Chapter 2
Centralized Online Algorithm for Optimal Energy Distribution in Connected Microgrid

Abstract The two-way energy and information flows in SG environment, together with the smart devices, bring new perspectives to energy management and demand response in MGs. This chapter investigates an online algorithm for electricity energy distribution in a connected MG. We first present a formulation that captures the key design factors such as user's utility and cost, grid load smoothing, dynamic pricing, and energy provisioning cost. The problem is shown to be convex and can be solved with an offline algorithm if future user and grid-related information are known a priori. We then develop an online algorithm that only requires past and present information about users and the grid, and prove that the online solution is asymptotically optimal. The proposed energy distribution framework and the online algorithm are quite general, suitable for a wide range of utility, cost, and pricing functions. It is evaluated with trace-driven simulations and shown to outperform a benchmark scheme.

Keywords Demand response · Energy management · Centralized online algorithm · Microgrid

2.1 Introduction

As stated in Chap. 1, DSM has been widely deployed in SG environment, which can be helpful for energy management in connected MGs. Researchers work mainly on demand profile shaping, user utility maximization, and cost reduction [1–10]. For example, machine learning is used in [3] to develop a learning algorithm for energy costs reduction and energy usage smoothing, while [6] aims to achieve a balance between user's cost and waiting time. In [7], the authors propose an optimal real-time pricing algorithm to maximize the social welfare, considering user utility maximization and energy provider cost minimization. In [9], the authors formulate a Stackelberg game between utility companies and end users aiming to maximize the revenue of each utility company and the payoff of each user. In [10], the authors discuss the architecture of home machine-to-machine (M2M) networks for energy management, which is an important component in the SG. In these works, convex

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programming, machine learning, and game theory are mostly used. A constrained multi-objective optimization problem is formulated in [11] to minimize energy consumption cost and to maximize a certain utility among a group of users. Lyapunov optimization is adopted in [1, 12] to stabilize the energy storage and user utility while reducing the operation cost of an MG. Lyapunov optimization is also used in [13] to optimally schedule the usage of all the energy resources in the system and minimize the long-term time-averaged expected total cost of supporting all users' load demand.

For the energy management in a single MG, research works cover several main topics, including interface or coupling between an MG and the Macrogrid, DER dispatching and power support, and energy management. In [14], the MG control strategies and energy management are examined from several aspects. In [15], a detailed report is presented to test the building and management of a hydrogen MG in Spain in a simple and reliable way. In [16], Tsikalakis and Hatziargyriou present a control operation for a centralized controller for MGs, which maximizes its value by optimizing the production of local DGs and power exchanges with the main distribution grid during interconnected operation. In [17], the authors introduce an economic power dispatching scheme for stable operation of an MG, while a multi-agent system is presented in [18] for DER energy management in an MG. The authors of [19] propose a multivariable digital control design methodology for the voltage regulation of an islanded single distributed generation (DG) unit MG and its dedicated load.

On the other hand, online algorithms [20] are widely used in wireless communications and networking, where precise channel and network information are hard to obtain. Recent research on solving wireless networking problems using online algorithms can be found in [21–24]. In [22], two online algorithms are developed from the optimal offline algorithms to maximize the amount of unit-length packets scheduled in a packet-switching mechanism. The authors of [23] address the energy-efficient uplink scheduling problem in a multiuser wireless system. With an online algorithm, an optimal scheduling is achieved without prior knowledge on arrival and channel statistics. In [24], online algorithm is applied to overcome the dynamic nature of the time-varying channels in wireless networks and then the throughput of the single transmitter is maximized by optimal power assignment. In [21], online algorithm is used for multiuser video streaming in a wireless system so that user's perceived video quality and its variations are jointly considered for a maximization with almost no statistical information about the congested channels.

Our work is inspired by the online algorithm works, which demonstrate the high potential of online algorithms for solving optimization problems with relatively limited information. In power systems, it is possible to use online algorithms to detect and control the grid load variance in real time. Motivated by this observation, we propose an energy distribution online algorithm to achieve utility maximization and load smoothing in a connected MG. We consider the key design factors from users, energy provider, and load variance in the problem formulation. The proposed online algorithm is quite effective as shown in Sect. 2.7.

In this chapter, we consider real-time energy distribution in connected MG. As shown in Fig. 2.1, MGCC collects real-time information from the three key

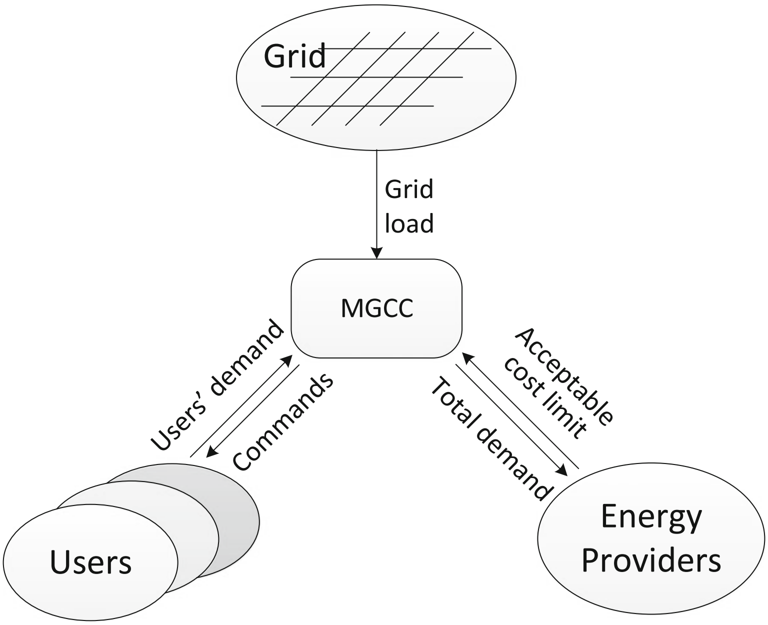

Fig. 2.1 Illustration of the key elements and interactions in the connected MG

components, i.e., the users, the grid, and the energy provider, makes decisions on, e.g., power transmission between MG and macrogrid, and then sends the decisions back to the key components to control their operations. The smart meters at the user side will be responsible for the information exchange with the MGCC and for enforcing the electricity schedule received from the MGCC. The information flows will be carried through a communications network infrastructure, such as a wireless network or a power line communication system.

For optimizing the performance of such a complex network system, the utilities and costs of the three key components, i.e., the users, the grid, and the energy provider, should be jointly considered. In this chapter, we take a holistic approach, to incorporate the key design factors including user's utility and cost, grid load smoothing, dynamic pricing, and energy provisioning cost in a problem formulation. To solve the real-time energy distribution problem, we first present an offline algorithm that can produce optimal solutions but assuming that the future user and grid information are known in advance. Based on the offline algorithm, we then develop an online algorithm that does not require any future information. As the name suggests, an online algorithm operates in an online setting, where the complete input is not known a priori [20]. It is very useful for solving problems with uncertainties [21]. We find the online algorithm particularly suitable in addressing the lack of accurate mathematical models and the lack of future information for electricity demand and supply in this problem. We also prove that the online algorithm converges to the optimal offline algorithm almost surely.

The proposed framework is quite general. It does not require any specific models for the electricity demand and supply processes, and only have some mild assumptions on the utility, cost, and price functions (e.g., convex and differentiable). The proposed algorithm can thus be applied to many different scenarios. The online algorithm also does not require any future information, making it easy to be implemented in a real MG system. It is also asymptotically optimal, a highly desirable property. Since there is no need for communications among the users, their privacy can be easily protected. The proposed algorithm is evaluated with trace-driven simulation using energy consumption traces recorded in the field. It outperforms a benchmark scheme that assumes global information.

The remainder of this chapter is organized as follows. We present the system model and problem formulation in Sect. 2.2. The offline algorithm is introduced in Sect. 2.3, and the online algorithm is developed and analyzed in Sect. 2.4. The communications protocol for supporting the online algorithm is discussed in Sect. 2.5. A practical online algorithm is presented in Sect. 2.6. We present the simulation studies in Sect. 2.7. Section 2.8 concludes this chapter. The notations used in this chapter are summarized in Table 2.1.

Table 2.1 Notations used in this chapter

Symbol	Description
$\mathbb{N}$	Set of electricity users in the system
$\mathbb{P}$	Set of power demand or consumption for users in a time slot
$\mathbb{C}$	Set of maximum cost for the energy provider at any time t
$\mathbb{U}$	Set of user utility functions
N	Number of users in the system
T	Total number of time slots in offline problem
$\mathbf{P}$	Power usage by the N users from time $1 : T$, offline
$\mathbf{P}_i$	Power usage by user i from time $1 : T$, offline
$\mathbf{P}(t)$	Power usage by the N users at time t, offline
$P_i(t)$	Power usage by user i at time t, offline
$\mathbf{P}^*$	Optimal solution of the offline problem
$\mathbf{P}_i^*$	Optimal power distribution for user i from time $1 : T$, offline
$\mathbf{P}^*(t)$	Optimal power distribution for N users at time t, offline
$P_i^*(t)$	Optimal power distribution for user i at time t, offline
$\eta(t)$	Lagrange multipliers associated with the offline problem
$\gamma_i(t)$	Lagrange multipliers variable associated with the offline problem
$\mathbf{p}(t)$	Power usage by the N users at time t, online
$p_i(t)$	Power usage by user i at time t, online
p_i	Power usage by user i at a fixed time, online
$\hat{\mathbf{p}}(t)$	Asymptotically convergent vector in the online problem
$\hat{p}_i(t)$	Asymptotically convergent variable in the online problem
$\mathbf{p}^*$	Optimal solution of the online problem

(continued)

Table 2.1 (continued)

Symbol	Description
$\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$	Optimal power distribution for N users at time t, online
$\mathbf{p}^*(t)$	Short term for $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$
$p_i^*(\hat{\mathbf{p}}, c(t))$	Optimal power distribution for user i at time t, online
$p_i^*(t)$	Short term for $p_i^*(\hat{\mathbf{p}}, c(t))$
$\lambda(t)$	Lagrange multipliers associated with the online problem
$\nu_i(t)$	Lagrange multiplier variable associated with the online problem
$p_{i,min}(t)$	Minimum power demand for user i at time t
$\omega_i(t)$	The flexibility of user i at time t
$L(t)$	Grid load at time t
$c(t)$	Maximum cost for the energy provider at time t
$U(\cdot)$	User utility function
$C(\cdot)$	Cost function of energy provider
$f(\cdot)$	Price function
$\Psi(\cdot)$	Optimal objective value of the offline problem
$\Phi(\cdot)$	Sum of online Lagrange dual function for t from $1 : T$
ρ	Modified parameter of p

2.2 Problem Statement

2.2.1 System Model

2.2.1.1 Network Structure

We consider a power distribution system in a connected MG within a SG environment where energy providers support the power usage of N users. The energy providers can be in MG of distributed generation or macrogrid of centralized generation, and the users could be residential, commercial, and industrial energy consumers. Each user has a *smart meter* that manages the schedule of electrical devices. As stated in Chap. 2, we envisage that the smart meters could be a controller of electrical appliances in a house and are connected to the MGCC through a communication network. At each time cycle, the smart meters update user information to, and receive control information from the MGCC, while the MGCC decides the power distribution among the users based on the real-time information of MG and macrogrid, such as grid load, user demand, and provider's cost. The MGCC manages the entire system as a whole to achieve an optimum distribution scheme that balances the users' utility, supply cost of the energy provider, and the variance of the grid.

