Chapter 2
Analysis by Manual Calculations

2.1 Introduction

In structural engineering, the term *analysis* usually refers to *force analysis* in which the distribution of force effects is determined in the various components of a structure. The responses of a structure such as deflections and bending moments are often referred to as *load effects*. Another infrequently used term in structural engineering is *strength analysis* which refers to the process of determining the strength of the whole structure or its components. The term *analysis* is used in this book only in the meaning of force analysis.

In bridge engineering the term *analysis* is also used for determining the effects of *load distribution* mainly in the longitudinal components of a bridge due to a vehicle. This chapter provides details of some methods which can be used to analyse a bridge for load distribution through manual calculations.

Notwithstanding the fact all calculations for bridge analysis are currently done with the help of computers, manual methods are important in permitting engineers to retain the physical feel of the distribution of load effects in a bridge, and confirming the results of computer analyses in a broad sense. The manual methods of calculations are also known as the 'simplified' methods. Bakht and Jaeger (1985) have written a book dealing with manual analysis of bridges, entitled as 'Bridge Analysis Simplified'.

It is very important to note that the specific manual methods of analysis presented in this chapter explicitly include: (a) the number of lines of wheels in the design vehicle and their transverse spacing, and (b) the reduction factors for multi-lane loadings specified by the design code. The manual methods of analysis presented in this chapter should be used only if the conditions of load placement are the same as those included in the methods.

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2.2 Distribution Coefficient Methods

The basis of many simplified/manual methods is the distribution coefficient methods prevalent in pre-computer days; the background to these methods is discussed in following. The distribution coefficient methods, e.g. Morice and Little (1956), were well known to bridge designers in the U.K. and in those Asian countries where bridge design practice is, or was, influenced significantly by the British practice. These methods can be applied manually to obtain the values of various load effects at any reference point on a transverse section of the bridge. For most of the load distribution coefficient methods, a right simply supported bridge is idealized as an orthotropic plate whose load distribution characteristics are governed by two dimensionless parameters α and θ, defined as follows:

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2\left(D_x D_y\right)^{0.5}} \tag{2.1}$$

$$\theta = \frac{b}{L}\left(\frac{D_x}{D_y}\right)^{0.25} \tag{2.2}$$

where the notation is as defined in the following:

x direction = the longitudinal direction, i.e., the direction of traffic flow
y direction = the transverse direction (perpendicular to the longitudinal direction)
D_x = the longitudinal flexural rigidity per unit width (corresponding to flexural rigidity EI in a longitudinal beam)
D_y = the transverse flexural rigidity per unit length (corresponding to flexural rigidity EI in a transverse beam)
D_{xy} = the longitudinal torsional rigidity per unit width (corresponding to torsional rigidity GJ in a longitudinal beam)
D_{yx} = the transverse torsional rigidity per unit length (corresponding to torsional rigidity GJ in a transverse beam)
D_1 = the longitudinal coupling rigidity per unit width (which is the contribution of transverse flexural rigidity to longitudinal torsional rigidity through Poisson's ratio)
D_2 = the transverse coupling rigidity per unit length (which is the contribution of longitudinal flexural rigidity to transverse torsional rigidity through Poisson's ratio)
b = half width of the idealised plate
L = the span of the idealised plate

In slab-on-girder bridges, D_1 and D_2 are small and have little effect on load distribution. It is customary to ignore these rigidities in the calculation of α for slab-on-girder bridges. Figure 2.1 illustrates some of the notation.

The Morice and Little method, which is originally due to Guyon and Massonnet reported by Bares and Massonnet (1966), is based upon the harmonic analysis of

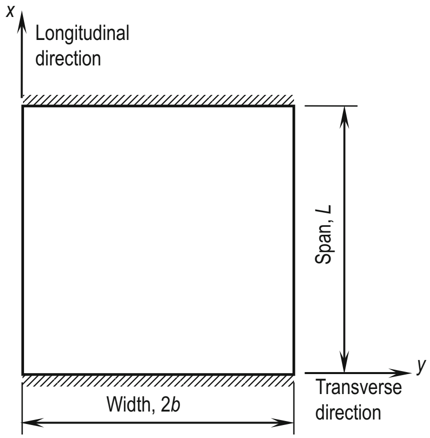


Fig. 2.1 Plan of a right bridge idealised as an orthotropic plate

orthotropic plates using only the first term of the harmonic series representing concentrated loads (discussed in Chap. 3). The basis of the method is the assumption that the deflected shape of a transverse section remains constant along the span irrespective of the longitudinal position of the load and the transverse section under consideration. The method uses charts of distribution coefficients K_α corresponding to nine different transverse reference stations and nine transverse positions of single concentrated loads. These coefficients are plotted in chart form against θ, and the charts are given for two values of α, namely 0.0 and 1.0.

Values of coefficients, K_α, for intermediate values of α are obtained by the following interpolation function:

$$K_\alpha = K_0 - (K_0 - K_1)(\alpha)^{0.5} \tag{2.3}$$

where K_0 and K_1 are the corresponding coefficients for α equal to 0.0 and 1.0, respectively. In using the method, the applied loads are converted into equivalent concentrated loads at the standard locations for which the charts are given. The distribution coefficients are then manually added for all the equivalent loads to give the final set of coefficients for the loading case under consideration. The exercise is, of course, repeated for each load case, and therefore requires extensive and tedious calculations.

To compensate for possible errors resulting from the representation of loads by only one harmonic, Morice and Little (1956) suggest that the computed longitudinal moments be increased by an arbitrary 10 %. Cusens and Pama (1975) improved the distribution coefficient method by taking seven terms of the harmonic series into account, and by extending the range of values of α up to 2.0. This method also uses an interpolation equation similar to Eq. (2.3).

It is interesting to examine the α-θ space with respect to various types of bridges. For practical bridges, α ranges between 0.0 and 2.0, and θ between 0.25 and 2.5. The ranges of α values for various types of bridges are shown in the α-θ space in

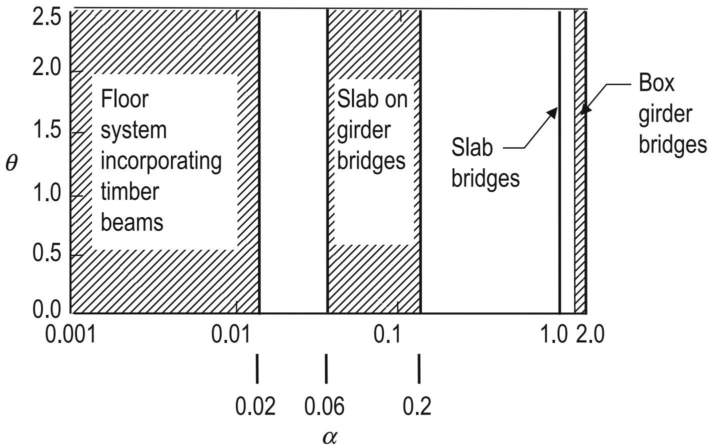


Fig. 2.2 The α-θ space for bridge superstructures idealized as orthotropic plates

Fig. 2.2. It can be seen that the various bridge types occupy distinctly separate zones. The space for bridges with timber beams occupies the space between $\alpha =$ nearly zero and 0.01; the space for slab-on-girder bridges is bracketed by values of α between 0.06 and 0.2; slab bridges have $\alpha = 1.0$; and box girder bridges have α close to 2.0.

In the Morice and Little method, since the values of α for slab-on-girder bridges are between 0.0 and 1.0, the distribution coefficients for these bridges have to be obtained by using Eq. (2.3). This equation is only an approximate design convenience and, irrespective of the accuracy of K_0 and K_1, can and does introduce significant errors, especially for bridges having smaller values of θ.

2.3 Simplified Methods of North America

Unlike the distribution coefficient methods, the simplified methods of bridge analysis used almost exclusively in North America provide only the maximum, i.e. the design, values of the various load effects at a given transverse section. Computation needed for these methods is only a fraction of that required for the distribution coefficient methods.

The North American simplified methods are permitted by the current and past design codes, being the AASHTO Specifications (1998, 2010), the CSA Code (1988), the Ontario Highway Bridge Design Code (1992) and the Canadian Highway Bridge Design Code (2000, 2006). These methods can be applied manually and can provide fairly reliable estimates of the design values of the various load effects in a very short period of time. The simplified methods of analysis are dependent upon the specification of the magnitude and placement of the design live loads, and accordingly are not always transportable between the various design codes.

