A New Selection Process Based on Granular Computing for Group Decision Making Problems

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Abstract. In Group Decision Making, there are situations in which the decision makers may not be able to provide his/her opinions properly and they could contain contradictions. To avoid it, in this contribution, we present a new selection process to deal with inconsistent information. As part of it, we use a method based on granular computing to increase the consistency of the opinions given by the decision makers. To do so, each opinion is articulated as a certain information granule instead of a single numeric value, offering the necessary flexibility to increase the consistency. Finally, the importance of the decision makers' opinions in the aggregation step is modeled by means of their consistency.

Keywords: Group decision making · Selection process · Granular computing · Consistency · Aggregation

1 Introduction

Group Decision Making (GDM) is a situation where there is a set of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$, to solve a problem and a group of decision makers, $E = \{e_1, e_2, \ldots, e_m\}$, $(m \geq 2)$, characterized by their own knowledge, trying to achieve a common solution. To do this, decision makers have to communicate their opinions by means of a set of assessments over the set of alternatives.

Preference relations are usually assumed to model decision makers' preferences in GDM problems [1]. According to the nature of the information expressed

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for every pair of alternatives, there exist many different representation formats of preference relations. In this contribution, we make use of fuzzy preference relations as they are one of the most employed because of their effectiveness as a tool for modelling decision processes and their utility and easiness of use when we want to aggregate decision makers' preferences into group ones [1,2].

The main advantage of pairwise comparison is that of focusing exclusively on two alternatives at a time, which facilitates decision makers when expressing their preferences. However, this way of providing preferences limits decision makers in their global perception of the alternatives and, as a consequence, the provided preferences could be not rational.

As consistent information, that is, information which does not imply any kind of contradiction, is more appropriate than information containing some contradictions, it is of great importance to provide decision makers with some tools that allow them to increase their level of consistency. To do so, information granularity may be used [3].

Information granularity is an important design asset offering to the decision makers some flexibility with the intent that their initial preferences can be adjusted in order to obtain a higher level of consistency. Assuming that each decision maker communicates his/her opinions using a fuzzy preference relation, this flexibility is brought into the fuzzy preference relations by allowing them to be granular rather than numeric. Therefore, the entries of the fuzzy preference relations are not considered plain numbers but information granules (fuzzy sets, probability density functions, rough sets, intervals, and so on).

The objective of this contribution is to present a new selection process based on granular computing for GDM. It is composed of three steps: (1) improvement of the consistency in the opinions given by the decision makers, (2) aggregation, and (3) exploitation. In the first step, an allocation of information granularity, as a key component to improve the consistency, is used. In such a way, some level of granularity is introduced in the realization of the granular representation of the fuzzy preference relations, supplying the required flexibility to increase the level of consistency. Then, assuming the choice scheme proposed in [4], aggregation following by exploitation, this new selection process is completed. On the one hand, the aggregation step consists in combining the decision makers' individual preferences into a collective one, which reflects the properties contained in all the individual preferences. On the other hand, the exploitation step transforms the global information about the alternatives into a global ranking of them. To do this, two quantifier-guided choice degrees of alternatives may be used: the dominance and the nondominance degree. The main advantages of this new selection process are that it supports the improvement of consistency, and it aggregates the decision makers' preferences giving more importance to the most consistent ones.

The rest of this contribution is set out as follows. Section 2 deals with how to obtain the level of consistency in a fuzzy preference relation. Section 3 provides a detailed description of the new selection process based on granular computing for GDM problems. An example of its application is shown in Sect. 4, and, finally, in Sect. 5, we point out some conclusions.

2 Obtaining the Consistency Level in a Fuzzy Preference Relation

In this section, we introduce both the definition of a fuzzy preference relation and a method to obtain its level of consistency.