Here, the time cycles or slots indexed by $t \in \{1, 2, \ldots\}$ could be, e.g., 1 h, 0.5 h, 15 min and even shorter, according to the updating period of the smart meters and the size of the MG. Usually, the MGCC takes a one-day operation cycle based on the daily periodical nature of electricity usage. Note that this is not a requirement for the model but a practical scenario in most cases, which will be applied in the performance evaluation section. Let $\mathbb{N} = \{1, 2, \ldots, N\}$ be the set of users. We denote the power consumption of user i at time t as $p_i(t)$. At each time slot, user i's minimum demand $p_{i,min}(t)$ should be guaranteed, i.e.,

$$p_i(t) \geq p_{i,min}(t), \forall\, i \in \mathbb{N}, t. \tag{2.1}$$

Besides, we assume that the users are rational, which means that at each time slot, power demand of each user has an upper bound, i.e., $p_i(t) \leq p_{i,max}(t)$. This will not become a constraint in our problem, because we aim to satisfy the user demand as much as possible under other constraints. However, this assumption together with (2.1) guarantees a closed set $\mathbb{P}$ which includes all the possible values of power demanded and used, that is, $p_i(t) \in \mathbb{P}$.

2.2.1.2 User Utility Function

We assume independent users with their own preferences of power usage. For example, each user could have its own time schedule for using different electrical appliances. Also, the user demand may vary as weather changes. Usually, the power consumption is larger in a hot summer day than that in a mild day in the spring. Besides, different users may have different reactions to different price schemes [5]. Therefore, it is difficult to characterize user preference with a precise mathematical model. In prior work, user preference is usually represented by a *utility function* [3]. Similarly, we use function $U(p_i(t), \omega_i(t))$ to represent user i's satisfaction on power consumption. We assume $U(\cdot, \cdot)$ to be a strictly increasing, concave function of the allocated power $p_i(t)$; its form could be general. One example is the widely used quadratic utility function [3, 5, 7]. For each user i, the other parameter $\omega_i(t)$ of the utility function indicates the user's flexibility at time t. A larger $\omega_i(t)$ means higher flexibility. $\omega_i(t)$ could be different for users or vary over time. Its values are sent to the MGCC at each updating cycle by the smart meter.

2.2.1.3 Energy Provisioning Cost

For energy providers, when demand is in the normal level, the generation cost increases only slowly as the demand grows. However, it will cost much more when the load peak is approaching the grid capacity, because the provider has to transmit more power from the macrogrid to avoid a shortage of power supply. Therefore, we use an increasing and strictly convex function to approximate the *cost function* for energy provisioning. Similar to [5, 7], we choose a quadratic function to model the provider's cost.

$$C(L(t)) = a \cdot L^2(t) + b \cdot L(t) + c, \tag{2.2}$$

where $a > 0$ and $b, c \geq 0$ are preselected for the MG and $L(t) = \sum_{i \in \mathbb{N}} p_i(t)$ denotes the grid load, i.e., the total power consumption for time slot t. From the provider's perspective, we assume that it aims to meet the user demand under an acceptable cost constraint $c(t)$ at time t, which shall not be exceeded.

$$C(L(t)) \leq c(t), \forall\, t \in \{1, 2, \ldots, T\}. \tag{2.3}$$

We call $c(t)$ *budget* in the rest of this chapter. Without loss of generality, we assume $c(t)$ to be an ergodic process, which is taken from a set $\mathbb{C}$, i.e., $c(t) \in \mathbb{C}$.

2.2.1.4 Price Model

Dynamic pricing like real-time pricing (RTP), critical peak pricing (CPP), and time of use pricing (TUP) [25] could be incorporated in the SG environment. However, real electricity market is still dominated by simple pricing schemes. In this chapter, we use a simple price model that can characterize most real electricity markets, especially for residential usage. As shown in [26, 27], without dynamic price demand, the price load curve has the shape of a *hockey stick*; it remains flat over a long range of grid load and then grows upward steeply as demand approaches the grid capacity. Let $f(\cdot)$ be the *price function* and $f(L(t))$ the price at time t. Therefore, we assume $f(\cdot)$ to be a twice-differentiable increasing convex function that maps the total load to a price. Similar to the utility function $U(\cdot)$, the price function $f(\cdot)$ could have a general form as well.

2.2.2 Problem Formulation

As mentioned in Sect. 2.1, we aim to minimize the load variance in the grid while maximizing user satisfaction. Large load variance is undesirable for grid operation. It brings about uncertainties that affect not only user satisfaction but also the stability of the power system. Furthermore, the energy provisioning cost should be bounded and users' necessary power needs should be guaranteed.

We first consider an offline scenario where the MGCC distributes the power to users during time $t = 1, 2, \ldots, T$, and all the information on users' flexibility $\omega_i(t)$ and provider's budget $c(t)$ are assumed to be known in advance. Let $P_i(t)$ denote the power usage for user i at time t, for $t \in \{1, 2, \ldots, T\}$. In this chapter, we use upper case P in the *offline problem* (see Sect. 2.3), where all the necessary constraints are known a priori. In the corresponding *online problem*, which will be examined in Sect. 2.4, we use lower case p for the corresponding variables. A vector with subscript i is used to denote a time sequence, e.g., $\mathbf{P}_i$ for the power usage by user i for $t \in \{1, 2, \ldots, T\}$. The offline problem can be formulated as follows:

$$\text{max:} \sum_{t=1}^{T}\sum_{i\in\mathbb{N}}\left[U(P_i(t),\omega_i(t)) - f\left(\sum_{i\in\mathbb{N}}P_i(t)\right)P_i(t)\right] - \frac{\alpha T}{2}\text{Var}\left(\sum_{i\in\mathbb{N}}\mathbf{P}_i\right) \tag{2.4}$$

subject to:

$$P_i(t) \geq P_{i,min}(t), \forall\, i \in \mathbb{N}, t \in \{1,2,\ldots,T\} \tag{2.5}$$

$$C\left(\sum_{i\in\mathbb{N}}P_i(t)\right) \leq c(t), \forall\, t \in \{1,2,\ldots,T\}, \tag{2.6}$$

where

$$\text{Var}\left(\sum_{i\in\mathbb{N}}\mathbf{P}_i\right) = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i\in\mathbb{N}}P_i(t) - \frac{1}{T}\sum_{k=1}^{T}\sum_{i\in\mathbb{N}}P_i(k)\right)^2.$$

The objective function (2.4) consists of two parts. The first part represents users' satisfaction and preference as the difference between user utility and cost. The second part represents the load variance of the grid. These two parts are integrated with a parameter $\alpha > 0$, allowing a trade-off between the two. Constraint (2.5) indicates that the minimum user demand should be guaranteed, while constraint (2.6) represents the cost upper bound for the energy provider. In Sect. 2.3, we present an algorithm that can solve this offline problem and explain how we can move from offline to online. In Sect. 2.4, we present an algorithm to solve the corresponding online problem that does not require any a priori user/grid information, and show that the online algorithm is asymptotically optimal.

2.3 Offline Algorithm

In the offline problem (2.4), the user power consumption $P_i(t)$ are independent. Hence the variance term can be rewritten as $\text{Var}(\sum_{i\in\mathbb{N}}\mathbf{P}_i) = \sum_{i\in\mathbb{N}}\text{Var}(\mathbf{P}_i)$ and the *price function* $f\left(\sum_{i\in\mathbb{N}}P_i(t)\right)$ is same for each user, which means

$$\sum_{i\in\mathbb{N}} f\left(\sum_{i\in\mathbb{N}}P_i(t)\right)P_i(t) = f\left(\sum_{i\in\mathbb{N}}P_i(t)\right)\sum_{i\in\mathbb{N}}P_i(t).$$

Therefore, we could depart the first term of (2.4) and rewrite the price term and variance term, respectively. Then the problem can be reformulated as follows (termed Prob-OFF).

$$\begin{aligned}\text{max: } \Psi(\mathbf{P}) = & \sum_{t=1}^{T} \sum_{i \in \mathbb{N}} U(P_i(t), \omega_i(t)) - \\ & \sum_{t=1}^{T} f\left(\sum_{i \in \mathbb{N}} P_i(t)\right) \sum_{i \in \mathbb{N}} P_i(t) - \frac{\alpha T}{2} \sum_{i \in \mathbb{N}} \mathrm{Var}\left(\mathbf{P}_i\right) \end{aligned} \tag{2.7}$$

subject to: (2.5) and (2.6),

where $\mathbf{P}$ is an $N \times T$ matrix that denotes the power allocated for each user i at time $t \in \{1, 2, \ldots, T\}$ and $\mathrm{Var}(\mathbf{P}_i) = \frac{1}{T} \sum_{t=1}^{T} \left(P_i(t) - \frac{1}{T} \sum_{k=1}^{T} P_i(k)\right)^2$.

In Prob-OFF, $U(\cdot)$ is concave and $C(\cdot)$ is convex. Since the price function $f(\cdot)$ is convex, $f(\sum_{i \in \mathbb{N}} P_i(t)) \sum_{i \in \mathbb{N}} P_i(t)$ is also convex. We only need to show the convexity of $\mathrm{Var}(\mathbf{P}_i)$ to establish a convex optimization problem. The convexity of $\mathrm{Var}(\mathbf{P}_i)$ can be easily proved by its definition.