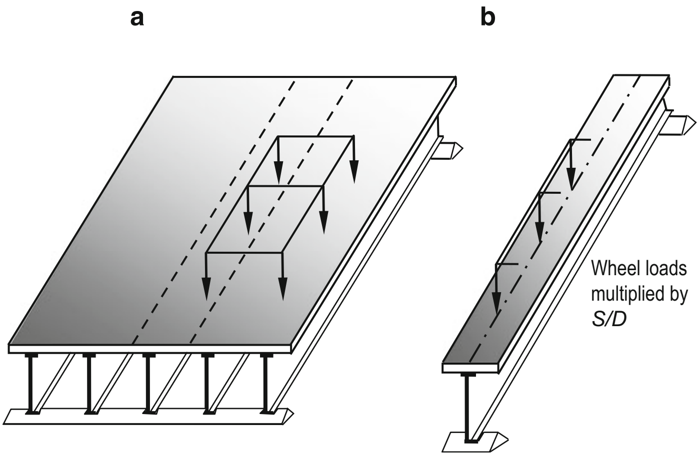


Fig. 2.3 Illustration of the simplified methods of North America: (**a**) actual bridge, (**b**) isolated 1-D model

Table 2.1 Some AASHTO (1989) *D* values

Bridge type	*D* in m	
	Bridge designed for one traffic lane	Bridge designed for two or more traffic lanes
Slab on steel or prestressed concrete girders	2.13	1.67
Slab on T-beams	1.98	1.83
Slab on timber girders	1.83	1.53

2.3.1 Old AASHTO Method

Most highway bridges in North America are designed by the AASHTO specifications. Design vehicles for these specifications consist of two- and three-axle vehicles having two lines of wheels the centres of which are 1.83 m apart. The old AASHTO specifications (1989) permitted a simplified method for obtaining live load longitudinal moments and shears, according to which a longitudinal girder, or a strip of unit width in the case of slabs, is isolated from the rest of the structure and treated as a one-dimensional beam. This beam, as shown in Fig. 2.3b, is subjected to loads comprising one line of wheels of the design vehicle multiplied by a load fraction (S/D), where S is the girder spacing and D, having the units of length, has an assigned value for a given bridge type. The resulting moments and shears are assumed to correspond to maximum girder moments and shears in the bridge. Values of D as specified in the AASHTO (1989) specifications for various cases of slab-on-girder bridges are given in Table 2.1.

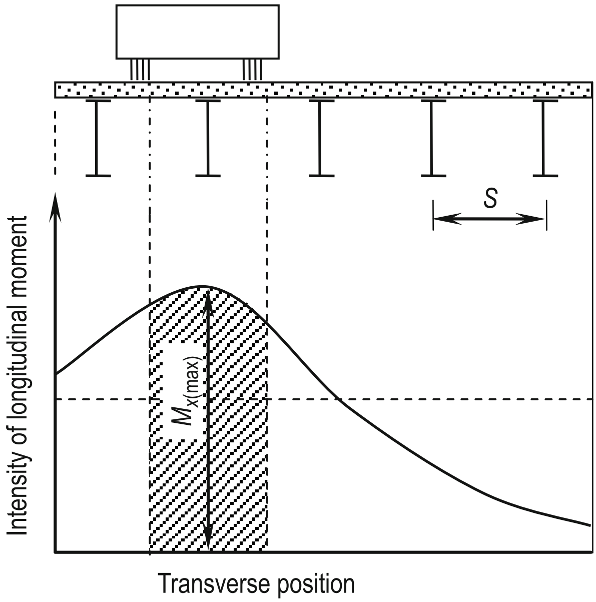


Fig. 2.4 Transverse distribution of longitudinal moment intensity

2.3.2 Concept of D Method

The concept of the factor D can be explained with reference to Fig. 2.4, which shows schematically the transverse distribution of live load longitudinal moment intensity in a slab-on-girder bridge at a cross-section due to one vehicle with two lines of wheels. The intensity of longitudinal moment, having the units of kN.m/m, is obtained by idealizing the bridge as an orthotropic plate.

It can be readily appreciated that the maximum girder moment, M_g, for the case under consideration occurs in the second girder from the left. The moment in this girder is equal to the area of the shaded portion under the moment intensity curve. If the intensity of maximum moment is $M_{x(max)}$ then this shaded area is approximately equal to $SM_{x(max)}$, so that:

$$M_g \simeq SM_{x(\max)} \tag{2.4}$$

It is assumed that the unknown quantity $M_{x(max)}$ is given by:

$$M_{x(\max)} = M/D \tag{2.5}$$

where M is equal to the total moment due to half a vehicle, i.e., due to one line of wheels. Substituting the value of $M_{x(max)}$ from Eq. (2.5) into Eq. (2.4):

$$M_g \simeq M(S/D) \tag{2.6}$$

Thus if the value of D is known, the whole process of obtaining longitudinal moments in a girder is reduced to the analysis of a 1-dimensional beam in which the loads of one line of wheels are multiplied by the load fraction (S/D).

2.3.3 New AASHTO Method

Earlier AASHTO D values for this extremely simple method were developed from results of extensive orthotropic plate analyses by Sanders and Elleby (1970). The simplicity of the method, however, does take its toll in accuracy. In the old AASHTO method, the value of D depends only on the bridge type; it is obvious, however, that the manner of load distribution in a long and narrow bridge is different from that in a short and wide bridge of the same type. The old AASHTO method is unable to cater for such factors as the aspect ratio of the bridge. It is noted that in its 1994 edition, the AASHTO specifications introduced another simplified method, which is similar in spirit to the D method, but is more accurate. In the new method, the ratio S/D is designated as g. For slab-on-girder bridges, the value of g is obtained as a function of (a) the girder spacing S in mm, (b) the span length L in mm, (c) the deck slab thickness t_S in mm, and (d) the longitudinal stiffness parameter K_g in mm^4, which is obtained as follows.

$$K_g = n\left(I_g + e_g^2 A\right) \tag{2.7}$$

where, n is the modular ratio E_{girder}/E_{deck}, I_g is the moment of inertia of the girder in mm^4, e_g is the girder eccentricity in mm, being the distance from the girder centroid to the middle of the deck slab, and A is the cross-sectional area of the girder in mm^2. For example, the value of g for moment in exterior girders of slab-on-girder bridges with steel or concrete girders and subjected to multiple lane loading is designated as mg_{moment} and is given by:

$$mg_{moment} = 0.075 + \left(\frac{S}{2900\,mm}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_S^3}\right)^{0.1} \tag{2.8}$$

2.3.4 Canadian Methods

2.3.4.1 Ontario Method I

When the development of the Ontario Highway Bridge Design Code (OHBDC) began in 1976, the committee which was entrusted with the task of writing the section on analysis was asked to develop and specify a method of analysis which was as simple as the old AASHTO method but far more accurate. The method, which was developed for the OHBDC and was specified in the editions published in 1979 and 1983, has come to be known as the α-θ method. As discussed by Bakht

Table 2.2 Values of D for longitudinal moments for the ultimate limit state specified by the CSA (1988) design code

Bridge type	D in metres for bridge with no. of lanes =		
	2	3	3 or more
Slab bridges and voided slab bridges	1.90	2.15	2.40
Concrete slab on girders	1.80	1.90	2.00
Timber flooring on girders	1.65	1.90	2.00
Multi cell box girders	1.80	2.05	2.40

and Jaeger (1985), in this method the contours of the values of D obtained from rigorous analyses are presented on charts which use the two characterizing parameters of orthotropic plates, being α and θ, as their axes. These parameters are the same as used in the distribution coefficient methods and are defined by Eqs. (2.1) and (2.2), respectively. Values of the plate rigidities used in these equations can be obtained by standard methods, e.g. Cusens and Pama (1975) and Bakht and Jaeger (1985).

The final value of D, which is used for analysing the bridge, denoted as D_d, is obtained from:

$$D_d = D\left\{\frac{1 + \mu C_f}{100}\right\} \tag{2.9}$$

where

$$\mu = \frac{W_e - 3.3}{0.6} \leq 1.0 \tag{2.10}$$

in which W_e is the design lane width in metres, and C_f is a factor, whose values are provided in chart-form on the α-θ space.

