Preface

It was the quest for a meaning, the praise of doubt, the wonderful fascination exerted by research in one of my diaries that seized me. The author was a mathematician from the school of Renato Cacciopoli. He spent his entire life among numbers, haunted by the febrile disquietude caused by the discovery of infinities. Astronomers rub shoulders with them, they seek them, they study them. Philosophers dream about them, talk about them, invent them. Mathematicians bring them to life, draw closer and closer and eventually touch them.

Walter Veltroni, *The discovery of dawn*[1]

This monograph aims at getting the reader acquainted with theories that play a central role in modern mathematics such as *integration* and *functional analysis*. Ultimately, these theories generalize notions that are treated in basic undergraduate courses—and even earlier, in high school—such as orthogonal vectors, linear transformations between Euclidean spaces, and the area delimited by the graph of a function of one real variable. Then, what is this generalization all about? It is about the more and more general nature of the environment in which these notions become meaningful: orthogonality in Hilbert spaces, linear transformations in Banach spaces, integration in measure spaces. These abstract structures are no longer restricted to a specific model like the real line or the Cartesian plane, but possess the least necessary properties to perform the operations we are interested in.

The reader should be warned that the above generalizations are not driven by mere search of abstraction or aesthetic pleasure. Indeed, on the one hand, this kind of procedure—typical in mathematics—allows to subsume a large body of results under few general theorems, the proof of which goes to the essence of the matter. On the other hand, in this way one discovers new phenomena and applications that

[1]Translated from Veltroni W., La scoperta dell'alba, p. 19. RCS Libri S.p.A., Milano (2006).

would be completely out of reach otherwise. In what follows, we have tried to share with the reader our interest for such an approach providing numerous examples, exercises, and some shortcuts to classical results, like our convolution-based proof of the Weierstrass approximation theorem for continuous functions.

We hope this textbook will be useful to graduate students in mathematics, who will find the basic material they will need in their future careers, no matter what they choose to specialize in, as well as researchers in other disciplines, who will be able to read this book without having to know a long list of preliminaries, such as Lebesgue integration in $\mathbb{R}^n$ or compactness criteria for families of continuous functions. The appendices at the end of the book cover a variety of topics ranging from the distance function to Ekeland's variational principle. This material is intended to render the exposition completely self-contained for whoever masters basic linear algebra and mathematical analysis.

Another aspect we would like to point out is that the two main subjects of this monograph, namely integration and functional analysis, are not treated as independent topics but as deeply intertwined theories. This feature is particularly evident in the large choice of problems we propose, the solution of which is often assisted with generous hints. Chapters 1–6 allow to cover both integration and functional analysis in a single course requiring a certain effort on the students' part.

If the material is split into two courses, then one can pick additional topics from the third part of the book, such as functions of bounded variation, absolutely continuous functions, signed measures, the Radon-Nikodym theorem, the characterization of the duals of Lebesgue spaces, and an introduction to set-valued maps. However, the two topics can be treated independently, as one is sometimes forced to do. In this case, Chaps. 1–4 provide the base for a course on integration theory for a broad range of students, not only for those with an interest in analysis. For instance, we have chosen an abstract approach to measure theory in order to quickly derive the extension theorem for countably additive set functions, which is a fundamental result of frequent use in probability. Chapters 5 and 6 are an essential introduction to functional analysis which highlights geometrical aspects of infinite-dimensional spaces. This part of the exposition is appropriate even for undergraduates once all examples requiring measure theory have been filtered out. Indeed, the new phenomena that occur in infinite-dimensional spaces are well exemplified in ℓ^p spaces, without need of any advanced measure-theoretical tools.

To conclude this preface, we would like to express our gratitude to all the people who made this work possible. In particular, we are deeply grateful to Giuseppe Da Prato who originated this monograph providing inspiration for both contents and methods. We would also like to thank our friend Ciro Ciliberto for encouraging us to turn our lecture notes into a book, getting us in touch with Francesca Bonadei who gave us all her valuable professional help and support. Many thanks are due to our students at the University of Rome "Tor Vergata", who read preliminary versions of our notes and solved most of the problems we propose in this textbook. We

wish to send special thanks, directly from our hearts, to Carlo Sinestrari and Francesca Tovena for standing by with their precious advice and invaluable patience. Finally, we would like to share with the reader our happiness for the increased set of names to whom this volume is dedicated, compared to the Italian edition. It is true that time does not go by in vain.

Rome, Italy
October 2014

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http://www.springer.com/978-3-319-17018-3

Introduction to Measure Theory and Functional Analysis
Cannarsa, P.; D'Aprile, T.
2015, XIV, 314 p. 8 illus., 1 illus. in color., Softcover
ISBN: 978-3-319-17018-3