Definition 1. *A fuzzy preference relation PR on a set of alternatives X is a fuzzy set on the Cartesian product $X \times X$, i.e., it is characterized by a membership function $\mu_{PR} : X \times X \to [0, 1]$.*

A fuzzy preference relation PR may be represented by the $n \times n$ matrix $PR = (pr_{ij})$, being $pr_{ij} = \mu_{PR}(x_i, x_j)$ $(\forall i, j \in \{1, \ldots, n\})$ interpreted as the preference degree or intensity of the alternative x_i over x_j: $pr_{ij} = 0.5$ indicates indifference between x_i and x_j $(x_i \sim x_j)$, $pr_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, and $pr_{ij} > 0.5$ indicates that x_i is preferred to x_j $(x_i \succ x_j)$. Based on this interpretation we have that $pr_{ii} = 0.5$ $\forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$. Since pr_{ii}'s (as well as the corresponding elements on the main diagonal in some other matrices) do not matter, we will write them as '–' instead of 0.5 [5].

The previous Definition 1 dealing with a fuzzy preference relation does not imply any kind of consistency property and, thus, the preference values of the pairwise comparisons can be contradictory. Obviously, because of an inconsistent source of information is not as useful as a consistent one, it is quite important to be able to measure the consistency of the information provided by the decision makers. To do so, different properties to be satisfied with the fuzzy preference relations have been proposed in the literature [6,7].

In this contribution, we make use of the additive transitivity property which facilitates the verification of consistency in the case of fuzzy preference relations. As it was shown in [7], additive transitivity for fuzzy preference relations may be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [8]. The mathematical formulation of the additive transitivity was given by [1]:

$$(pr_{ij} - 0.5) + (pr_{jk} - 0.5) = (pr_{ik} - 0.5), \forall i, j, k \in \{1, \ldots, n\}. \tag{1}$$

Additive transitivity implies additive reciprocity. Indeed, because $pr_{ii} = 0.5$ $\forall i$, if we make $k = i$ in (1), then we have: $pr_{ij} + pr_{ji} = 1$, $\forall i, j \in \{1, \ldots, n\}$.

Equation (1) can be rewritten as follows:

$$pr_{ik} = pr_{ij} + pr_{jk} - 0.5, \forall i, j, k \in \{1, \ldots, n\}. \tag{2}$$

A fuzzy preference relation is considered to be "additively consistent" when for every three options encountered in the problem, say $x_i, x_j, x_k \in X$, their associated preference degrees, $pr_{ij}, pr_{jk}, pr_{ik}$, fulfil (2).

Given a fuzzy preference relation, (2) can be used to calculate an estimated value of a preference degree using other preference degrees. In fact, the following estimated value of pr_{ik} $(i \neq k)$ can be calculated in three different ways using an intermediate alternative x_j [5]:

– From $pr_{ik} = pr_{ij} + pr_{jk} - 0.5$, we obtain the estimated value $(epr_{ik})^{j1}$:

$$(epr_{ik})^{j1} = pr_{ij} + pr_{jk} - 0.5. \tag{3}$$

– From $pr_{jk} = pr_{ji} + pr_{ik} - 0.5$, we obtain the estimated value $(epr_{ik})^{j2}$:

$$(epr_{ik})^{j2} = pr_{jk} - pr_{ji} + 0.5. \tag{4}$$

– From $pr_{ij} = pr_{ik} + pr_{kj} - 0.5$, we obtain the estimated value $(epr_{ik})^{j3}$:

$$(epr_{ik})^{j3} = pr_{ij} - pr_{kj} + 0.5. \tag{5}$$

Then, the estimated value, epr_{ik}, of a preference degree, pr_{ik}, is calculated according to the following expression:

$$epr_{ik} = \frac{\sum_{\substack{j=1 \\ j \neq i,k}}^{n} \left((epr_{ik})^{j1} + (epr_{ik})^{j2} + (epr_{ik})^{j3}\right)}{3(n-2)}. \tag{6}$$