2.3.4.2 CSA Method

Despite the simplicity of the Ontario method, some designers were not happy with having to calculate the values of α and θ. When the 1988 edition of the Canadian Standards Association (CSA) bridge code was being developed by using the OHBDC as its model, it was decided to heed the above concern of the designers and present conservative estimates of the values of D depending upon the type and width of the bridge. A selection of the CSA (1988) values of D is presented in Table 2.2. It can be appreciated that in terms of accuracy, the CSA method lies between the AASHTO and Ontario methods.

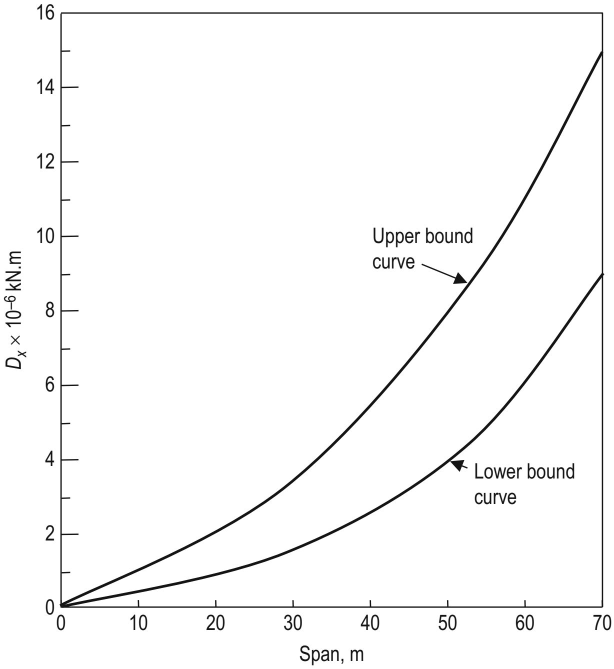


Fig. 2.5 D_x plotted against span length for slab-on-girder bridges in North America

2.3.4.3 Ontario Method II

Research done by Bakht and Moses (1988) has shown that the simplified method of Ontario incorporated in the 1979 and 1983 editions can be "simplified" further by recognizing mainly that the longitudinal flexural rigidity per unit width, D_x, of girder bridges in North America lies between the two bounds defined as follows:

$$D_x = 59,575L + 2,275L^2 \text{(upper bound)} \tag{2.11}$$

$$D_x = 9,250L + 1,790L^2 \text{(lower bound)} \tag{2.12}$$

where the span of the bridge L is in metres and D_x in kN.m. Figure 2.5 shows the upper and lower bounds of D_x for slab-on-girder bridges in North America.

The advantage of determining the upper and lower bound values of D_x for a bridge of a given span can be explained with the example of a specific four-lane slab-on-girder bridge, which has $L = 30$ m, and $2b = 14.4$ m; this bridge is analyzed by the semi-continuum method in Sect. 3.3.4. With reference to Fig. 2.2 it can be seen that α for a slab on girder bridge lies between 0.06 and 0.1. From Eqs. (2.11) and (2.12), the upper and lower bound values of D_x are found to be 3,834,750 and 1,888,500 kN.m, respectively. The value of D_y for this bridge with a 175 mm thick concrete deck slab is calculated to be 8930 kN.m, so that from Eq. (2.1) the upper

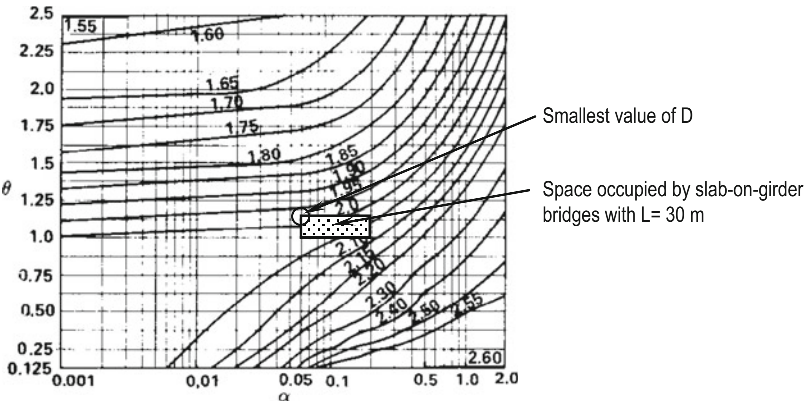


Fig. 2.6 *D* charts for four-lane bridges for ULS, OHBDC loading

and lower bound values of θ are found to be 1.17 and 0.98, respectively. The contours of the *D* values for four-lane bridges for internal portions specified in the OHBDC (1983) for the ULS are reproduced in Fig. 2.6, which also shows the very small rectangular area occupied by slab-on-girder bridges with a span of 30 m. From the trend of the contours of the *D* values plotted in this figure, it can be appreciated that within the small rectangular area, the smallest value of D, i.e. the most conservative value, is at the corner where the value α is the smallest and the value of θ the largest. If the values of *D* are obtained by rigorous analysis for α and θ representing this corner of the rectangle, then all other values of *D* corresponding to all other combinations of α-θ within the rectangle would be larger but by only a small amount. This principle was used to calculate the values of *D* and C_f as functions of span length *L*.

The expressions for *D* and C_f for longitudinal moments in two types of bridge on Class A highways are listed in Table 2.3 corresponding to the Ultimate Limit State (ULS).

Having obtained the values of *D* and C_f from the expressions given in Table 2.3, the design value of *D*, i.e. D_d, is obtained from Eq. (2.9).

Recognizing that the distribution of longitudinal moments is more benign than that of longitudinal shears, the OHBDC (1992) has specified that the values of D_d for longitudinal shears be obtained from a separate table which is reproduced herein as Table 2.4.

2.3.5 CHBDC Method

The successors to the OHBDC, the Canadian Highway Bridge Design Code (CHBDC 2000, 2006) have specified a slightly different simplified method of analysis. The purpose of this new method was to permit the designers to have a

Table 2.3 Expressions for D and C_f for longitudinal moments at the ULS in bridges on Class A highways

No. of design lanes	External/internal portion or girder	D(m)		C_f (%)
		$3 < L \leq 10$ m	$L > 10$ m	
(a) Slab bridges and voided slab bridges				
1	External	2.10	2.10	$16 - (36/L)$
	Internal	$2.00 + (3L/100)$	2.30	$16 - (36/L)$
2	External	2.05	2.05	$20 - (40/L)$
	Internal	$2.10 - (1/L)$	$2.10 - (1/L)$	$20 - (40/L)$
3	External	$1.90 + (L/20)$	$2.60 - (2/L)$	$16 - (30/L)$
	Internal	$1.45 + (L/10)$	$2.65 - (2/L)$	$16 - (30/L)$
(b) Slab-on-girder bridges				
1	External	2.00	$2.10 - (1/L)$	$5 - (12/L)$
	Internal	$1.75 + (L/40)$	$2.30 - (3/L)$	$5 - (12/L)$
2	External	1.90	$2.00 - (1/L)$	$10 - (25/L)$
	Internal	$1.40 + (3L/100)$	$2.10 - (4/L)$	$10 - (25/L)$
3	External	1.90	$2.00 - (1/L)$	$10 - (25/L)$
	Internal	$1.60 + (2L/100)$	$2.30 - (5/L)$	$10 - (25/L)$

Table 2.4 Values of D_d in metres for longitudinal shear for ultimate limit state for bridges on Class A highways

Bridge type	D_d in m for number of design lanes =			
	1	2	3	4 or more
Slab	2.05	1.95	1.95	2.15
Voided slab	2.05	1.95	1.95	2.15
Slab-on-girder	1.75	1.70	1.85	1.90
Stress-laminated wood decks	1.75	1.70	1.85	1.90

better feel of the load distribution characteristics of bridge than could be afforded by the somewhat abstract D method. The CHBDC method provides a multiplier to the average longitudinal moment or shear; this multiplier, which has to be always greater than 1.00 and which is denoted as F_m for moments and F_s for shears, gives an indication of the load distribution characteristics of a given bridge. If the multiplier is much larger than 1.00, the bridge has poor load distribution characteristics. On the other hand, a multiplier closer to 1.00 indicates that the bridge has good load distribution characteristics.

