Preface

To the Student

This textbook offers a primer in general topology (point-set topology), together with an introduction to algebraic topology. It is meant primarily for students with a mathematical background that is usually taught in the first year of undergraduate degrees in Mathematics and Physics.

Point-set topology is the language in which a considerable part of mathematics is written. It is not an accident that the original name 'analytic topology' was replaced by 'general topology', a more apt term for that part of topology that is used by the vast majority of mathematicians and is fundamental in many areas of mathematics. Over time its unabated employment has had a constant polishing effect on its theorems and definitions, thus rendering it an extraordinarily elegant subject. There is no doubt that point-set topology has a significant formative value, in that it forces the brain—and trains it at the same time—to handle extremely abstract objects, defined solely by axioms. In studying on this book, you will experience hands-on that the point-set topology resembles a language more than a theory. There are endless terms and definitions to be learnt, a myriad of theorems whose proof is often rather easy, only occasionally exceeding 20 lines. There are, obviously, also deep and far-from-trivial results, such as the theorems of Baire, Alexander and Tychonov.

The part on algebraic topology, details of which we will give in Chap. 9 together with the mandatory motivations, is devoted to the study of homotopy, fundamental groups and covering spaces.

I included around 500 exercises in the text: trying to solve them with dedication is the best way to attain a firm hold on the matter, adapt it to your own way of thinking and also learn to develop original ideas. Some exercises are solved directly in the text, either in full or almost. They are called '*Examples*', and their importance should not be underestimated: understanding them is the correct way to make abstract notions concrete. Exercises marked with ♡, instead, are solved in Chap. 16.

It is a matter of fact that the best way to learn a new subject is by attending lectures, or studying on books, and trying to understand definitions, theorems and the interrelationships properly. At the same time you should solve the exercises, without the fear of making mistakes, and then compare the solutions with the ones in the text, or those provided by teacher, classmates or the internet.

This book also proposes a number of exercises marked with ☕, which I personally believe to be harder than the typical exam question. These exercises should therefore be taken as endeavours to intelligence, and incentives to be creative: they require that we abandon ourselves to new synergies of ideas and accept to be guided by subtler analogies, rather than trail patiently along a path paved by routine ideas and standard suggestions.

To the Lecturer

In the academic years 2004–2005 and 2005–2006, I taught a lecture course called 'Topology' for the Bachelor's degree in Mathematics at University of Rome 'La Sapienza'. The aim was to fit the newly introduced programme specifications for mathematical teaching in that part of the syllabus traditionally covered in 'Geometry 2' course of the earlier 4-year degrees. The themes were carefully chosen so to keep into account on one side the formative and cultural features of the single topics, on the other their usefulness in the study of mathematics and research alike. Some choices certainly break with a long-standing and established tradition of topology teaching in Italy, and with hindsight I suspect they might have been elicited by my own research work in algebra and algebraic geometry. I decided it would be best to get straight to the point and state key results and definitions as early as possible, thus fending off the terato(po)logical aspects.

From the initial project to the final layout of my notes, I tried to tackle the conceptual obstacles gradually, and make both theory and exercises as interesting and entertaining as possible for students. Whether I achieved these goals the reader will tell.

The background necessary to benefit from the book is standard, as taught in first-year Maths and Physics undergraduate courses. Solid knowledge of the language of sets, of linear algebra, basic group theory, the properties of real functions, series and sequences from 'Calculus' are needed. The second chapter is dedicated to the arithmetic of cardinal numbers and Zorn's lemma, two pivotal prerequisites that are not always addressed during the first year: it will be up to the lecturer to decide—after assessing the students' proficiency—whether to discuss these topics or not.

The material present here is more than sufficient for 90 hours of lectures and exercise classes, even if, nowadays, mathematics syllabi tend to allocate far less time to topology. In order to help teachers decide what to skip I indicated with the symbol ↷ ancillary topics, which may be left out at first reading. It has to be said, though, that Chaps. 3–6 (with the exception of the sections displaying ↷), form the backbone of point-set topology and, as such, should not be excluded.

The bibliography is clearly incomplete and lists manuals that I found most useful, plus a selection of research articles and books where the willing student can find further information about the topics treated, or mentioned in passing, in this volume.

Acknowledgments

I am grateful to Ciro Ciliberto and Domenico Fiorenza for reading earlier versions and for the tips they gave me. I would like to thank Francesca Bonadei of Springer-Verlag Italia for helping with the final layout, and all the students ('victims') of my lectures on topology for the almost-always-useful and relevant observations that allowed me to amend and improve the book.

This volume is based on the second Italian edition. Simon G. Chiossi has done an excellent work of translation; he also pointed out a few inaccuracies and proposed minor improvements. To him and the staff at Springer, I express here my heartfelt gratitude.

Future updates can be found at
http://www.mat.uniroma1.it/people/manetti/librotopology.html.

Rome, February 2015 Marco Manetti

http://www.springer.com/978-3-319-16957-6

Topology
Manetti, M.
2015, XII, 309 p. 72 illus., Softcover
ISBN: 978-3-319-16957-6