Lemma 2.1 *Prob-OFF is a convex optimization problem and has a unique solution.*

The complete proof of Lemma 2.1 is presented in 2.8. As Lemma 2.1 holds, we can carefully choose $P_{i,min}(t)$ to meet Slater's condition [28], and thus the KKT conditions [28] are sufficient and necessary for the optimality of Prob-OFF. Let $\mathbf{P}^*$ be an optimal solution to Prob-OFF. Let $\eta(t)$ and $\gamma_i(t)$ be the Lagrange multipliers and variables, respectively, for $i \in \mathbb{N}$ and $t \in \{1, 2, \ldots, T\}$. We have

$$\begin{cases} U'(P_i^*(t), \omega_i(t)) - h\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right) - \alpha\left(P_i^*(t) - \bar{P}_i^*\right) - \\ \qquad \eta(t) C'\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right)/c(t) + \gamma_i(t) = 0 \\ \eta(t)\left(C\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right)/c(t) - 1\right) = 0 \\ \gamma_i(t)\left(P_i^*(t) - P_{i,min}(t)\right) = 0 \\ \eta(t), \gamma_i(t) \geq 0, \forall\, i \in \mathbb{N},\ t \in \{1, 2, \ldots, T\}, \end{cases} \tag{2.8}$$

where

$$h\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right) = f'\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right) \sum_{i \in \mathbb{N}} P_i^*(t) + f\left(\sum_{i \in \mathbb{N}} P_i^*(t)\right)$$

and

$$\bar{P}_i^* = \frac{1}{T} \sum_{k=1}^{T} P_i^*(k). \tag{2.9}$$

From the above equations, we can solve for $\eta(t)$ as

$$\eta(t) = \frac{\alpha\left(\bar{P}_i^* - P_i^*(t)\right) + U'(P_i^*(t), \omega_i(t)) - h(\sum_{i \in \mathbb{N}} P_i^*(t)) + \gamma_i(t)}{C'(\sum_{i \in \mathbb{N}} P_i^*(t))/c(t)} \tag{2.10}$$

Therefore, to achieve optimality, there is an identical $\eta(t)$ for all users in a time slot t. The optimal solution guarantees that the right-hand side (RHS) of (2.10) has the same value for all users. Furthermore, we observe that only the $\bar{P}_i^*$ term requires information from other time slots. This implies that if $\bar{P}_i^*$ could be accurately estimated, the optimal energy distribution $\mathbf{P}^*$ could be determined using only information in the current time slot, such as $c(t)$ and $P_{i,min}(t)$. This is essential, because in the offline scenario, our assumption that future information are known a priori is not a possible case in the real MG. Based on this observation, we are able to present an *online algorithm* for the energy distribution problem in the next section which requires no future information.

2.4 Online Algorithm

In this section, we present an online algorithm for energy distribution, and prove that the online solution is asymptotically convergent to the offline optimal solution, i.e., *asymptotically optimal*. The online energy distribution algorithm consists of the following three steps.

Step 1: For each $i \in \mathbb{N}$, initialize $\hat{p}_i(0) \in \mathbb{P}$.

Step 2: In each time slot t, the MGCC solves the following convex optimization problem (termed Prob-ON).

$$\text{max:} \quad \sum_{i\in\mathbb{N}} U(p_i(t), \omega_i(t)) - f\left(\sum_{i\in\mathbb{N}} p_i(t)\right)\sum_{i\in\mathbb{N}} p_i(t) - \frac{\alpha}{2}\sum_{i\in\mathbb{N}}(p_i(t) - \hat{p}_i(t-1))^2 \tag{2.11}$$

$$\text{subject to:} \quad p_i(t) \geq p_{i,min}(t), \forall\, i \in \mathbb{N} \tag{2.12}$$

$$C\left(\sum_{i\in\mathbb{N}} p_i(t)\right) \leq c(t), \forall\, t. \tag{2.13}$$

Let $\mathbf{p}^*(t)$ denote the solution to Prob-ON, where each element $p_i^*(t)$ represents the optimal power allocation to user i.

Step 3: Update $\hat{p}_i(t)$, for all $i \in \mathbb{N}$ as follows:

$$\hat{p}_i(t) = \hat{p}_i(t-1) + \frac{\alpha}{t+\alpha} \cdot (p_i^*(t) - \hat{p}_i(t-1)). \tag{2.14}$$

$\mathbf{p}^*(t)$ is indeed the short term of $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$. For brevity, we use $\mathbf{p}^*(t)$ instead in the chapter when it is clear in context. Comparing to (2.7), the variance term is approximated by $\sum_{i\in\mathbb{N}}(p_i(t) - \hat{p}_i(t-1))^2$ in (2.11). In Prob-ON, (2.14) can be viewed as a stochastic approximation updating equation, if the budget of the energy provider, $c(t)$, is viewed as a stationary stochastic process. This interpretation can be justified because $c(t)$ is assumed to be ergodic, and thus is stationary.

Similar to Prob-OFF, problem Prob-ON is also a convex optimization problem satisfying Slater's condition. Its KKT conditions with KKT multipliers $\lambda(t)$ and KKT variables $\nu_i(t)$, for $i \in \mathbb{N}$, are as follows:

$$
\begin{cases}
U'(p_i^*(t), \omega_i(t)) - h\left(\sum_{i \in \mathbb{N}} p_i^*(t)\right) - \alpha\left(p_i^*(t) - \hat{p}_i(t-1)\right) \\
\qquad\qquad -\lambda(t) C'\left(\sum_{i \in \mathbb{N}} p_i^*(t)\right)/c(t) + \nu_i(t) = 0 \\
\lambda(t)\left(C\left(\sum_{i \in \mathbb{N}} p_i^*(t)\right)/c(t) - 1\right) = 0 \\
\nu_i(t)\left(p_i^*(t) - p_{i,min}(t)\right) = 0 \\
\lambda(t), \nu_i(t) \geq 0, \quad \forall\, i, t.
\end{cases}
\tag{2.15}
$$

In the remainder of this section, we first prove that $\hat{p}_i(t)$ approaches a limit for t goes to infinity and then we show that $\hat{p}_i(t)$ converges to the mean of the power allocated to each user $i \in \mathbb{N}$ over time, as given in (2.9).

We begin with the definition of the function $g(\hat{\mathbf{p}}, c(t))$:

$$
\begin{aligned}
g(\hat{\mathbf{p}}, c(t)) = \sum_{i \in \mathbb{N}} & U(p_i^*(\hat{\mathbf{p}}, c(t)), \omega_i(t)) - \\
& f\left(\sum_{i \in \mathbb{N}} p_i^*(\hat{\mathbf{p}}, c(t))\right) \sum_{i \in \mathbb{N}} p_i^*(\hat{\mathbf{p}}, c(t)) - \\
& \frac{\alpha}{2} \sum_{i \in \mathbb{N}} (p_i^*(\hat{\mathbf{p}}, c(t)) - \hat{p}_i)^2.
\end{aligned}
\tag{2.16}
$$

Note that the optimized function $g(\hat{\mathbf{p}}, c(t))$ share the same form with (2.11), but with a different meaning. Here we regard the optimizer $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ and the optimized objective $g(\hat{\mathbf{p}}, c(t))$ as stochastic processes. We need to show that the process $\hat{\mathbf{p}}(t)$ converges almost surely, for given stationary stochastic process $c(t)$. We have the following immediate properties of $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ and $g(\hat{\mathbf{p}}, c(t))$.

Property 2.1 Continuity of $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ and $g(\hat{\mathbf{p}}, c(t))$.
For any $c(t) \in \mathbb{C}$, we have
(i) $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ and $g(\hat{\mathbf{p}}, c(t))$ are continuous functions of $\hat{\mathbf{p}}$;
(ii) $E[\mathbf{p}^*(\hat{\mathbf{p}}, c(t))]$, $E[g(\hat{\mathbf{p}}, c(t))]$ are continuous functions of $\hat{\mathbf{p}}$.

Property 2.2 Differentiability of $g(\hat{\mathbf{p}}, c(t))$ and $E[g(\hat{\mathbf{p}}, c(t))]$.
For any $c(t) \in \mathbb{C}$ and each $i \in \mathbb{N}$, we have
(i) $\nabla_{\hat{p}_i} g(\hat{\mathbf{p}}, c(t)) = \alpha(p_i^*(\hat{\mathbf{p}}, c(t)) - \hat{p}_i)$;
(ii) $\nabla_{\hat{p}_i} E[g(\hat{\mathbf{p}}, c(t))] = \alpha(E[p_i^*(\hat{\mathbf{p}}, c(t))] - \hat{p}_i)$.

With Properties 2.1 and 2.2, we are able to show the following result, which is an important step to the proof of the convergence of process $\hat{\mathbf{p}}$. We next show the convergence of $\hat{p}_i(t)$ as stated in the following Lemmas 2.2 and 2.3. The complete proofs of Properties 2.1 and 2.2, Lemmas 2.2 and 2.3 are shown in the Appendix.

Lemma 2.2 *The solution of the following fixed point equation is unique*

$$
E[\mathbf{p}^*(\hat{\mathbf{p}}, c(t))] = \hat{\mathbf{p}}. \tag{2.17}
$$

Lemma 2.3 *$\hat{p}_i(t)$ converges almost surely to the unique solution $\hat{\mathbf{p}}$ of the fixed point equation $E[\mathbf{p}^*(\hat{\mathbf{p}}, c(t))] = \hat{\mathbf{p}}$.*

Based on the convergence of $\hat{p}_i(t)$, we are ready to prove the asymptotic optimality of the online algorithm, which indicates that for a sufficiently long time period, the time averaged difference between the online and offline objective values will become negligible. We introduce the following lemma for the optimality proof.

Lemma 2.4 *The following limit exits and converges for $i \in \mathbb{N}$:*

$$\lim_{T\to\infty}\left(\frac{1}{T}\sum_{t=1}^{T} p_i^*(t) - \hat{p}_i(T)\right) = 0.$$

Proof Rewrite (2.14) and sum from $t = 1$ to T. We have

$$\sum_{t=1}^{T}\left(\frac{t+\alpha}{\alpha}\right)(\hat{p}_i(t) - \hat{p}_i(t-1)) = \sum_{t=1}^{T}(p_i^*(t) - \hat{p}_i(t-1)).$$

Expanding the sum on the LHS, it follows that

$$\begin{aligned}&\frac{1}{\alpha}\left(T\cdot\hat{p}_i(T) - \sum_{t=1}^{T}\hat{p}_i(t-1)\right) - (\hat{p}_i(T) - \hat{p}_i(1))\\ &= \sum_{t=1}^{T}(p_i^*(t) - \hat{p}_i(T) + \hat{p}_i(T) - \hat{p}_i(t-1)).\end{aligned}$$

Take limit over T on both sides and it follows that

$$\begin{aligned}&\lim_{T\to\infty}\frac{T\cdot\hat{p}_i(T) - \sum_{t=1}^{T}\hat{p}_i(t-1)}{\alpha\cdot T} - \lim_{T\to\infty}\frac{\hat{p}_i(T) - \hat{p}_i(1)}{T}\\ &= \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}(p_i^*(t) - \hat{p}_i(T) + \hat{p}_i(T) - p_i^*(t-1)).\end{aligned}$$

The second term of the LHS is zero as $T \to \infty$. Rearranging the terms, we have

$$\begin{aligned}&\lim_{T\to\infty}\left(\frac{1-\alpha}{\alpha}\right)\left(\hat{p}_i(T) - \frac{1}{T}\sum_{t=1}^{T}\hat{p}_i(t-1)\right)\\ &= \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}(p_i^*(t) - \hat{p}_i(T)).\end{aligned}$$

Since the sequence $\hat{p}_i(t)$ converges as shown in Lemma 2.3,

$$\lim_{T\to\infty}\left(\hat{p}_i(T) - \frac{1}{T}\sum_{t=1}^{T}\hat{p}_i(t-1)\right) = 0,$$

and the LHS will be zero. Thus the limit on the RHS will also be zero. □

Based on Lemma 2.4, we have the following theorem.