For girder-type bridges, the longitudinal moment M_g, or longitudinal shear $V_{g,}$ in a girder due to design live loads is obtained by multiplying the average girder moment $M_{g\ avg}$, or the average girder shear $V_{g\ avg}$ by the multiplier F_m or F_v. Values of $M_{g\ avg}$, or $V_{g\ avg}$ for each girder are obtained by placing the design live loads in all

Table 2.5 Modification factors, R_L, for multi-lane loading specified by CHBDC (2006)

Number of loaded lanes	Highway class		
	A	B	C or D
1	1.00	1.00	1.00
2	0.90	0.90	0.85
3	0.80	0.80	0.70
4	0.70	0.70	–

design lanes and multiplying the loads by the appropriate modification factor for multi-lane loading, so that:

$$M_{g\,avg} = \frac{nM_tR_L}{N} \tag{2.13}$$

and

$$V_{g\,avg} = \frac{nV_tR_L}{N} \tag{2.14}$$

where

M_t = the maximum moment per design lane at the transverse section of the span under consideration
V_t = the maximum shear per design lane at the transverse section of the span under consideration
n = the number of design lanes in the bridge
R_L = the modification factor for multi-lane loading as shown in Table 2.5
N = the number of girders

The Highway classes referred to in Table 2.5 relate to the volume of average daily truck traffic (ADTT) per lane of the bridge. The ADTT for Class A, B, C and D Highways is more than 1000, between 250 and 1000, between 50 and 250, and less than 50, respectively.

The maximum girder moments M_g and shears V_g are obtained by multiplying the average moments or shears by the amplification factors F_m or F_v, respectively; these factors, which account for the transverse variation in maximum longitudinal moment or shear intensities, as compared to the average responses, are obtained by the following equations.

$$F_m = \frac{SN}{F\left(1 + \frac{\mu C_f}{100}\right)} \geq 1.05 \tag{2.15}$$

and

$$F_v = \frac{SN}{F} \geq 1.05 \tag{2.16}$$

Table 2.6 Expressions for F and C_f for longitudinal moments in slab-on-girder bridges for the ultimate limit state of CHBDC

Class of highway	No. of design lanes	External/internal girders	F, m		C_f, %
			For $L \leq 10$ m, but > 3 m	For L > 10 m	
A or B	1	External	3.30	$3.50 - (2/L)$	$5 - (12/L)$
		Internal	$3.30 + 0.05\,L$	$4.40 - (6/L)$	$5 - (12/L)$
	2	External	6.50	$6.80 - (3/L)$	$5 - (15/L)$
		Internal	$4.80 + 0.10\,L$	$7.20 - (14/L)$	$5 - (15/L)$
	3	External	8.30	$8.70 - (4/L)$	$10 - (25/L)$
		Internal	$6.70 + 0.08\,L$	$9.60 - (21/L)$	$10 - (25/L)$
	4	External	9.50	$10.00 - (5/L)$	$10 - (25/L)$
		Internal	$7.60 + 0.14\,L$	$11.20 - (22/L)$	$10 - (25/L)$
C or D	1	External	3.30	$3.50 - (2/L)$	$5 - (12/L)$
		Internal	$3.30 + 0.05\,L$	$4.40 - (6/L)$	$5 - (12/L)$
	2	External	6.10	$6.40 - (3/L)$	$5 - (15/L)$
		Internal	$4.80 + 0.10\,L$	$7.20 - (14/L)$	$5 - (15/L)$
	3	External	7.70	$8.10 - (4/L)$	$10 - (25/L)$
		Internal	$6.60 + 0.04\,L$	$8.80 - (18/L)$	$10 - (25/L)$

Table 2.7 Values for F for longitudinal shears in slab-on-girder bridges for the ultimate limit state of CHBDC

Class of highway	No. of design lanes	F, m
A or B	1	3.50
	2	6.10
	3	8.20
	4	9.50
C or D	1	3.50
	2	6.10
	3	7.60

where

S = centre-to-centre girder spacing in metres
$\mu = (W_e - 3.3)/0.6$
W_e = width of the design lane in metres

F is a width dimension that characterises load distribution for a bridge, and C_f is a correction factor in %. Both F and C_f for moments and shears are obtained from Tables 2.6 and 2.7, respectively, for slab-on-girder bridges for the ultimate limit state. It is noted that the values of F for external girders noted in Table 2.6 are applicable when the deck slab overhang is equal to or less than $0.5S$.

The Ontario II and CHBDC methods discussed above can also be used in conjunction with other design codes provided that the following conditions are met.

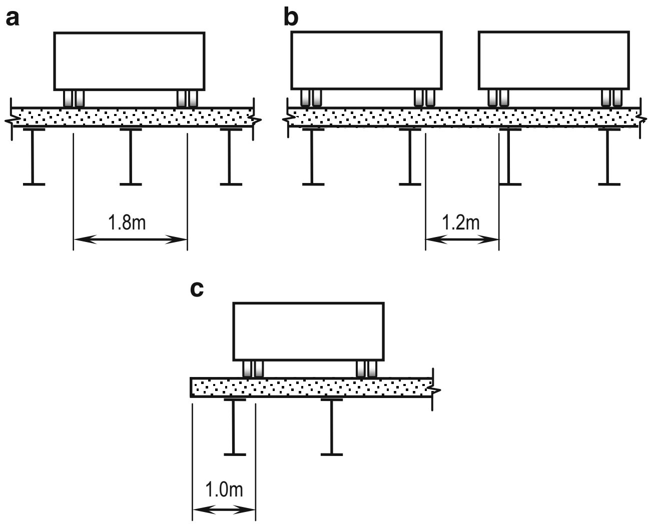


Fig. 2.7 Transverse spacing of longitudinal lines of wheels: (**a**) spacing between two lines of wheels of a vehicle, (**b**) spacing between two lines of wheels of adjacent vehicles, (**c**) minimum vehicle edge distance

(a) The design vehicle has two longitudinal lines of wheels the centres of which are transversely about 1.8 m apart, as illustrated in Fig. 2.7a.
(b) When two design vehicles are present on the bridge side-by-side, their adjacent longitudinal lines of wheels are about 1.2 m apart, centre to centre, as illustrated in Fig. 2.7b.
(c) The transverse distance between a longitudinal free edge of the bridge and the centre of the closest longitudinal line of wheels of the design vehicle, i.e. the vehicle edge distance, is not less than about 1.0 m.
(d) The reduction factors for multiple presences in more than one lane of the bridge are as shown in Table 2.5.

It is noted that the Ontario and CHBDC methods are applicable only to those bridges where the values of D_x lie below the upper-bound values defined by Eq. (2.11). In some jurisdictions, the bridges are considerably stiffer. Figure 2.8, for example, compares the values of D_x for slab-on-girder bridges in Hong Kong with those in North America (Chan et al. 1995). For such bridges, the Ontario and CHBDC methods should not be used as they will lead to unsafe results. When the conditions noted above are not met, a set of new simplified methods can be developed readily as explained in Sect. 2.4.

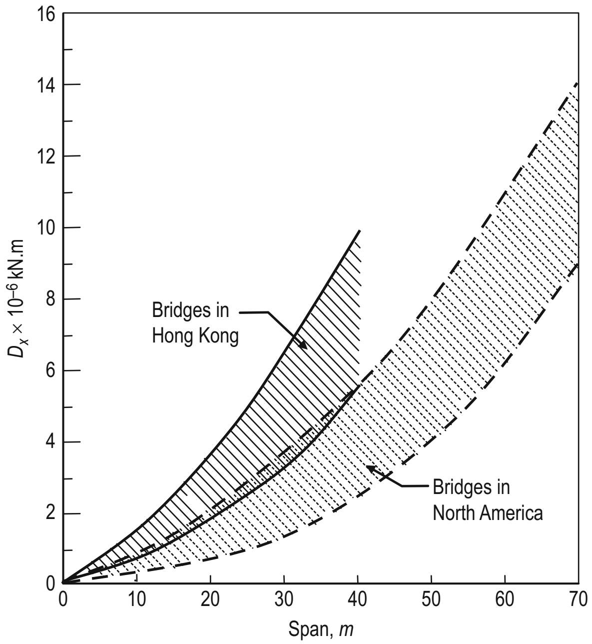
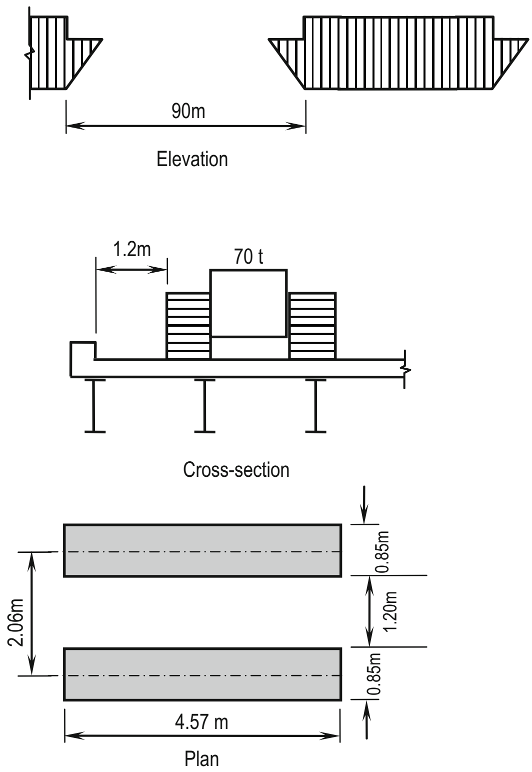