When information provided is completely consistent then $(epr_{ik})^{jl} = pr_{ik}$ $\forall j, l$. However, because decision makers are not always fully consistent, the assessment made by a decision maker may not verify (2) and some of the estimated preference degree values $(epr_{ik})^{jl}$ may not belong to the unit interval $[0, 1]$. We note, on a basis of (3)–(5), that the maximum value of any of the preference degrees $(epr_{ik})^{jl}$ $(l \in \{1, 2, 3\})$ is 1.5 while the minimum one is -0.5. In such a way, the error, εpr_{ik}, between a preference degree and its estimated one in $[0, 1]$ is computed as follows [5]:

$$\varepsilon pr_{ik} = \frac{2}{3} \cdot |epr_{ik} - pr_{ik}|. \tag{7}$$

This error can be used to define the consistency degree cd_{ik} associated to the preference degree pr_{ik} as follows:

$$cd_{ik} = 1 - \varepsilon pr_{ik}. \tag{8}$$

When $cd_{ik} = 1$, then $\varepsilon pr_{ik} = 0$ and there is no inconsistency at all. The lower the value of cd_{ik}, the higher the value of εpr_{ik} and the more inconsistent is pr_{ik} with respect to the rest of information.

Finally, the consistency degrees associated with individual alternatives and the overall fuzzy preference relation are defined as follows:

– The consistency degree, $cd_i \in [0, 1]$, associated to a particular alternative x_i of a fuzzy preference relation is defined as:

$$cd_i = \frac{\sum_{k=1; i \neq k}^{n} (cd_{ik} + cd_{ki})}{2(n-1)}. \tag{9}$$

– The consistency degree, $cd \in [0, 1]$, of a fuzzy preference relation is defined as:

$$cd = \frac{\sum_{i=1}^{n} cd_i}{n}. \tag{10}$$

When the fuzzy preference relation is given by a decision maker e_h, the consistency degree of the fuzzy preference relation is represented as cd^h.

3 A Selection Process Based on Granular Computing

In this section, we present the new selection process based on granular computing for GDM problems. It consists of three steps: (1) improvement of the consistency, (2) aggregation, and (3) exploitation. An allocation of information granularity, as a key component to increase the level of consistency in the fuzzy preference relations, is used in the first step. The aggregation phase defines a collective fuzzy preference relation indicating the global preference between every pair of alternatives, while the exploitation step transforms the global information about the alternatives into a global ranking of them.

3.1 Improvement of the Consistency

The improvement of consistency when the decision makers communicate their opinions by means of fuzzy preference relations becomes a very important aspect in order to avoid misleading solutions. As we have already aforementioned, the improvement of consistency calls for some flexibility exhibited by the decision makers with respect their initial opinions.

These changes of preferences are articulated through modifications of the entries of the fuzzy preference relations. That is, if the pairwise comparisons of the fuzzy preference relations are not managed as single numeric values, which are inflexible, but rather as information granules, it will bring the indispensable factor of flexibility.

The notation $\boldsymbol{G}(PR)$ is here used to accentuate that we are interested in granular fuzzy preference relations. $\boldsymbol{G}(.)$ represents the specific granular formalism which is used, say intervals, probability density functions, fuzzy sets, rough sets, and alike. In particular, in this contribution, the granularity of information is articulated through intervals. Therefore, $\boldsymbol{G}(PR) = \boldsymbol{P}(PR)$, where $\boldsymbol{P}(.)$ denotes a family of intervals. The length of such intervals (entries of the fuzzy preference relations) is sought as a level of granularity α, which is treated as synonymous of the level of flexibility, facilitating the improvement of the consistency. The higher level of granularity is allowed to the decision maker, the higher the feasibility of arriving at a higher level of consistency.

This flexibility given by the level of granularity may be used to optimize a certain objective function in order to increase the level of consistency. In the interval-valued granular model of fuzzy preference relations, it is supposed that each decision maker feels equally comfortable when selecting any fuzzy preference relation whose values are placed within the bounds fixed by the level of granularity α. In such a way, the improvement of the consistency is effectuated at the level of individual decision makers using the following optimization index:

$$Q = \frac{1}{m} \sum_{h=1}^{m} cd^h. \tag{11}$$

Therefore, the overall optimization problem reads as follows:

$$\mathrm{Max}_{PR^1, PR^2, \ldots, PR^m \in \boldsymbol{P}(PR)} Q. \tag{12}$$

The aforementioned maximization is conducted for all acceptable interval-valued fuzzy preference relations because of the introduced level of granularity α. This fact is underlined by including a granular form of the fuzzy preference relations allowed in the problem, that is, PR^1, PR^2, ..., PR^m, are elements of the family of interval-valued fuzzy preference relations, namely, $\boldsymbol{P}(PR)$.