Theorem 2.1 *The online optimal solution converges asymptotically and almost surely to the offline optimal solution.*

Proof The proof is equivalent to showing that $\lim_{T\to\infty}\frac{1}{T}(\Psi(\mathbf{p}^*) - \Psi(\mathbf{P}^*)) = 0$ holds true almost surely, where $\mathbf{p}^*$ is online optimal solution and $\mathbf{P}^*$ is the offline optimal solution. Recall that $\lambda^*(t)$ and $\nu_i^*(t)$ are the nonnegative multipliers that satisfy the KKT conditions of the online problem (see (2.15)). We define a new differentiable concave function $\Phi(\cdot)$ as follows:

$$\begin{aligned}\Phi(\mathbf{P}^*) = &\sum_{t=1}^{T}\sum_{i\in\mathbb{N}} U(P_i^*(t), \omega_i(t)) - \sum_{t=1}^{T} f\left(\sum_{i\in\mathbb{N}} P_i^*(t)\right)\sum_{i\in\mathbb{N}} P_i^*(t) - \\ &\frac{\alpha T}{2}\sum_{i\in\mathbb{N}}\mathrm{Var}(\mathbf{P}_i^*) - \sum_{t=1}^{T}\lambda^*(t)\left(\frac{C(\sum_{i\in\mathbb{N}} P_i^*(t))}{c(t)} - 1\right) + \\ &\sum_{t=1}^{T}\sum_{i\in\mathbb{N}}\nu_i^*(t)(P_i^*(t) - P_{i,min}(t)).\end{aligned} \tag{2.18}$$

Note that the sum of the first three terms on the RHS of (2.18) is equal to $\Psi(\mathbf{P}^*)$, while the last two terms on the RHS of (2.18) are both nonnegative. It follows that

$$\Psi(\mathbf{P}^*) \le \Phi(\mathbf{P}^*). \tag{2.19}$$

Furthermore, with the concave and differentiable properties of function $\Phi(\cdot)$, we have [28]

$$\Phi(\mathbf{P}^*) \le \Phi(\mathbf{p}^*) + \triangledown\Phi(\mathbf{p}^*) \bullet (\mathbf{P}^* - \mathbf{p}^*), \tag{2.20}$$

where $\bullet$ denotes the inner product operation. Combining (2.19) and (2.20), we have

$$\begin{aligned}\Psi(\mathbf{P}^*) \le \Phi(\mathbf{P}^*) \le \Phi(\mathbf{p}^*) + \triangledown\Phi(p^*) \bullet (P^* - p^*) \quad (2.21)\\ = \sum_{t=1}^{T}\sum_{i\in\mathbb{N}} U(p_i^*(t), \omega_i(t)) -\end{aligned}$$

$$\sum_{t=1}^{T} f\left(\sum_{i\in\mathbb{N}} p_i^*(t)\right)\sum_{i\in\mathbb{N}} p_i^*(t) - \frac{\alpha T}{2}\sum_{i\in\mathbb{N}} \mathrm{Var}(\mathbf{p}_i^*) -$$
$$\sum_{t=1}^{T} \lambda^*(t)\left(\frac{C(\sum_{i\in\mathbb{N}} p_i^*(t))}{c(t)} - 1\right) +$$
$$\sum_{t=1}^{T}\sum_{i\in\mathbb{N}} \nu_i^*(t)(p_i^*(t) - p_{i,min}(t)) +$$
$$\sum_{t=1}^{T}\sum_{i\in\mathbb{N}}(P_i^*(t)-p_i^*(t))\left(U'(p_i^*(t),\omega_i(t))-g\left(\sum_{i\in\mathbb{N}} p_i^*(t)\right)+\right.$$
$$\left.\frac{\alpha}{T}\sum_{k=1}^{T} p_i^*(k)-\alpha p_i^*(t)-\lambda^*(t)\frac{C(\sum_{i\in\mathbb{N}} p_i^*(t))}{c(t)}+\nu_i^*(t)\right).$$

As $\lambda^*(t)$ and $\nu_i^*(t)$ are the Lagrange multipliers and variables of Prob-ON, we can substitute (2.15) into the above inequality (2.21) to have

$$\Psi(\mathbf{P}^*) \leq \sum_{t=1}^{T}\sum_{i\in\mathbb{N}} U(p_i^*(t),\omega_i(t)) -$$
$$\sum_{t=1}^{T} f\left(\sum_{i\in\mathbb{N}} p_i^*(t)\right)\sum_{i\in\mathbb{N}} p_i^*(t) - \frac{\alpha T}{2}\sum_{i\in\mathbb{N}} \mathrm{Var}(\mathbf{p}_i^*) +$$
$$\sum_{t=1}^{T}\sum_{i\in\mathbb{N}} \alpha(P_i^*(t) - p_i^*(t))\left(\frac{1}{T}\sum_{k=1}^{T} p_i^*(k) - \hat{p}_i(t-1)\right).$$

Adding $-\hat{p}_i(T) + \hat{p}_i(T)$ to the last component of the RHS of the above inequality, we have

$$\frac{1}{T}\sum_{k=1}^{T} p_i^*(k) - \hat{p}_i(t-1) = \frac{1}{T}\sum_{k=1}^{T} p_i^*(k) - \hat{p}_i(T) + \hat{p}_i(T) - \hat{p}_i(t-1). \qquad (2.22)$$

From Lemma 2.4, the limit of the above equation is zero for all users. We can take limit of (2.22) and it follows that

$$\lim_{T\to\infty}\frac{\Psi(\mathbf{P}^*)}{T} \leq \lim_{T\to\infty}\frac{1}{T}\left(\sum_{t=1}^{T}\sum_{i\in\mathbb{N}} U(p_i^*(t),\omega_i(t))-\right.$$
$$\left.\sum_{t=1}^{T} f\left(\sum_{i\in\mathbb{N}} p_i^*(t)\right)\sum_{i\in\mathbb{N}} p_i^*(t)-\frac{\alpha T}{2}\sum_{i\in\mathbb{N}}\mathrm{Var}(\mathbf{p}_i^*)\right)=\lim_{T\to\infty}\frac{\Psi(\mathbf{p}^*)}{T}.$$

Thus $\lim_{T\to\infty} \frac{1}{T}(\Psi(\mathbf{P}^*) - \Psi(\mathbf{p}^*)) \leq 0$ holds for all users. Because $\mathbf{P}^*$ is optimal to the offline problem and $\Psi(\mathbf{P}^*)$ is the offline objective value, we also have $\Psi(\mathbf{P}^*) \geq \Psi(\mathbf{p}^*)$. We conclude that Theorem 2.1 holds true. □

2.5 Communication Network Protocol

Information exchange is an important element in SG and the emerging MG. Communications between smart meters and the MGCC are essential for both control and distribution. The online algorithm is also based on such information exchanges. As more advances are made in SG, there is a compelling need for network architectures, standards, and protocols for communications in SG. We hereby introduce a basic protocol for communications network support in the SG, which is simple but sufficient to support the real-time online power distribution algorithm.

In the online power distribution algorithm, the users' basic demand for power and the maximum acceptable cost of EP should be updated in every decision period at the MGCC for real-time execution, because these are the constraints and are sometimes unpredictable. As we try to smooth the total power consumption of all users in the system, grid stability is another objective. We have four entities in the system: the MGCC is the core and Users, EP, and Grid are also important participants. At the beginning of each time slot, users send their demands to the MGCC through their smart meters, while EP informs the MGCC its acceptable cost limit. The MGCC also collects other information from the grid, such as the actual grid load. Then the MGCC executes the online power distribution algorithm using the updated information. It sends the allocated amounts to the users and the total usage or demand to the EP. Moreover, MGCC is able to send other control information to the EP or users for regulation, accounting, emergency response and alerts, etc.

Figure 2.2 illustrates the information flows in the network system. At each updating slot, the MGCC sends a grid information request to the grid, which returns relevant real-time grid parameters such as load condition and capacity. Meanwhile, users send their basic power demands to the MGCC to request power for the time period. Also, the EP sends its cost limit to the MGCC to get their energy provisioning cost controlled within an acceptable range. After the MGCC have gathered these necessary information, it applies the optimal online algorithm and then sends the results to the EP and users, so that the EP could supply the corresponding amount of power to the users. Finally, the grid will update the actual grid load to the MGCC for grid inspection and control. Note that there will be no information exchange among the users, so that their privacy (e.g., electricity usage habit) could be protected. Note that the update intervals are at the order of hour or tens of minutes. Given the data rate of existing wireless networks, such exchange of control information only takes a negligible fraction of the interval.

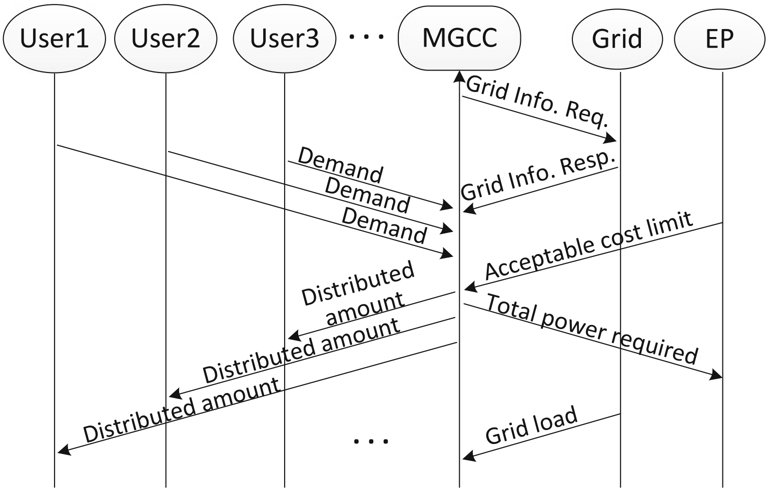

Fig. 2.2 Information flows in the connected MG

2.6 Practical Online Algorithm

In the communication network discussed in Sect. 2.5, we notice that the MGCC has to communicate with all the Users, the Grid, and the EP. It is a large burden when the grid system becomes larger. This only brings more users, but will increase the time for both calculation and communication. With modern network infrastructure and protocol, time for information exchange could be well controlled. However, in complicated practical situations, the *utility function*, the *cost function*, and the *price function* may have different realizations, some of which are very complex. This will no doubt bring much difficulty for the MGCC to solve the Prob-ON. In some cases, the KKT conditions (see (2.15)) are very difficult to solve especially in short intervals. Therefore,in this section, we present a practical online algorithm for energy distribution in connected MG, motivated by the online algorithm stated in Sect. 2.4.

Step 1: For each $i \in \mathbb{N}$, initialize $\hat{p}_i(0) \in \mathbb{P}$.