Fig. 2.8 D_x plotted against L for slab-on-girder bridges in Hong Kong and North America

2.4 Two Proposed Methods for Two-Lane Slab-On-Girder Bridges

The previous manual methods of bridge analysis, such as those by Morice and Little and by Cusens and Pama, were made simple by simplifying assumptions. On the other hand, the simplified methods presented in this chapter are based on the results of rigorous analyses, and accordingly are dependent upon the specification of design live loading. The design loadings used elsewhere in the world are significantly different from those of North America. Two simplified methods are presented in this section for the analysis of slab-on-girder bridges with two design lanes for two specific design loads. It is important to note that these methods give approximate values of maximum moments and shears in girders that should preferably be used only to verify the results of rigorous analyses. One of the presented methods was developed by analysing a selected number of bridges by the orthotropic plate method, and the other was developed by analysing the selected bridges by the semi-continuum method; both these rigorous methods of analyses are discussed in Chap. 3 along with their computer programs, which are can be downloaded from http://extras.springer.com. Other similar simplified methods could be developed by using these programs.

Fig. 2.9 70-*R* track loading



2.4.1 Simplified Method for Indian Road Congress Bridge Design Loads

The Indian Road Congress (IRC) specifies three design loads, being 70-R loading, Class A loading and Class B loading. Details of the 70-R loading are shown in Fig. 2.9, and those for Class A loading in Fig. 2.10. Class B loading is the same as Class A loading, except that its axle weights are 60 % of those of Class A loading, and its wheel contact areas are smaller than those for Class B loading. The transverse position of the 70R loading with respect to the nearest curb is shown in Fig. 2.9, and those for Class A and B loadings in Fig. 2.11. The latter figure also shows the transverse distance between the lines of wheels of two adjacent design loads.

A simplified method was developed for slab-on-girder bridges having a clear distance of 7.5 m between the curbs, so that each design lane has a width of 3.75 m.

The lane width was assumed to be 3.75 m, and the minimum curb width, *B*, was assumed to be 0.225 m, resulting in a total bridge width of 7.95 m. The upper limit of the curb width was assumed to be 0.5 m, which resulted in a total bridge width of 8.5 m. Fifteen bridges corresponding to each of these two widths, and having spans

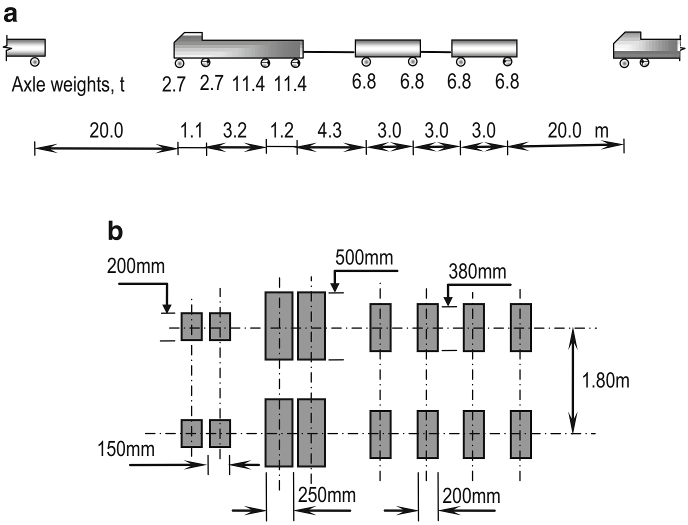


Fig. 2.10 Class A loading: (**a**) elevation, (**b**) plan, not to scale

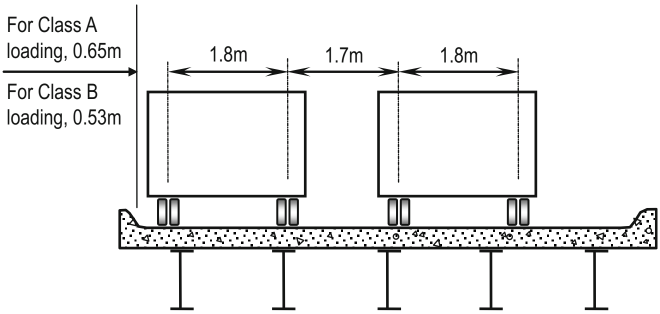


Fig. 2.11 Transverse vehicle positions for Class A and B loadings

of 20.0 m were analysed for *70-R*, *Class A* and *Class B* loadings by the orthotropic plate method of Cusens and Pama (1975), which is incorporated into a computer program PLATO, discussed in Chap. 3. The 15 bridges effectively covered the entire α-θ space for slab-on-girder bridges, as can be seen in Fig. 2.12.

For determining the values of D for longitudinal moments, the vehicles were placed in such longitudinal positions as would induce maximum total longitudinal moments. Similarly for shear D values, the vehicles were positioned to induce maximum longitudinal shears. The transverse vehicle positions were as shown in Figs. 2.9 and 2.11. The orthotropic plate method takes into account the finite size of a concentrated load, which has a significant effect on moment and shear intensities directly under a wheel load. To make the representation of loads as realistic as possible, a deck slab thickness of 200 mm was assumed. It was further assumed that the effective

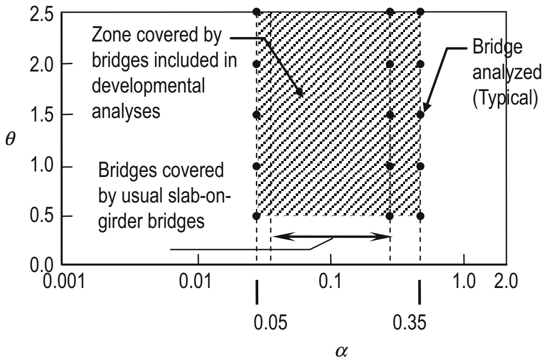


Fig. 2.12 Bridges analysed for the development of the proposed method for IRC loading

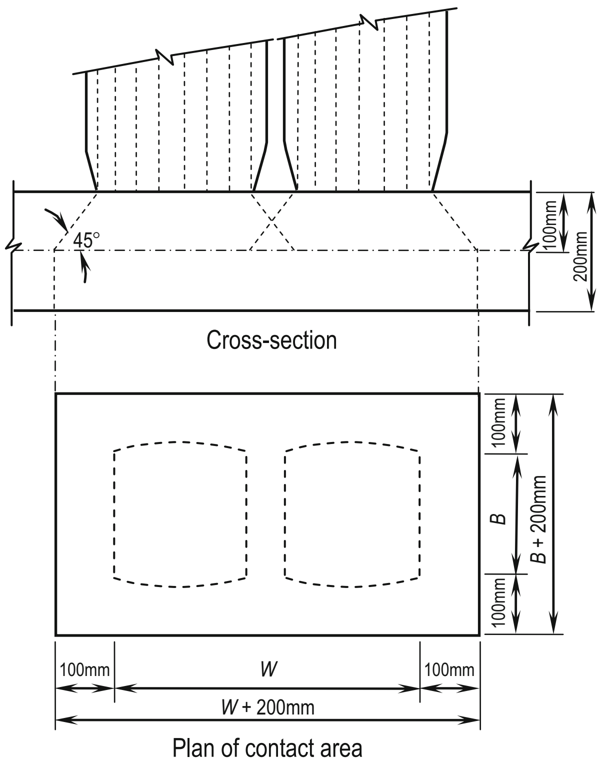


Fig. 2.13 Effective contact area of concentrated loads

size of a concentrated load is obtained by dispersing the surface contact area of the wheel load by 45° to the slab middle surface, as shown in Fig. 2.13.