In this contribution, the optimization of the fuzzy preference relations, coming from the space of interval-valued fuzzy preference relations, is carried out by means of the Particle Swarm Optimization (PSO) framework [9]. PSO is here used because it is especially attractive given its less significant computing overhead in comparison with other techniques of global optimization [10]. However, other optimization mechanisms could be used as well.

The PSO is well documented in the existing literature with numerous modifications and augmentations. Refer to the generic flow of computing in which velocities and positions of the particles are updated. What is important in this setting is a formation of the particle. In our framework, each particle represents a vector whose entries are located in the unit interval. When it comes to the representation of the solutions, the particle is composed of "$m \cdot n(n-1)$" entries positioned in the $[0, 1]$ interval which corresponds to the search space.

Assuming a given level of granularity α (located in the unit interval) and starting with the initial fuzzy preference relation PR, provided by the decision maker, let us consider an entry pr_{ij} of PR. The interval of admissible values of this entry of $\boldsymbol{P}(PR)$ implied by the level of granularity α is equal to:

$$[a, b] = [\max(0, pr_{ij} - \alpha/2), \min(1, pr_{ij} + \alpha/2)]. \tag{13}$$

Considering that the entry of interest of the particle is x, an entry pr_{ij} is transformed linearly according to the expression $z = a + (b - a)x$. For instance, suppose that pr_{ij} is equal to 0.3, the admissible level of granularity α is equal to 0.1, and the corresponding entry of the particle is $x = 0.7$. According to it, the corresponding interval of the granular fuzzy preference relation calculated as given by (13) becomes equal to $[a, b] = [0.25, 0.35]$. Therefore, $z = 0.32$, and the modified value of pr_{ij} becomes equal to 0.32.

Finally, it is important to note that the overall particle is composed of the individual segments, where each of them is concerned with the optimization of the parameters of the fuzzy preference relations. Hence, the fitness function, f, associated with the particle is defined as $f = Q$, being Q the optimization index presented previously. The higher the value of f, the better the particle is.

3.2 Aggregation

Once the consistency of the fuzzy preference relations has been increased, a collective fuzzy preference relation $PR^c = (pr^c_{ij})$ must be obtained by aggregating all of the m individual fuzzy preference relations $\{PR^1, \ldots, PR^m\}$. Here, each value $pr^c_{ij} \in [0, 1]$ will represent the preference of the alternative x_i over the alternative x_j according to the majority of the most consistent decision makers.

A logical assumption in the resolution process of a GDM problem is that of associating more importance to the decision makers who provide the most consistent opinions. Approaches for the inclusion of these values of importance in the aggregation process involve the transformation of the preference values under the importance degree by means of a transformation function to generate a new value [11,12]. In this contribution, we apply an alternative approach consisting of using consistency levels as the order inducing values of the Induced Ordered Weighted Averaging (IOWA) operator [13] to be applied in the aggregation step of the selection process.

Definition 2. *An IOWA operator of dimension n is a function $\Phi_W : (\mathbb{R} \times \mathbb{R})^n \rightarrow \mathbb{R}$, to which a set of weights or weighting vector is associated, $W = (w_1, \ldots, w_n)$, with $w_i \in [0,1]$, $\sum_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n two-tuples $\{\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\}$ according to the following expression:*

$$\Phi_W(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}, \tag{14}$$

being σ a permutation of $\{1, \ldots, n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)}\rangle$ is the two-tuple with $u_{\sigma(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$.

In the above definition, the reordering of the set of values to be aggregated, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated with them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order inducing variable and $\{p_1, \ldots, p_n\}$ the values of the argument variable [13].

An essential question in the definition of the IOWA operator is how to obtain the associated weighting vector. To do so, the approaches proposed to calculate the weighting vector of an Ordered Weighted Averaging (OWA) operator can be applied [14].