In each time slot t, do the next two steps:

Step 2: For each $i \in \mathbb{N}$, initialize $p_i(t) = p_{i,min}(t)$. Let the set $\mathbb{S} = \{1, 2, ..., N\}$. Then take the following loop:

while $\mathbb{S} \neq \emptyset$

For each $i \in \mathbb{S}$, take

$$\lambda_i = \frac{\alpha(\hat{p}_i(t-1) - p_i(t)) - h(\sum_{i \in \mathbb{N}} p_i(t)) + U'(p_i(t), \omega_i(t))}{(1/c(t))C'(\sum_{i \in \mathbb{N}} p_i(t))};$$

if $\max_{i \in \mathbb{S}} \lambda_i < 0$, then $\mathbb{S} = \emptyset$;

else $j = \text{argmax}_{i \in \mathbb{S}} \lambda_i$;

$p_j(t) = p_j(t) + step$;

if $p_j(t) > \max\{p : p \in \mathbb{P}\}$ or $C(\sum_{i \in \mathbb{N}} p_i(t)) > c(t)$

then $p_j(t) = p_j(t) - step$;

delete j from $\mathbb{S}$;

end

end

end

Step 3: In each time slot t, update $\hat{p}_i(t)$, for all $i \in \mathbb{N}$ as follows:

$$\hat{p}_i(t) = \hat{p}_i(t-1) + \frac{\alpha}{t+\alpha} \cdot (p_i(t) - \hat{p}_i(t-1)).$$

At each time slot t, $p_i(t)$, for all $i \in \mathbb{N}$ is the distribution power to user i. In the above practical algorithm, the derivative could be replaced by the difference equation, when the analytic function form of the function is difficult to be acquired. For example, use $\frac{C(\sum_{i \in \mathbb{N}} p_i(t)) - C(\sum_{i \in \mathbb{N}} p_i(t-1))}{\sum_{i \in \mathbb{N}}[p_i(t) - p_i(t-1)]}$ instead of $C'(\sum_{i \in \mathbb{N}} p_i(t))$, when the *cost function* cannot be formulated. From the practical algorithm, we see clearly the allocation process in evaluating λ_i in Step 2, which is a natural expression from (2.10). In this way, MGCC distributes the energy uniformly while not giving a user too much. So we would expect a more smoothy allocation.

The parameter $step$ controls the incremental precision and the running number (and time) of the loop in Step 2. When the updating interval is short, it is safe to set $step$ very small, which leads to a longer running time and vice versa. The complexity of the practical algorithm is roughly proportional to $N \cdot \max(p)/step$, i.e., the number of users times the maximum distributed energy over the increment. $step$ also decides the error between the practical solution and the theoretical solution to KKT conditions. So $step$ is an important parameter in the practical online algorithm. The MGCC could choose $step$ according to the length of updating periods and the number of users. With controlled $step$, the MGCC could support a large number of users. Although the good power distribution from the practical algorithm is not the optimal one, it is more practical as it can be used with complicated functions and its running time and precision could be controlled.

2.7 Performance Evaluation

2.7.1 Simulation Configuration

In this section, we evaluate the proposed online algorithm with trace-driven simulations. The simulation data and parameters are acquired from the traces of power consumption in the Southern California Edison (SCE) area recorded in 2011 [29].

We first study the performance of the proposed algorithm on convergence, grid load variance, and peak reduction. We then compare the online algorithm with an existing scheme under different numbers of users.

Consider a power distribution system in a small area with $N = 20$ users and 15 min updating periods. Note that a quarter is a practical set which allows MGCC to have sufficient time to coordinate all the users so that the system could support more users and that in most cases, 15 min is short enough to show the users' change of demand. We will show results within a 24-h time pattern for an evaluation of the daily operations. We choose users' utility function from a function set $\mathbb{U}$ in which the functions are generated as widely used quadratic expressions (see [3, 5, 7]), with $\omega_i(t) \in (0, 1)$ randomly selected.

$$U(p_i(t), \omega_i(t)) = \begin{cases} \omega_i(t)p_i(t) - \frac{1}{8}p_i(t)^2, & \text{if } 0 \le p_i(t) \le 4\omega_i(t) \\ 4\omega_i(t), & \text{if } p_i(t) \ge 4\omega_i(t). \end{cases} \tag{2.23}$$

We also assume that the basic user demand $p_{i,min}(t)$ and the initial value $p_i(0)$ are selected from the set of $\mathbb{P} = [0.5, 3]$, for all i. The parameters in the energy provisioning cost function (2.2) are set as $a = 0.05$, $b = c = 0$, and $c(t)$ is selected randomly from the set $\mathbb{C} = [1, 20]$ for each time slot. These parameters are carefully determined after studying the characteristics of the SCE trace. In addition, we choose the price function as

$$f(L(t)) = 0.047 \cdot L(t)^2 - 0.38 \cdot L(t) + 27.67. \tag{2.24}$$

It is a quadratic function and also a twice-differentiable increasing convex function as discussed in Sect. 2.2.1.4. This model is formulated from the predicted and actual prices from the SCE trace [30]. We simulate two scenarios with α set as 1 and 0.01, respectively, to examine how it affects the result.

2.7.2 Algorithm Performance

We first study the convergence of $\hat{p}_i(t)$. Earlier discussions in Sect. 2.4 show that $\hat{p}_i(t)$ is convergent. Figure 2.3 illustrates that for $\alpha = 1$, one day is sufficient for $p_i^*(t)$ to converge to steady state values. In Fig. 2.4, it takes more time to converge. In the online problem Prob-ON, α is not only a parameter integrating different objectives, but also an important coefficient affecting the convergence of the algorithm. In the online updating Eq. (2.14), it is clear that a large α will cause relatively a large disturbance, especially at the very beginning. However, a large α will also lead to fast convergence, and vice versa, as shown in Figs. 2.3 and 2.4. Besides, α also affects the impact of the variance (or, smoothness) on the overall objective value (2.11). It shapes the grid load curve to some degree, as we will see in Sect. 2.7.3.

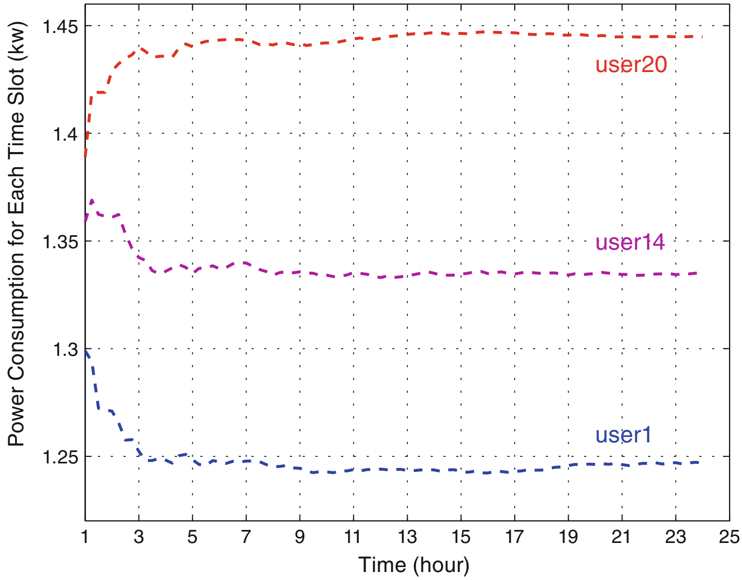

Fig. 2.3 Convergence of $\hat{p}_i(t)$ for different users ($\alpha = 1$)

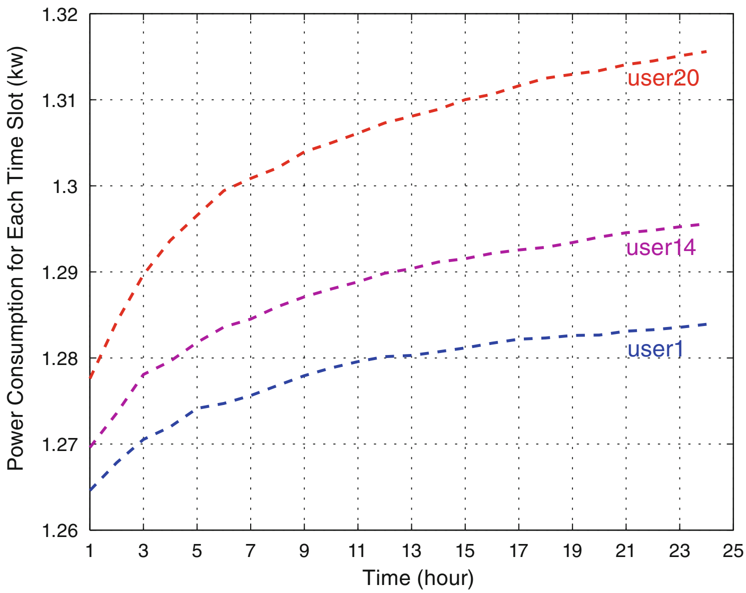

Fig. 2.4 Convergence of $\hat{p}_i(t)$ for different users ($\alpha = 0.01$)

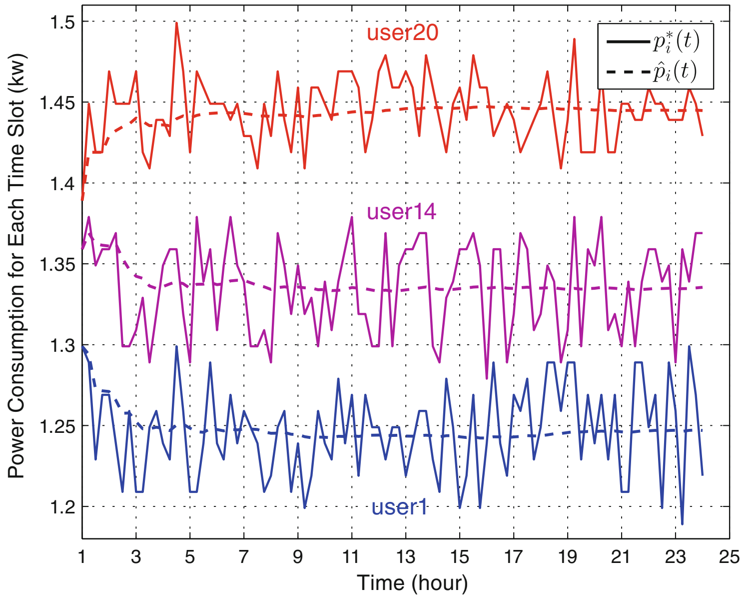

Fig. 2.5 Online power distribution $p_i^*(t)$ and $\hat{p}_i(t)$ for different users when $\alpha = 1$

Lemma 2.4 states that $\hat{p}_i(t)$ will converge to the time averaged $p_i^*(t)$ if we run the simulation sufficiently long. For a larger α, the convergence will be faster, shown in Fig. 2.5, where we find that $\hat{p}_i(t)$ fluctuates uniformly along the $p_i^*(t)$ curve for different users. For a smaller α, the convergence could be very slow. Figure 2.6 demonstrates the slow convergence when $\alpha = 0.01$. However, the convergence of $\hat{p}_i(t)$ is proved to be true as $T \to \infty$ (see the proof of Lemma 2.4). In Fig. 2.6, it can be seen that $\hat{p}_i(t)$ is still approaching $p_i^*(t)$, although slowly. Therefore, the value of α should be carefully chosen to trade-off between convergence and other objectives.