From Fig. 2.4, it can be readily appreciated that the quantity $SM_{x\ (max)}$ is slightly more than the quantity represented by the shaded area under the curve, which is equal to the moment sustained by the girder. Such over-estimation of live load moments can be eliminated by taking an average of longitudinal moment intensity

Table 2.8 D values in m for longitudinal moments corresponding to *70-R Loading* and $B = 0.2$ m

	D in metres for $\theta =$				
α	0.5	1.0	1.5	2.0	2.5
0.05	2.72	2.37	2.19	2.14	2.13
0.20	2.90	2.46	2.22	2.18	2.18
0.35	3.03	2.53	2.29	2.22	2.20
Mean D values	2.88	2.45	2.23	2.18	2.17
Max. variation from mean	±5 %	±3 %	±3 %	±2 %	±2 %

over the width S and then multiplying this average moment intensity by S to obtain the girder moment. For all the analysed cases, the average maximum longitudinal moments and shears, $M'_{x(max)}$ and $V'_{x(max)}$ respectively, were obtained by averaging the corresponding quantities over a width of 2.0 m. It should be appreciated that this 2.0 m width is only to reduce the effect of "peakiness" of $M_{x(max)}$. A departure of actual girder spacing from this quantity would have negligible effect on $M'_{x(max)}$. From computer-calculated values of $M'_{x(max)}$, the governing value for longitudinal moments was obtained by the following equation, which is a rearranged form of Eq. (2.5), in which $M_{x(max)}$ is replaced by $M'_{x(max)}$:

$$D = M/M'_{x(\max)} \tag{2.17}$$

where M is the total moment due to one line of wheels or one track of the design loading. Similarly, D for longitudinal shear is given by:

$$D = V/V'_{x(\max)} \tag{2.18}$$

where V is the total shear due to one line of wheels or one track of the design loading. Each of the cases were analysed for 31 harmonics. Spot checks of results with 41 harmonics confirmed that the solutions were fully converged under 31 harmonics.

All D values for longitudinal moments were found to be relatively insensitive to small variations in α values, as may be seen for example in Table 2.8 which shows the D values corresponding to the *70-R* loading. Adopting a mean value of D for given θ values results in maximum errors or ±5 %. In the light of this observation, it was decided to eliminate α from consideration. Changes in curb widths did have a noticeable effect on D values for longitudinal moment, especially for *Class A* and *Class B* loadings, for which the code-specified minimum edge distances are unusually small.

The D values for moments are plotted in Fig. 2.14 for the three loadings, and for curb widths $B = 0.2$ and 0.5 m, respectively. A few spot checks indicated that a linear interpolation for intermediate curb widths provided results of reasonable accuracy.

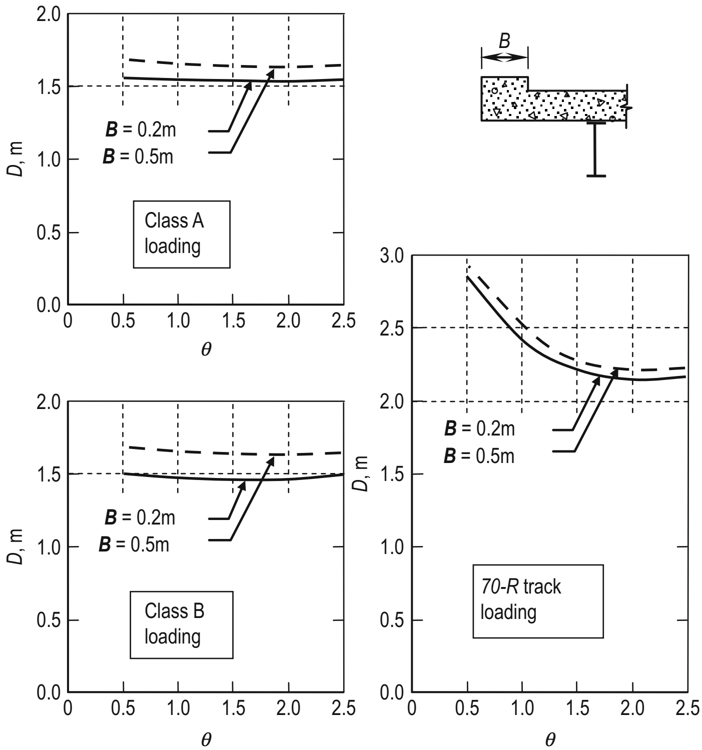


Fig. 2.14 *D* values for longitudinal moments for IRC design loads

The θ values of 0.5, 1.0, 1.5, 2.0, and 2.5 for which bridges were analysed, correspond to γ of 0.141, 0.035, 0.016, 0.009 and 0.006, respectively, where γ is given as follows:

$$\gamma = \left(D_y/D_x\right)^{0.5} \tag{2.19}$$

It was found that for the *70-R* loading, the *D* value for shear varied almost linearly with γ, resulting in the following simple relationship which gives the *D* value in metres:

$$D = 2.16 + 1.6\gamma \tag{2.20}$$

For *Class A* and *Class B* loadings, *D* values for shear were little affected by γ. It was found to be sufficiently accurate to adopt the single value of 1.6 m for *D* for all slab-on-girder bridges for both *Class A* and *Class B* loadings.

The method described above can be used for analysing slab-on-girder bridges with two design lanes, and subjected to *70-R, Class A* or *Class B* design loads, it being noted that the bridges concerned must satisfy the following conditions.

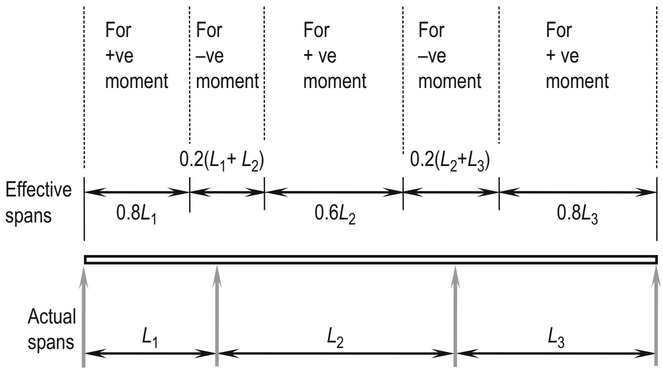


Fig. 2.15 Effective span lengths for calculating θ

(a) The width is constant or nearly constant and there are at least three girders in the bridge;
(b) the skew parameter $\varepsilon = (S \tan \psi)/L$ does not exceed 1/18 where S is girder spacing, L is span and ψ is the angle of skew;
(c) for bridges curved in plan, L^2/bR does not exceed 1.0, where R is the radius of curvature; L is span length; b is half width of the bridge;
(d) the total flexural rigidity of transverse cross-section remains substantially the same over at least the central 50 % length of each span;
(e) girders are of equal flexural rigidity and equally spaced, or with variations from the mean of not more than 10 % in each case; and
(f) the deck slab overhang does not exceed 60 % of S, and is not more than 1.8 m.

In cases where the above conditions are not fully met, engineering judgement should be exercised to ascertain if a bridge meets them sufficiently closely for the simplified method to be applicable.

The proposed method requires the following steps:

(a) Calculate values of D_x and D_y using Eqs. (2.21) and (2.22), respectively.
(b) Obtain the value of θ from Eq. (2.2). The effective spans for continuous bridges can be obtained from Fig. 2.15 for the purpose of calculating θ.
(c) Corresponding to the type of loading and values of θ and curb width B, read the value of D for moments from Fig. 2.14. For curb widths larger than 0.5 m, use the D value corresponding to $B = 0.5$ m.
(d) Calculate live load longitudinal moment at any section by multiplying the total moment at that section due to one line of wheels, or one track, of the relevant loading by the load fraction S/D.
(e) For longitudinal shear, use a D value of 1.60 m for *Class A* and *Class B* loadings. For class *70-R* loading, obtain the value of D from Eq. (2.20) corresponding to the γ value given by Eq. (2.19). The same value of D for longitudinal shear is applicable for both single and continuous spans.
(f) Similarly to longitudinal moments, obtain longitudinal shear per girder by multiplying the total shear for half the design loading by the load fraction (S/D).