Definition 3. *An OWA operator of dimension n is a function $\phi_W : \mathbb{R}^n \rightarrow \mathbb{R}$, which has a set of weights or weighting vector associated with it, $W = (w_1, \ldots, w_n)$, with $w_i \in [0,1]$, $\sum_i w_i = 1$, and it is defined to aggregate a list of n values $\{p_1, \ldots, p_n\}$ according to the following expression:*

$$\phi_W(p_1, \ldots, p_n) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}, \tag{15}$$

being $\sigma : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ a permutation such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $p_{\sigma(i)}$ is the i-th highest value in the set $\{p_1, \ldots, p_n\}$.

In the process of quantifier-guided aggregation, given a collection of n criteria represented as fuzzy subsets of the alternatives X, the OWA operator is used to implement the concept of fuzzy majority in the aggregation phase by means of a fuzzy linguistic quantifier [15], indicating the proportion of satisfied criteria "necessary for a good solution" [16]. This implementation is done by using the quantifier to calculate the OWA weights. According to it, in the case of a regular increasing monotone (RIM) quantifier Q, the procedure to evaluate the overall satisfaction of Q criteria (or decision makers) by the alternative x_j is carried out calculating the IOWA weights as follows:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n. \tag{16}$$

When a fuzzy linguistic quantifier Q is used to compute the weights of the above aggregation operators, then it is symbolized by Φ_Q and ϕ_Q, respectively.

Definition 2 allows the construction of many different IOWA operators. Here, we use an IOWA operator in which the ordering of the preference values to be aggregated is induced by ordering the decision makers from the most to the least consistent one. Therefore, the collective fuzzy preference relation is obtained as:

$$pr^c_{ij} = \Phi_Q(\langle cd^1, pr^1_{ij}\rangle, \ldots, \langle cd^m, pr^m_{ij}\rangle), \tag{17}$$

where Q is the fuzzy linguistic quantifier used to implement the fuzzy majority concept and, using (16), to compute the weighting vector of the IOWA operator.

3.3 Exploitation

At this point, in order to select the alternative(s) "best" acceptable for the majority (Q) of the most consistent decision makers, two quantifier-guided choice degrees of alternatives can be employed [17]: a dominance degree ($QGDD$), and a nondominance degree ($QGNDD$).

- $QGDD_i$: This quantifier-guided dominance degree evaluates the dominance that the alternative x_i has over all the others in a fuzzy majority sense. It is computed as follows:

$$QGDD_i = \phi_Q(pr^c_{i1}, pr^c_{i2}, \ldots, pr^c_{i(i-1)}, pr^c_{i(i+1)}, \ldots, pr^c_{in}). \tag{18}$$

- $QGNDD_i$: This quantifier-guided nondominance degree gives the degree in which the alternative x_i is not dominated by a fuzzy majority of the remaining alternatives. It is calculated as follows:

$$QGNDD_i = \phi_Q(1-p^s_{1i}, 1-p^s_{2i}, \ldots, 1-p^s_{(i-1)i}, 1-p^s_{(i+1)i}, \ldots, 1-p^s_{ni}), \tag{19}$$

  where $p^s_{ji} = max\{pr^c_{ji} - pr^c_{ij}, 0\}$ represents the degree in which x_i is strictly dominated by x_j.

The application of the above choice degrees of alternatives over X may be carried out according to two different policies: (1) sequential policy, and (2) conjunctive policy [5]. On the one hand, in the sequential policy, one of the choice degrees is selected and applied to X according to the preference of the decision makers, obtaining a selection set of alternatives. If there is more than one alternative in this selection set, then, the other choice degree is applied to select the alternative of this set with the best second choice degree. One the other hand, in the conjunctive policy, both choice degrees are applied to X, obtaining two selection sets of alternatives. The final selection set of alternatives is obtained as the intersection of these two selection sets of alternatives. The latter conjunction selection process is more restrictive than the former sequential selection process because it is possible to obtain an empty selection set.

4 Illustrative Example

In this section, we present an illustrative example which helps quantifying the performance of the selection process proposed in this contribution.