More importantly, our main objective is to develop an optimal online algorithm to reduce the variance of the grid load and to balance electricity demand and supply. In Fig. 2.7, we plot the total power consumption achieved with the online algorithm and the actual load. The real power usage is the summation of 20 independent users' consumption generated by the average real load in the SCE trace on a hot day (i.e., September 1, 2011) [29]. The constraints are derived from the real load in the 2011 SCE trace. For better presentation, we only plot the result of the online algorithm with $\alpha = 1$. The results with $\alpha = 0.01$ will be shown in Sect. 2.7.3.

In Fig. 2.7, we find that the online algorithm achieves a well smoothed grid load. Interestingly, although the power usage of each user varies over time (as shown in Fig. 2.5), the total power usage is effectively smoothed out by the online algorithm. This result demonstrates the effectiveness of variance detection and reduction of the online algorithm. Although the controlled curve lies slightly above the average level of the real load, it reduces the cost of energy provisioning by achieving a considerable peak reduction, which is about 35 % in this scenario with only 20 users.

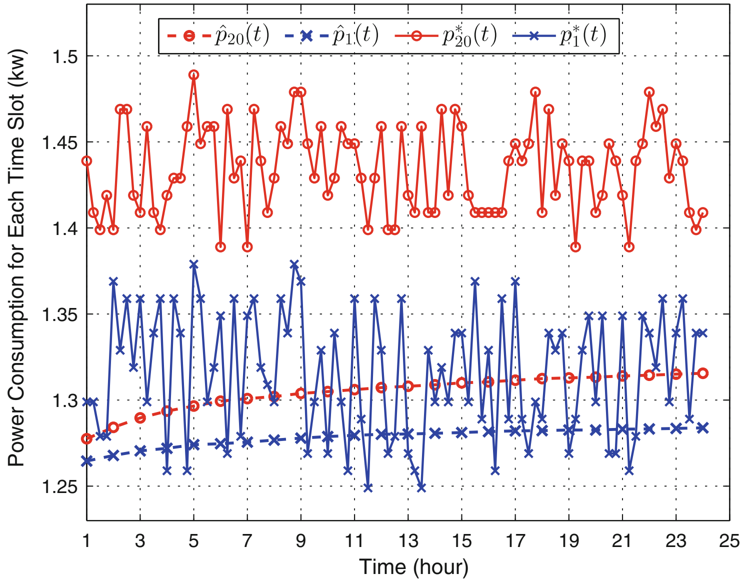

Fig. 2.6 Online power distribution $p_i^*(t)$ and $\hat{p}_i(t)$ for different users when $\alpha = 0.01$

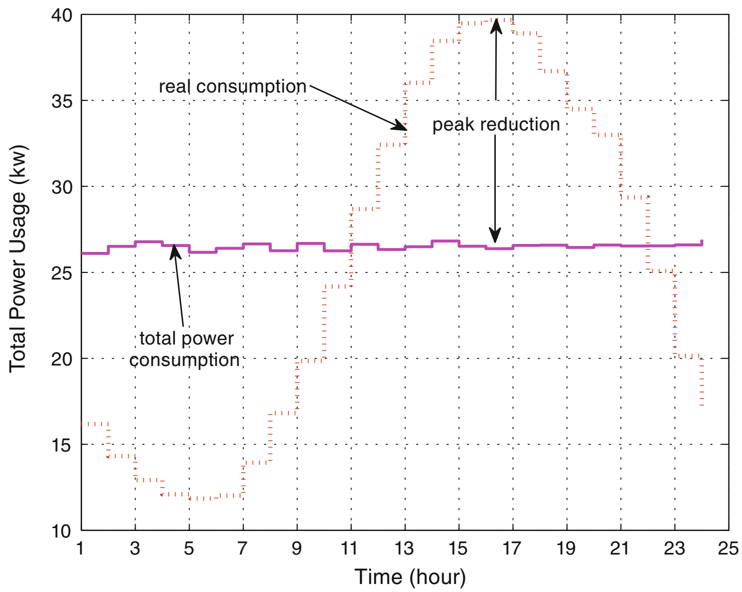

Fig. 2.7 Real power usage and total power usage by the online algorithm when $\alpha = 1$

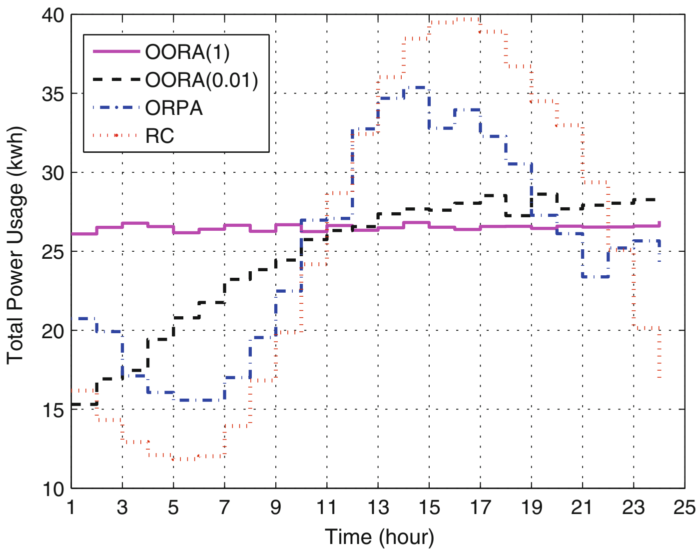

Fig. 2.8 Total power consumption for OORA(1), OORA(0.01), ORPA, and RC

2.7.3 Comparison with a Benchmark

We next compare the online algorithm with the Optimal Real-time Pricing Algorithm (ORPA) presented in [7] as a Benchmark. Comparing to prior work, this one formulates a similar but simpler problem to our problem. It adopts a real-time pricing strategy to maximize social welfare of the smart grid, as

$$\max \sum_{i \in \mathbb{N}} \left(U(p_i(t), \omega_i(t)) - C\left(\sum_{i \in \mathbb{N}} p_i(t) \right) \right), \tag{2.25}$$

for $t \in \{1, 2, \ldots, T\}$ and for all independent user i. As we can see, (2.25) is similar to but simpler than (2.11). With the same parameters as in the online algorithm, this is also a convex optimization problem. We can solve (2.25) with a centralized interior-point method as discussed in [7].

First, we show the total power consumption of different algorithms in Fig. 2.8. From the aspect of smoothness, we could see clearly that the online optimal real-time energy distribution algorithm with $\alpha = 1$ (termed OORA(1)) achieves the best performance. The figure also shows that the online algorithm with $\alpha = 0.01$ (termed OORA(0.01)) also outperforms the benchmark ORPA. All the three algorithms achieve smoother total loads than the real consumption (RC). The peak reductions over RC are 35 % for OORA(1), 28 % for OORA(0.01), and 12.5 % for ORPA. Therefore, OORA(1) achieves the largest peak reduction, while OORA(0.01) still outperforms ORPA with considerable gains.

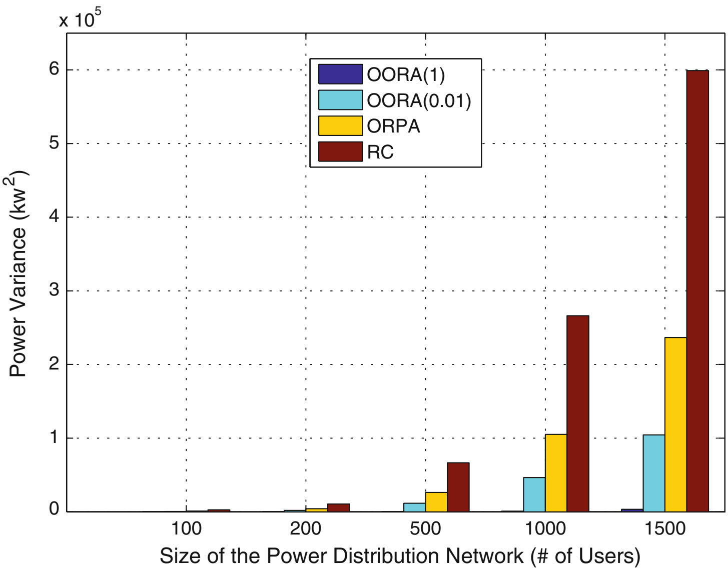

Fig. 2.9 Total power variance by OORA(1), OORA(0.01), ORPA, and RC

Next, we plot the variance of the total load in Fig. 2.9 for different system settings. These results are consistent with that in Fig. 2.8. We find that OORA(1) achieves the minimum variance for all the cases simulated, while OORA(0.01) still outperforms ORPA with a much smaller variance. This is because variance is explicitly incorporated into the objective function in the online problem formulation, while ORPA is designed mainly to maximize the social welfare as in (2.25) and cannot guarantee a smooth total grid load.

Finally, we provide a more detailed comparison of the three schemes in Table 2.2, where the simulation results of several individual performance measures are listed for networks of 200, 500, and 1000 users. Note that the price function is different for different network sizes, which is a function of the total load. As defined in (2.26), $\overline{V}$, $\overline{U}$, $\overline{F}$, and $\overline{PK}$ denote the averages across users of the total power variance, users' utility, users' cost, and the peak of the total load, respectively, while c is the total energy provisioning cost for the entire period.

$$\begin{cases} \overline{V} = \frac{1}{N} \sum_{i \in \mathbb{N}} \mathrm{Var}(p_i^*(t)) \\ \overline{U} = \frac{1}{N} \sum_{t=1}^{T} \sum_{i \in \mathbb{N}} U(p_i^*(t), \omega_i(t)) \\ \overline{F} = \frac{1}{N} \sum_{t=1}^{T} f(\sum_{i \in \mathbb{N}} p_i^*(t))(\sum_{i \in \mathbb{N}} p_i^*(t)) \\ \overline{PK} = \frac{1}{N} \max_{t \in [1:T]} \sum_{i \in \mathbb{N}} p_i^*(t) \\ c = \sum_{t=1}^{T} C(\sum_{i \in \mathbb{N}} p_i^*(t)). \end{cases} \tag{2.26}$$

For $\overline{V}$, the best performer is OORA(1), which is consistent with the earlier results. Also, the variance is increasing as the user number grows. For $\overline{F}$, we observe a

Table 2.2 Simulation results of individual performance measures for different algorithms

Algorithm	N	$\overline{V}$	$\overline{U}$	$\overline{F}$	c $(\times 10^3)$	$\overline{PK}$
OORA(1)	200	**0.02**	3.52	3.41	1.69	**1.35**
OORA(0.01)	200	9.3	3.59	3.27	1.61	1.46
ORPA	200	21.5	3.56	3.43	1.54	1.79
RC	200	53.5	3.86	3.65	1.86	2.07
OORA(1)	500	**0.05**	3.53	3.31	10.5	**1.37**
OORA(0.01)	500	23.2	3.63	3.42	10.1	1.51
ORPA	500	52.6	3.54	3.28	9.54	1.83
RC	500	113	3.88	3.61	14.0	2.27
OORA(1)	1000	**0.10**	3.51	3.25	42.2	**1.41**
OORA(0.01)	1000	44.1	3.59	3.30	40.2	1.59
ORPA	1000	105	3.54	3.25	38.1	1.93
RC	1000	266	3.87	4.23	54.1	2.58

relatively stable number of the averaged cost on daily electricity consumption for each user. In the first three algorithms, $\overline{F}$ is almost the same while RC always has the largest number because in reality where the RC curve was recorded, supply was always matched to the user demand. This is confirmed by the results of users' utility $\overline{U}$: as users could use electricity freely, they should have the highest satisfaction level. Observing $\overline{U}$ and $\overline{F}$, we see that a higher satisfaction level is achieved with a higher cost. Moreover, it is interesting to see that utility $\overline{U}$ of OORA(1), OORA(0.01), ORPA are almost the same for different numbers of users, with OORA(0.01) being slightly better. This is because, as in ORPA, the utility is incorporated in the objective function of OORA. When α is small, the first two terms in (2.11) will have larger weights.