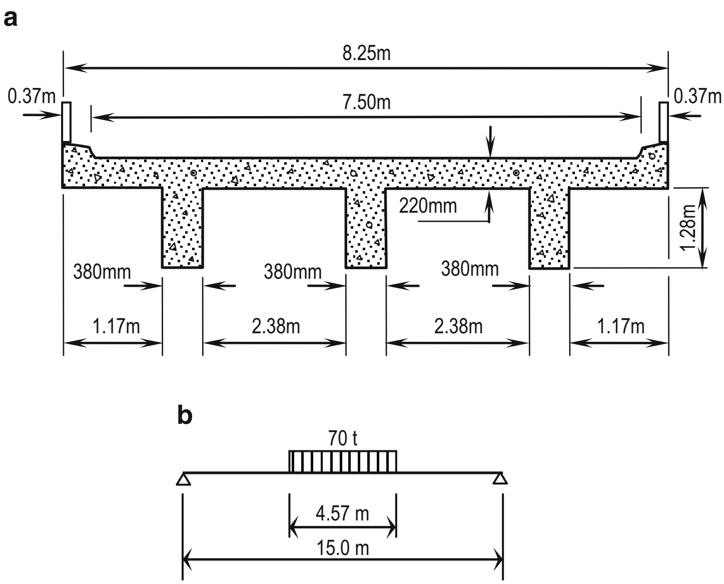


Fig. 2.16 Details of example: (**a**) cross-section, (**b**) simply supported beam under 70-R track loading

The longitudinal flexural rigidity D_x, of a bridge is the product of E and i, where E is modulus of elasticity of deck slab concrete, and i is the longitudinal moment of inertia per unit width in the units of deck slab concrete. For obtaining a value for i, the total moment of inertia of the cross-section of the bridge, I, should be calculated in terms of deck slab concrete. The parameter i is then obtained by dividing I by the bridge width. Thus:

$$D_x = \frac{EI}{2b} \tag{2.21}$$

For bridges having fewer than five intermediate diaphragms per span, the transverse flexural rigidity is obtained by ignoring contributions from diaphragms, so that for slab thickness t:

$$D_y = Et^3/12 \tag{2.22}$$

The contribution of diaphragms to transverse flexural rigidity should be taken into consideration only when engineering judgement shows that their contribution can be realistically assumed to be uniformly distributed along the span. Neglecting contributions of diaphragms is a safe-side assumption for nearly all practical bridges.

To illustrate the use of the method, the example of a single-span T-beam bridge is presented; the cross-section of the bridge is shown in Fig. 2.16a, and the 70-R loading on the 15 m simply supported span in Fig. 2.16b.

Table 2.9 Steps in calculation of maximum moments due to 70-R loading

S, m	D, m (from Fig. 2.14)	S/D	Mid-span moment due to one line of tracks, t.m	Max. girder moment at mid-span, t.m
2.76	2.57	2.76/ 2.57 = 1.07	111.25	1.07 × 111.25 = 119.0

The total moment of inertia, I, of the bridge cross-section is calculated to be 0.66 m^4. Hence $D_x = E \times 0.66/8.25 = 0.08E$. The transverse rigidity, D_y, is calculated by ignoring contributions of any diaphragms, so that: $D_y = E \times 0.22^3/12 = 0.00087E$. The half width b and span L for the bridge are 4.125 and 15.0 m, respectively. Therefore, θ is given by:

$$\theta = \frac{4.125}{15}\left[\frac{E \times 0.08}{E \times 8.87 \times 10^{-4}}\right]^{0.25} = 0.85$$

The 70-R track loading, shown in Fig. 2.16b leads to the mid-span moment = 70 (7.5 − 1.14) = 222.5 t.m, so that the mid-span moment due to one line of tracks = 0.5 × 222.5 = 111.25 t.m. The steps in calculating the maximum mid-span moment due to the 70-R loading are listed in Table 2.9, which shows that the maximum live load longitudinal moment per girder is equal to 119.0 t·m.

The above-cited bridge is similar to the one used by Krishna and Jain (1977) to illustrate the use of the Morice and Little method, according to which the maximum longitudinal moment per girder due to the 70-R loading is found to be 110 t·m.

2.4.2 *Simplified Method for HB Design Loads*

A simplified method for two-lane slab-on-girder bridges in Hong Kong was presented by Chan et al. (1995); details of this method, which was developed for the British HB loading, are presented in the following with the hope that it would prove useful for preliminary design of bridges designed by the British document BS5400.

As shown in Fig. 2.17, the HB loading comprises four axles, with four wheels in each axle. The weights of the wheels are governed by the units of the HB loading. In Great Britain, most highway bridges are designed for 45 units of HB loading, so that each wheel has a load of 112.5 kN, giving a load of 45 kN per axle. Each unit of HB loading is equal to 10 kN. It should, however, be noted that the units of the design loading does not affect the simplified method presented in the following.

Chan et al. (1995) have shown that D_x, the longitudinal flexural rigidity per unit width, of slab-on-girder bridges in Hong Kong lies between two bounds defined by the following equations.

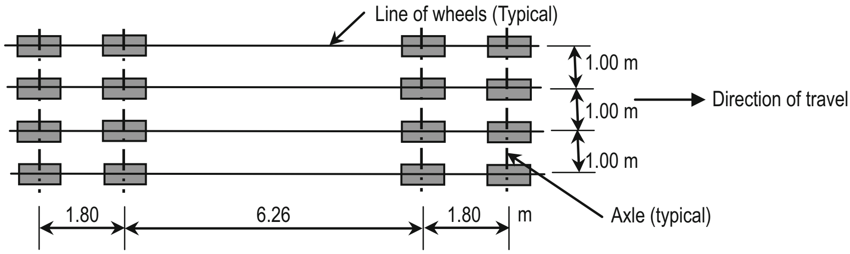


Fig. 2.17 Plan of HB loading

$$D_x = 48{,}000L + 5{,}100L^2 \text{(upper bound)} \tag{2.23}$$

$$D_x = 2{,}000L + 3{,}650L^2 \text{(lower bound)} \tag{2.24}$$

where the span of the bridge L is in metres and D_x in kN.m. The two bounds defined by the above equations are illustrated in Fig. 2.8. From a study of a large number of slab-on-girder highway bridges in Hong Kong, it was determined that the ranges of the various parameters which influence the transverse load distribution characteristics of a bridge are as follows.

(a) The deck slab thickness, t, varies between 150 and 230 mm, with the usual value being 200 mm.
(b) The centre-to-centre spacing of girders, S, varies between 0.2 and 2.0 m, with the usual value being 1.0 m.
(c) The lane width, W_e, varies between 3.2 and 3.8 m, with the usual value being 3.5 m.
(d) The vehicle edge distance, VED, being the transverse distance between the centre of the outermost line of wheels of the HB loading and the nearest longitudinal free edge of the bridge, varies between 0.75 and 5.00 m, with the usual value being 1.00 m.

From the above observations, the following values of the various parameters were adopted for the developmental analyses conducted for developing the simplified method: $t = 200$ mm; $S = 1.0$ m; $W_e = 3.5$ m; and VED = 1.00 m. In addition, it was assumed that the deck slab overhang beyond the outer girders was 0.55 m.

Bridges with spans of 10, 20, 30 and 40 m were selected for the developmental analyses. The flexural rigidities of their girders were calculated from the mean values of D_x given by Eqs. (2.22) and (2.23). The selected idealised bridges under the HB loading were analysed by the semi-continuum method, which is incorporated in computer program SECAN, discussed in Chap. 3. For each bridge, the HB vehicle was placed so as to induce maximum moments at the mid-span. The maximum moment, designated as $M_{g(max)}$ was calculated for each case for each of the external and internal girders.

From Eq. (2.6), it can be shown that D, which has the units of length and provides a measure of the transverse load distribution characteristics of bridge as discussed earlier, is given by:

$$D = MS/M_{g\,(max)} \tag{2.25}$$

Chan et al. (1995), having plotted the values of D from the above analyses against the span length L, found that these values of D are related to L according to the following equations with a reasonable degree of accuracy, provided that the design value of D, i.e. D_d, is corrected by Eq. (2.30).

For internal girders having $L < 25$ m:

$$D = 1.20 - 3.50/L \tag{2.26}$$

For internal girders having $L \geq 25$ m:

$$D = 1.06 \tag{2.27}$$

For external girders having $L < 25$ m:

$$D = 0.95 + 2.10/L \tag{2.28}$$

For external girders having $L \geq 25$ m:

$$D = 1.03 \tag{2.29}$$

The correcting equation for obtaining D_d is as follows.

$$D_d = D\left(1 + \frac{\mu C_w}{100}\right) \tag{2.30}$$

in which μ is given by:

$$\mu = \frac{3.5 - W_e}{0.25} \tag{2.31}$$

where W_e is the width of the design lane in meters.