Let us suppose that the supermarket manager wants to buy 500 bottles of Spanish wine from among four possible brands of wine: $\{x_1 =$ Marqués de Cáceres, $x_2 =$ Los Molinos, $x_3 =$ Somontano, $x_4 =$ René *Barbier*$\}$. The manager decide to inquire four decision makers, $E = \{e_1, e_2, e_3, e_4\}$, about their opinions on what Spanish wine should be bought. The decision makers provide the following fuzzy preference relations:

$$PR^1 = \begin{pmatrix} - & 0.60 & 0.30 & 0.50 \\ 0.10 & - & 0.70 & 0.70 \\ 0.80 & 0.10 & - & 0.10 \\ 0.10 & 0.40 & 0.60 & - \end{pmatrix} \qquad PR^2 = \begin{pmatrix} - & 0.20 & 0.50 & 0.10 \\ 0.40 & - & 0.20 & 0.80 \\ 0.50 & 0.40 & - & 0.90 \\ 0.90 & 0.10 & 0.40 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.20 & 0.20 & 0.70 \\ 0.30 & - & 0.60 & 0.90 \\ 0.10 & 0.40 & - & 0.30 \\ 0.10 & 0.40 & 0.70 & - \end{pmatrix} \qquad PR^4 = \begin{pmatrix} - & 0.70 & 0.10 & 0.50 \\ 0.50 & - & 0.50 & 0.30 \\ 0.90 & 0.70 & - & 0.40 \\ 0.30 & 0.70 & 0.70 & - \end{pmatrix}$$

Once the decision makers have expressed their opinions, the selection process is applied in order to rank the Spanish wines from best to worst.

4.1 First Step: Improvement of the Consistency

Proceeding with the details of the optimization environment, in this contribution, a generic version of the PSO is used. The parameters in the update equation for the velocity of the particle were set as $c_1 = c_2 = 2$, as these values are usually encountered in the existing literature. The size of the swarm consists of 100 particles, and the algorithm was run for 200 generations (or iterations). These values were selected as a result of intensive experimentation.

Considering a given level of granularity α, Table 1 shows the performance of the PSO quantified in terms of the fitness function. To put the achieved optimization results in a certain context, we report the performance obtained when no granularity is allowed ($\alpha = 0$), that is, when considering the entries of the fuzzy preference relations are single numeric values. In such a case, the corresponding consistency degrees of the four fuzzy preference relations are: $cd^1 = 0.73$, $cd^2 = 0.76$, $cd^3 = 0.82$, and $cd^4 = 0.81$. Therefore, the value of the fitness function f is 0.78.

Table 1. Performance of the PSO for selected values of α

	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$	$\alpha = 2.0$
cd^1	0.73	0.77	0.79	0.83	0.88	0.93	0.96	0.98	1.00	1.00
cd^2	0.76	0.78	0.82	0.85	0.88	0.91	0.94	0.97	1.00	1.00
cd^3	0.83	0.84	0.86	0.89	0.91	0.94	0.97	0.99	1.00	1.00
cd^4	0.82	0.83	0.85	0.88	0.91	0.94	0.97	0.98	1.00	1.00
f	0.79	0.80	0.83	0.86	0.90	0.93	0.96	0.98	1.00	1.00

Comparing with the values obtained by the PSO, the fitness function f takes on now lower values. As we can see in Table 1, the higher the admitted level of granularity α, the higher the values obtained by the fitness function f. It is logical, as the higher the level of granularity α, the higher the level of flexibility introduced in the fuzzy preference relations and, hence, the possibility of achieving higher level of consistency. Furthermore, when each entry of the granular preference relation is treated as the whole $[0, 1]$ interval ($\alpha = 2.0$), the value of the fitness function is 1, the maximum one. Nevertheless, in this case, we have to take into account that whether the level of granularity is very high, the values of the entries of the fuzzy preference relation could be very different in comparison with the original ones given by the decision maker and, hence, he/she could reject them.