For energy provisioning cost c, ORPA exhibits its advantage as it include this term in the objective function. Also, if we take $\overline{U} - c$, ORPA is also the best performer, which could be expected from its objective function (2.25). However, this advantage becomes insignificant when the variance $\overline{V}$ and the peak $\overline{PK}$ are considered. OORA has unique advantages on variance control and peak reduction. It is also worth noting that OORA is an online algorithm that requires minimal exchange of control/state information within the grid, while the ORPA results are obtained with a centralized solver assuming accurate global information.

2.8 Conclusion

In this chapter, we present a study of optimal real-time energy distribution for a connected MG. With a formulation that captures the key design factors of the system, we first present an offline algorithm that can solve the problem with optimal solutions.

We then develop an online algorithm that requires no future information about users and the grid. We also show that the online solution converges to the offline optimal solution asymptotically and almost surely. The proposed online algorithm is evaluated with trace-driven simulations and is shown to outperform an existing benchmark scheme.

Appendix

Proof of Property 2.1

Proof (i) From (2.11), $g(\hat{\mathbf{p}}, c(t))$ is a continuous function of $\hat{\mathbf{p}}$. The continuity of $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ could be guaranteed if all the four conditions of Theorem 2.2 from [31] are satisfied. The conditions are verified because Prob-ON is always feasible on a closed set and $\hat{\mathbf{p}}$ is bounded on a set $\mathbb{P}$ in our case. Therefore, $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ is continuous with respect to $\hat{\mathbf{p}}$.

(ii) Take $\hat{\mathbf{p}}_n$ as any sequence such that $\lim_{n\to\infty} \hat{\mathbf{p}}_n = \hat{\mathbf{p}}$. Then we have

$$\begin{aligned}\lim_{m\to\infty} E[p_i^*(\hat{\mathbf{p}}_n, c(t))] &= E[\lim_{m\to\infty} p_i^*(\hat{\mathbf{p}}_n, c(t))] \\ &= E[p_i^*(\hat{\mathbf{p}}, c(t))],\end{aligned}$$

which follows the Bounded Convergence Theorem since we already have the continuity of $\mathbf{p}^*(\hat{\mathbf{p}}, c(t))$ and the closed set $\mathbb{P}$ of p_i^* (see 2.2.1.1). Consequently, $E[g(\hat{\mathbf{p}}, c(t))]$ is also continuous. □

Proof of Property 2.2

Proof (i) The differentiability of $g(\hat{\mathbf{p}}, c(t))$ follows directly from Theorem 4.1 in [32].

(ii) Similar to the proof in Part (ii) of Property 2.1, take any sequence $\hat{p}_{i,n}$ such that $\lim_{n\to\infty} \hat{p}_{i,n} = 0$. We have that

$$\left|\frac{g(\hat{\mathbf{p}} + \hat{p}_{i,n}\mathbf{e}) - g(\hat{\mathbf{p}}, c(t))}{\hat{p}_{i,n}}\right| = \alpha \left|p_i^*(\hat{\mathbf{p}} + \hat{p}_{0,n}\mathbf{e}, c(t)) - \hat{p}_i - p_{0,n}\right| \le \alpha p_{max},$$

for $0 < p_{0,n} < \hat{p}_{i,n}$, which follows the Mean Value Theorem and part (ii) of Property 2.2. For each $i \in \mathbb{N}$,

$$\begin{aligned}\frac{\partial}{\partial \hat{p}_i}E[g(\hat{\mathbf{p}}, c(t))] &= \lim_{n\to\infty} E\left[\frac{g(\hat{\mathbf{p}}+\hat{p}_{i,n}\mathbf{e})-g(\hat{\mathbf{p}}, c(t))}{\hat{p}_{i,n}}\right]\\ &= E\left[\lim_{n\to\infty}\frac{g(\hat{\mathbf{p}}+\hat{p}_{i,n}\mathbf{e})-g(\hat{\mathbf{p}}, c(t))}{\hat{p}_{i,n}}\right]\\ &= \alpha(E[p_i^*(\hat{\mathbf{p}}, c(t))]-\hat{p}_i). \qquad \square\end{aligned}$$

Proof of Lemma 2.1

Proof For two vectors $\mathbf{P}_i^1$, $\mathbf{P}_i^2$ and for any $i \in \mathbb{N}$, $0 < \theta < 1$, it follows from the variance definition and the strict convexity of quadratic function $f(x) = x^2$ that

$$\mathrm{Var}(\theta\mathbf{P}_i^1 + (1-\theta)\mathbf{P}_i^2) \leq \theta\mathrm{Var}(\mathbf{P}_i^1) + (1-\theta)\mathrm{Var}(\mathbf{P}_i^2).$$

We conclude that $\mathrm{Var}(\mathbf{P}_i)$ is strictly convex unless $\mathrm{Var}(\mathbf{P}_i^1) = \mathrm{Var}(\mathbf{P}_i^2)$. Since all the constraints of Prob-OFF are also convex, we conclude that Prob-OFF is a convex problem.

We next prove that Prob-OFF has a unique solution. Assume $\mathbf{P}_i^1$ and $\mathbf{P}_i^2$ are two optimal solutions to Prob-OFF. Because the objective function is concave, $\theta\mathbf{P}_i^1 + (1-\theta)\mathbf{P}_i^2$ is also optimal, for $0 < \theta < 1$. Note that we have three terms that are all concave (or convex) in (2.7). Thus $\theta\mathbf{P}_i^1 + (1-\theta)\mathbf{P}_i^2$ is optimal only if

$$U(\theta P_i^1(t)+(1-\theta)P_i^2(t)) = \theta U(P_i^1(t))+(1-\theta)U(P_i^2(t)) \tag{2.27}$$

$$\begin{aligned}&f\left(\theta\mathbf{P}_i^1 + (1-\theta)\mathbf{P}_i^2\right)\cdot\left(\theta\sum_{i\in\mathbb{N}}P_i^1(t) + (1-\theta)\sum_{i\in\mathbb{N}}P_i^2(t)\right)\\ &= \theta f(\mathbf{P}_i^1)\sum_{i\in\mathbb{N}}P_i^1(t) + (1-\theta)f(\mathbf{P}_i^2)\sum_{i\in\mathbb{N}}P_i^2(t)\end{aligned} \tag{2.28}$$

$$\mathrm{Var}(\theta\mathbf{P}_i^1 + (1-\theta)\mathbf{P}_i^2) = \theta\mathrm{Var}(\mathbf{P}_i^1) + (1-\theta)\mathrm{Var}(\mathbf{P}_i^2), \forall\, i \in \mathbb{N}. \tag{2.29}$$

Since $U(\cdot)$ is assumed to be a strictly increasing function in Sect. 2.2.1.2, (2.27) holds true if and only if $P_i^1(t) = P_i^2(t)$, for all $i \in \mathbb{N}$, $t \in \{1, 2, \ldots, T\}$. Equations (2.28) and (2.29) are also sufficient for this result. Therefore, we conclude that Prob-OFF is a convex problem with a unique solution. $\square$

Proof of Lemma 2.2

Proof We define several notations to be used in this proof. Define $\rho_i = \sqrt{\alpha}p_i$ and $\rho_i^*(\hat{\boldsymbol{\rho}}, c(t)) = \sqrt{\alpha}p_i^*(\hat{\mathbf{p}}, c(t))$ for each $i \in \mathbb{N}$ and $p_i \in \mathbb{P}$, and function $\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}, c(t))$. Also define

$$\mathrm{dist}(\mathbf{p}^1, \mathbf{p}^2) = \sqrt{\sum_{i \in \mathbf{N}} (p_i^1 - p_i^2)^2} = \left| \sum_{i \in \mathbf{N}} (p_i^1 - p_i^2) \right|, \text{ for any } \mathbf{p}^1, \mathbf{p}^2 \in \mathbb{P}^N.$$

We next show the following two intermediate results that will be used to prove the lemma. The first result is that the solution of the next fixed point equation exits.

$$E[\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}, c(t))] = \hat{\boldsymbol{\rho}}. \tag{2.30}$$

It follows Property 2.1 that $E[\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}, c(t))]$ is a continuous function and it maps a convex compact subset of $\mathbb{P}^N$ to itself. Hence from Brouwer's Fixed Point Theorem in [33], the existence of the solution to (2.30) can be shown.

Second, we show that $E[\boldsymbol{\rho}_\mathbf{i}{}^*(., c(t))]$ is a pseudocontraction. Since $\mathbb{P}^N$ is a compact set, we need to show equivalently that for any two different $\hat{\mathbf{p}}^1$ and $\hat{\mathbf{p}}^2 \in \mathbb{P}^N$,

$$\mathrm{dist}(E[\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}^1, c(t))], E[\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}^2, c(t))]) < \mathrm{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2).$$

Here, let $\hat{\boldsymbol{\rho}}^1$ be a solution to (2.30) and $\hat{\boldsymbol{\rho}}^2 \neq \hat{\boldsymbol{\rho}}^1$.

To prove this, we modify the Prob-ON to obtain a new problem New-Prob-ON as

$$\begin{aligned} \max: \quad & g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}) \\ \text{subject to:} \quad & \frac{\rho_i}{\sqrt{\alpha}} \geq p_{i,min}, \forall\, i \in \mathbb{N} \\ & C\left(\sum_{i \in \mathbb{N}} \frac{\rho_i}{\sqrt{\alpha}}\right) \leq c(t), \forall\, t, \end{aligned} \tag{2.31}$$

where

$$g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}) = \sum_{i \in \mathbb{N}} U(\frac{\rho_i}{\sqrt{\alpha}}, \omega_i) - f(\sum_{i \in \mathbb{N}} \frac{\rho_i}{\sqrt{\alpha}}) \sum_{i \in \mathbb{N}} \frac{\rho_i}{\sqrt{\alpha}} - \frac{\alpha}{2} \sum_{i \in \mathbb{N}} (\frac{\rho_i}{\sqrt{\alpha}} - \frac{\hat{\rho}_i}{\sqrt{\alpha}})^2.$$

For brevity, we drop the time index (t) in the remainder of this proof, when their meanings are clear in the context. Note that $\boldsymbol{\rho}_\mathbf{i}{}^*(\hat{\boldsymbol{\rho}}, c(t))$ is the optimal solution for New-Prob-ON.

