzzle 7 Suppose the teacher, Alice, had only said that the exam would take place next week, but without saying that the exam would come as a surprise.

During lunch break, her pupil Rineke walks past the staffroom and overhears the teacher saying to a colleague, “I am going to give my class an exam next week, and the day of the exam will be a surprise to them.” The teacher did not realize that Rineke was overhearing her. What can Rineke conclude on the basis of this information about the day of the examination?

But the plot thickens. Because after lunch break Rineke says to the teacher, “I heard you say that the day of our exam next week will come as a surprise.” The teacher confirms this. However, later that day, while getting her parked bike from the bikeshed, the teacher meets the staffroom colleague again, who is about to go home as well, and tells him that Rineke had overheard them earlier that day, and says, “But the day of the exam will still come as a surprise!” Unfortunately, Rineke overhears this again. What can Rineke now conclude about the day of the exam?

2.4 History

During the Second World War, the Swedish mathematician Lennart Ekbom overheard a radio message announcing a military training exercise next week. The training exercise would, of course, come as a surprise. It occurred to him that this message seemed paradoxical (Kvanvig 1998; Sorensen 1988, p. 253). The paradox was then published by O’Connor (1948). One of the responses to this publication then mentions that the exercise could still take place (Scriven 1951), what makes it even more paradoxical.

There are many versions of the paradox, the best known is the “surprise exam” version where a schoolmaster announces to his class that an examination will be given next week, but that the day will be a surprise (this first appeared in Weiss 1952). The “hangman” version of our presentation first appeared in Quine (1953).

The treatment of the puzzle differs depending on how “surprise” is interpreted. This can be done in many different ways. It can be in terms of derivability (the precise day *does not follow from* what the judge says). This approach was followed by Shaw (1958). But of course “surprise” can also be interpreted as “ignorance”: lack of knowledge. This is what we have done here. “The prisoner will be surprised” then means that the prisoner does not *know* in advance when the hanging will take place.

Since 1948, more than 100 publications have appeared on the hangman paradox. They contain even more interpretations. A detailed overview of treatments of the paradox and its history is given by Sorensen (1988).

It is remarkable that all this “scientific work” has not resulted in a universally accepted solution of the paradox. Chow (1998) even calls it a meta-paradox:

The meta-paradox consists of two seemingly incompatible facts. The first is that the surprise exam paradox seems easy to resolve. [. . .] The second (astonishing) fact is that to date nearly a hundred papers on the paradox have been published, and still no consensus on its correct resolution has been reached.

The solution given in this chapter is based on the work of Gerbrandy (1999, 2007). It is also treated by van Ditmarsch and Kooi (2005, 2006).

One Hundred Prisoners and a Light Bulb

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