Following with the example, we are going to consider that the level of granularity α is equal to 0.6 and, therefore, the consistency level achieved among all the decision makers is 0.83 which is better than the consistency obtained when no granularity is admitted (0.78). Then, using this level of granularity, the new fuzzy preference relations obtained using the PSO are:

$$PR^1 = \begin{pmatrix} - & 0.30 & 0.10 & 0.20 \\ 0.10 & - & 0.40 & 0.40 \\ 0.50 & 0.10 & - & 0.10 \\ 0.10 & 0.10 & 0.30 & - \end{pmatrix} \qquad PR^2 = \begin{pmatrix} - & 0.26 & 0.52 & 0.21 \\ 0.42 & - & 0.26 & 0.76 \\ 0.52 & 0.42 & - & 0.81 \\ 0.81 & 0.21 & 0.42 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.19 & 0.19 & 0.63 \\ 0.23 & - & 0.53 & 0.75 \\ 0.15 & 0.33 & - & 0.23 \\ 0.15 & 0.33 & 0.63 & - \end{pmatrix} \qquad PR^4 = \begin{pmatrix} - & 0.56 & 0.10 & 0.36 \\ 0.36 & - & 0.36 & 0.16 \\ 0.70 & 0.56 & - & 0.26 \\ 0.16 & 0.56 & 0.56 & - \end{pmatrix}$$

4.2 Second Step: Aggregation

Once the consistency of the fuzzy preference relations have been increased, we aggregate them by means of the IOWA operator presented in Sect. 3.2. We make use of the linguistic quantifier "most of", represented by the RIM quantifier $Q(r) = r^{1/2}$, which applying (16) generates a weighting vector of four values to obtain each collective preference value pr^c_{ij}. As example, the collective preference value pr^c_{12} is calculated in the following way:

$$\begin{aligned}
&w_1 = Q(1/4) - Q(0) = 0.5 - 0 = 0.5\\
&w_2 = Q(2/4) - Q(1/4) = 0.71 - 0.5 = 0.21\\
&w_3 = Q(3/4) - Q(2/4) = 0.87 - 0.71 = 0.16\\
&w_4 = Q(1) - Q(3/4) = 1 - 0.87 = 0.13\\
&cd^1 = 0.79,\ cd^2 = 0.82,\ cd^3 = 0.86,\ cd^4 = 0.85\\
&\sigma(1) = 3,\ \sigma(2) = 4,\ \sigma(3) = 2,\ \sigma(4) = 1\\
&pr^c_{12} = w_1 \cdot pr^3_{12} + w_2 \cdot pr^4_{12} + w_3 \cdot pr^2_{12} + w_4 \cdot pr^1_{12} = 0.21
\end{aligned}$$

Then, the collective fuzzy preference relation is:

$$PR^c = \begin{pmatrix} - & 0.21 & 0.17 & 0.45 \\ 0.27 & - & 0.43 & 0.58 \\ 0.37 & 0.36 & - & 0.31 \\ 0.25 & 0.33 & 0.54 & - \end{pmatrix}$$

4.3 Third Step: Exploitation

Using again the same linguistic quantifier "most of" and (16), we obtain the following weighting vector $W = (w_1, w_2, w_3)$:

$$\begin{aligned}
&w_1 = Q(1/3) - Q(0) = 0.58 - 0 = 0.58\\
&w_2 = Q(2/3) - Q(1/3) = 0.82 - 0.58 = 0.24\\
&w_3 = Q(1) - Q(2/3) = 1 - 0.82 = 0.18
\end{aligned}$$

Using, for example, the quantifier-guided dominance degree, we obtain the following values: $\{QGDD_1 = 0.34, QGDD_2 = 0.49, QGDD_3 = 0.36, QGDD_4 = 0.44\}$. Then, applying, for instance, the sequential policy, the following ranking of alternatives is obtained: $x_2 \succ x_4 \succ x_3 \succ x_1$. Using this information, the supermarket manager should buy 500 bottles of Los Molinos wine.

5 Conclusions

In this contribution, we have presented a new selection process based on granular computing to be used to solve GDM problems. As main novelty, it incorporates a first step in order to increase the consistency achieved by the decision makers in their opinions. To do so, we have proposed the concept of granular fuzzy preference relation and we have emphasized a role of information granularity as a conceptual vehicle to facilitate admissible changes to the results of pairwise comparisons. It has offered a badly needed flexibility to increase the consistency. In addition, the aggregation of the opinions provided by the decision makers has been carried out by giving more importance to the most consistent ones.

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