Now, we use Proposition 6.1 from [32] to achieve the Lipschitz continuity and acquire the Lipschitz constant of $\boldsymbol{\rho}_\mathbf{i}{}^*(., c(t))$ in a neighborhood of $\hat{\boldsymbol{\rho}}^1$. Two conditions are necessary to hold the proposition: the Lipschitz continuity of the difference function in a neighborhood of $\hat{\boldsymbol{\rho}}^1$ and the second-order growth condition.

We define the difference function $\Delta g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2)$ as

$$\begin{aligned} \Delta g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2) &= g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^2) - g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^1) \\ &= \frac{1}{2} \sum_{i \in \mathbb{N}} (\hat{\rho}_i^1 - \hat{\rho}_i^2)(2\rho_i - \hat{\rho}_i^1 - \hat{\rho}_i^2). \end{aligned}$$

Then it follows that

$$\begin{aligned}&\text{dist}(\Delta g_0(\boldsymbol{\rho}^1, \hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2), \Delta g_0(\boldsymbol{\rho}^2, \hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2))\\ =&\left|\sum_{i\in\mathbb{N}}(\hat{\rho}_i^1 - \hat{\rho}_i^2)(\rho_i^1 - \rho_i^2)\right| \le \text{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2)\text{dist}(\boldsymbol{\rho}^1, \boldsymbol{\rho}^2),\end{aligned} \tag{2.32}$$

where the inequality holds from Cauchy–Schwarz inequality. Hence, the first condition of Proposition 6.1 in [32] holds.

Next, we show that the second condition also holds. In our case, the second-order growth condition requires that there exists a positive constant a such that

$$g_0(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \hat{\boldsymbol{\rho}}^1) - g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^1) \ge a(\text{dist}(\boldsymbol{\rho}, \boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t))))^2.$$

We find a sufficient condition for this second-order growth condition in [34], in which Theorem 6.1 states that if the Slater qualification hypothesis holds, the second-order growth condition (Theorem 6.1 (v)) is equivalent with three other conditions (Theorem 6.1 (vi)–(viii)). Because the Slater qualification hypothesis could be satisfied if we carefully choose $p_{i,min}$ (see Sect. 2.3). We thus verify that an equivalent condition Theorem 6.1 (vii) is satisfied. For this, define:

$$\begin{aligned}&L(\boldsymbol{\rho}, \lambda, (\nu_i : i \in \mathbb{N})) = g_0(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \hat{\boldsymbol{\rho}}^1) - g_0(\boldsymbol{\rho}, \hat{\boldsymbol{\rho}}^1)\\ &+\lambda\left(\frac{1}{c(t)}C\left(\sum_{i\in\mathbb{N}}\frac{\rho_i}{\sqrt{\alpha}}\right) - 1\right) - \sum_{i\in\mathbb{N}}\nu_i\left(\frac{\rho_i}{\sqrt{\alpha}} - p_{i,min}\right).\end{aligned} \tag{2.33}$$

Then, we can write the function φ in Theorem 6.1 (vii), for any $\mathbf{d} \in \mathbb{R}^N$, as

$$\varphi_{\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t))}(\mathbf{d}) = \mathbf{d}'\frac{\partial^2}{\partial \boldsymbol{\rho}^2}L(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \lambda^*, (\nu_i^* : i \in \mathbb{N}))\mathbf{d},$$

where λ^* and $(\nu_i^* : i \in \mathbb{N})$ are the optimal Lagrange multipliers and variables. Substituting (2.33), we have that

$$\begin{aligned}\varphi_{\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t))}(\mathbf{d}) = \sum_{i\in\mathbb{N}} d_i^2\Bigg(1 - \frac{1}{\alpha}U''\left(\frac{\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t))}{\sqrt{\alpha}}, \omega_i\right)&\\ +\frac{1}{\alpha}\Bigg(2f'\left(\sum_{i\in\mathbb{N}}\frac{\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t))}{\sqrt{\alpha}}\right)&\\ +f''\left(\sum_{i\in\mathbb{N}}\frac{\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t))}{\sqrt{\alpha}}\right)\left(\sum_{i\in\mathbb{N}}\frac{\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t))}{\sqrt{\alpha}}\right)\Bigg)&\\ +\frac{\lambda^*}{\alpha}C''\left(\sum_{i\in\mathbb{N}}\frac{\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t))}{\sqrt{\alpha}}\right)/c(t)\Bigg).&\end{aligned}$$

Since λ^* is the optimal Lagrange multiplier, $\lambda^* \geq 0$. Also U is a strictly increasing, concave function, and C and f are strictly increasing, convex functions. Moreover, $p_i \in \mathbb{P}$ so that ρ_i lies in a closed set $\mathbb{P}_0$, for $i \in \mathbb{N}$. Therefore, there exist positive constants $\xi_{U''}$, $\xi_{f'}$, and $\xi_{f''}$ such that $U''(\rho_i) \leq -\xi_{U''}$, $f'(\rho_i) \geq \xi_{f'}$, and $f''(\rho_i) \geq \xi_{f''}$, for all $\rho_i \in \mathbb{P}_0$. So we have that for any $\mathbf{d} \in \mathbb{R}^N$,

$$\varphi_{\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t))}(\mathbf{d}) \geq \left(1 + \frac{\xi}{\alpha}\right) \sum_{i \in \mathbb{N}} d_i^2 > \sum_{i \in \mathbb{N}} d_i^2, \tag{2.34}$$

where $\xi = \xi_{U''} + 2\xi_{f'} + \xi_{f''}$ is a positive constant. Now, we have verified the condition of Theorem 6.1 (vii) and hence from Theorem 6.1 of [34], Theorem 6.1 (v) is satisfied, which equals to the second-order growth condition. Thus, for proposition 6.1 of [32], both conditions are satisfied. We could use it safely and conclude that:

$$\operatorname{dist}(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^2, c(t))) \leq \left(1 + \frac{\xi}{\alpha}\right)^{-1} \operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2) < \operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2).$$

Thus, we can conclude that

$$E\left[\left(\operatorname{dist}(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^2, c(t)))\right)^2\right] < \left(\operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2)\right)^2.$$

Further, we have that

$$\begin{aligned}
&\operatorname{dist}(E[\boldsymbol{\rho}_{\mathbf{i}}{}^*(\hat{\boldsymbol{\rho}}^1, c(t))], E[\boldsymbol{\rho}_{\mathbf{i}}{}^*(\hat{\boldsymbol{\rho}}^2, c(t))]) \\
&= \sqrt{\sum_{i \in \mathbb{N}} \left(E\left[\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t)) - \rho_i^*(\hat{\boldsymbol{\rho}}^2, c(t))\right]\right)^2} \\
&\leq \sqrt{\sum_{i \in \mathbb{N}} E\left[\left(\rho_i^*(\hat{\boldsymbol{\rho}}^1, c(t)) - \rho_i^*(\hat{\boldsymbol{\rho}}^2, c(t))\right)^2\right]} \\
&= \sqrt{E\left[\left(\operatorname{dist}(\boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^1, c(t)), \boldsymbol{\rho}^*(\hat{\boldsymbol{\rho}}^2, c(t)))\right)^2\right]} < \operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2).
\end{aligned}$$

The first inequality is due to Jensen's inequality. This proves our second intermediate result.

Then, suppose that the fixed point Eq. (2.30) has two distinct solutions $\hat{\boldsymbol{\rho}}^1$ and $\hat{\boldsymbol{\rho}}^2$. We have that

$$\operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2) = \operatorname{dist}(E[\boldsymbol{\rho}_{\mathbf{i}}{}^*(\hat{\boldsymbol{\rho}}^1, c(t))], E[\boldsymbol{\rho}_{\mathbf{i}}{}^*(\hat{\boldsymbol{\rho}}^2, c(t))]) < \operatorname{dist}(\hat{\boldsymbol{\rho}}^1, \hat{\boldsymbol{\rho}}^2),$$

which is an contradiction. This implies that (2.30) has at most one solution. We conclude that (2.17) has a unique solution. □

Proof of Lemma 2.3

Proof Given that $c(t)$ is an ergodic process, the updating function (2.14) can be considered as a stochastic approximation update equation. We can apply Theorem 1.1 of Chap. 6 in [35] for the convergence proof. We verify the assumptions in Theorem 1.1 of Chap. 6 in [35] in the following.

We first list the variables used in Theorem 1.1 and correspond them to our problem and our notation style: $\boldsymbol{\theta}_t = \hat{\mathbf{p}}^*(t)$, $\xi_t = c(t+1)$, $(Y_t)_i = \alpha(p_i^*(\hat{\mathbf{p}}^*(t), c(t+1)) - \hat{p}_i^*(t)), \forall\ i \in \mathbb{N}$, $\epsilon_t = \frac{1}{t+\alpha}$, $g(\hat{\mathbf{p}}, c(t)) = \alpha(p_i^*(\hat{\mathbf{p}}, c(t)) - \hat{p}_i^*), \forall\ i \in \mathbb{N}$, $\delta\mathbf{M} = \mathbf{0}$, $\boldsymbol{\beta}_t = \mathbf{0}$ and $\mathbf{Z}_t = \mathbf{0}$ for each t.

Now we need to verify that all the assumptions in Chap. 6 of [35] from (A.1.1) to (A.1.8) are satisfied. According to Property 2.1, $E[Y_t]$ is a continuous function of $\hat{p}_i^*(t)$ and $\mathbf{p}_i^*(\hat{\mathbf{p}}^*(t-1), c(t)) \in \mathbb{P}^N$ for any t. Thus, (A.1.1) is satisfied. (A.1.2) also follows from Property 2.1 that $g(\hat{\mathbf{p}}, c(t))$ is a continuous function of $\hat{\mathbf{p}}$, which guarantees (A.1.7) as well. For (A.1.3), we can take the following form of the function

$$(\bar{g}(\hat{\mathbf{p}}^*(t)))_i = \alpha(E[p_i^*(\hat{\mathbf{p}}^*(t), c(t+1))] - \hat{p}_i^*(t)).$$

According to [35], (A.1.3) holds due to the strong law of large numbers, because $c(t)$ is an ergodic process. Since $\boldsymbol{\beta}_t = \mathbf{Z}_t = \mathbf{0}$ for each t, we have both (A.1.4) and (A.1.5) hold true. For (A.1.6), it holds because $g(\hat{\mathbf{p}}, c(t))$ is bounded. Hence, all the assumptions are satisfied. It follows Theorem 1.1 in [35] and Property 2.2 that $\hat{p}_i(t)$ converges almost surely to the unique solution of $E[\mathbf{p}^*(\hat{\mathbf{p}}, c(t))] = \hat{\mathbf{p}}$. □

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