Chan et al. (1995) have provided charts in which C_w is plotted as functions of L for internal and external girders. However, C_w can also be obtained fairly accurately by the following equations.

For internal girders:

$$C_w = \frac{6L}{40} \tag{2.32}$$

For external girders:

$$C_w = \frac{17.5 - 8.5(L - 10)}{10} \tag{2.33}$$

It is noted that in the above equations, both D and L are in metres.

The following conditions must be met for applying the above simplified method.

(a) The value of D_x, calculated from Eq. (2.21), lies between the upper and lower bound values given by Eqs. (2.23) and (2.24), respectively.
(b) The bridge has two design lanes, with the lane width, W_e, being $\geq$3.5 m.
(c) The width is constant or nearly constant, and there are at least three girders in the bridge.
(d) The skew parameter $\varepsilon = (S \tan \psi)/L$ does not exceed 1/18 where S is girder spacing, L is span and ψ is the angle of skew.
(e) For bridges curved in plan, L^2/bR does not exceed 1.0, where R is the radius of curvature; L is span length; $2b$ is the width of the bridge.
(f) The total flexural rigidity of transverse cross-section remains substantially the same over at least the central 50 % length of each span.
(g) Girders are of equal flexural rigidity and equally spaced, or with variations from the mean of not more than 10 % in each case.
(h) The deck slab overhang does not exceed 0.6 S, and is not more than 1.8 m.

In cases where the above conditions are not fully met, engineering judgement should be exercised to ascertain if the bridge meets them sufficiently closely for the simplified method to be applicable.

The following steps of calculation are required in calculating maximum moments in internal an external girders due to HB loading.

(a) Calculate the value of D from the relevant of Eqs. (2.26, 2.27, 2.28, and 2.29); for simply supported spans, L is the actual span length, and for continuous span bridges, the effective L for different spans is obtained from Fig. 2.15.
(b) For the design lane width, W_e, obtain μ from Eq. (2.31) and C_w from the relevant of Eqs. (2.32) and (2.33), and thereafter obtain D_d from Eq. (2.30).
(c) Isolate one girder and the associated portion of the deck slab, as illustrated in Fig. 2.3b, and analyse it by treating it as a one-dimensional beam under one line of wheels of the HB loading, shown in Fig. 2.17. The moment thus obtained at any transverse section of the beam is designated as M.
(d) For of the internal and internal girders, obtain the maximum moment at the transverse section under consideration by multiplying M with (S/D_d), where D_d is as obtained in Step (b) for the relevant of the internal and external girders.

The use of the above method is illustrated by analysing an actual two-span continuous steel girder bridge in Hong Kong, the Canton Road Duplication Bridge (Chan et al. 1995). Since this bridge was included in determining the upper and lower bound values of D_x for bridges in Hong Kong (Fig. 2.8), there is no need to check if the value of its D_x falls within the two bounds. In the following example, maximum positive, i.e. sagging, moments in the internal girders of the bridge under 45 units of HB loading are determined by the simplified method. The various parameters of the bridge are as noted in the following.

Actual span lengths, $L = 22.0$ and 22.0 m
Bridge width, $2b = 8.7$ m
Design lane width $W_e = 3.35$ m
Deck slab thickness $= 0.18$ m
Girder spacing $S = 1.48$ m
Deck slab overhang $= 0.65$ m
Vehicle edge distance VED $= 1.25$ m

The parameters for the simplified analysis are calculated as follows.

From Fig. 2.15, effective span $L = 0.8 \times 22.0 = 17.6$ m
From Eq. (2.26), $D = 1.20 - 3.5/17.6 = 1.001$ m
From Eq. (2.31), $\mu = (3.5 - 3.35)/0.25 = 0.6$
From Eq. (2.32), $C_w = 2.6$ %
From Eq. (2.30), $D_d = 1.001 \times (1 + 0.6 \times 2.5/100) = 1.017$ m
Load fraction $S/D_d = 1.48/1.017 = 1.455$

From beam analysis, the maximum positive moment in the two-span beam due to one line of 45 units of HB loading, M, is found to be 1160 kN.m. By multiplying this moment with load fraction 1.445, the maximum positive moment in any of the internal girders of the bridge was found to be 1688 kN.m.

A rigorous analysis of the above problem by the semi-continuum and grillage methods gave the maximum moments of 1503 and 1636 kN.m, respectively (Chan et al. 1995). This observation confirms that the moments given by the simplified method are 12 % larger than those given by the semi-continuum method and 3 % larger than those given by the grillage method. It is discussed in Chap. 3 that while the two rigorous methods of analysis are similar in idealisation, the semi-continuum method is believed to be more accurate than the grillage method.

2.5 Analysis of Two-Girder Bridges

In two-girder bridges and those bridges which comprise two main longitudinal members, such as trusses, the transverse distribution analysis is usually done by simple static apportioning of the loads to the two main longitudinal members. For example, considering the cross-section of the two-girder bridge shown in Fig. 2.18a, with a girder spacing S and a concentrated load P located on the deck slab at a distance eS from the left girder, it is usual to assume, as shown in Fig. 2.18b, that the left and right girders receive loads $(1 - e)\,P$ and eP, respectively. It is recalled that these loads are the same as the reactions of a beam simply supported by the two girders.

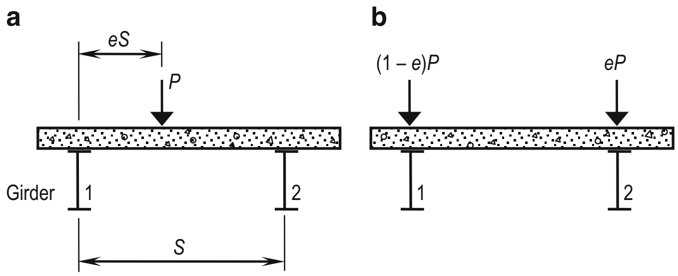


Fig. 2.18 The usual method of apportioning loads to girders in a two-girder bridge (**a**) single load between two girders, (**b**) static apportioning of load without considering torsional rigidities

Transverse load distribution analysis by static apportioning as described above is based upon the implicit assumption that the bridge has negligible torsional rigidities in both the longitudinal and transverse directions of the bridge. In practice, even though the torsional rigidities of girders themselves may be negligible, the torsional rigidity of the deck slab, in both the longitudinal and transverse directions, can be substantial. The neglect of these torsional rigidities can make the analysis by static apportioning somewhat conservative. The objective of this section is to present a very simple, yet accurate, method of apportioning loads to the girders of two-girder, right bridges, based upon the semi-continuum method, the general treatment of which is described in Chap. 3. It is emphasized that knowledge of the general semi-continuum method is not needed in order to be able to use the proposed method.

2.5.1 Two-Girder Bridges

The case of a right, simply-supported bridge with two main girders supporting a concrete deck slab, is considered first. Consistent with usual practice, transverse deflections of the solid concrete deck slab due to shear are assumed to be negligible.

Figure 2.18a shows a two-girder bridge carrying a longitudinal line load at a distance *eS* from girder 1. By using the general semi-continuum method described by Jaeger and Bakht (1989), it can be shown that the distribution coefficients for longitudinal bending moment and torsion are given by:

$$\rho_1 = \frac{\frac{\eta}{2}(1 - e + \lambda + 2\mu) + \mu(1 + \lambda - 3e^2 + 2e^3)}{\frac{\eta}{2}(1 + 2\lambda + 4\mu) + \mu(1 + 2\lambda)} \tag{2.34}$$

$$\rho_2 = \frac{\frac{\eta}{2}(e + \lambda + 2\mu) + \mu(\lambda + 3e^2 - 2e^3)}{\frac{\eta}{2}(1 + 2\lambda + 4\mu) + \mu(1 + 2\lambda)} \tag{2.35}$$

$$\rho_1^* = \frac{e(1-e)}{2\left(\frac{\eta}{6} + \mu\right)} + \frac{\left(\frac{1}{2} - e\right)\{(1 + 2\lambda)e(1 - e) - \eta\}}{\frac{\eta}{2}(1 + 2\lambda + 4\mu) + \mu(1 + 2\lambda)} \tag{2.36}$$

$$\rho_2^* = \frac{-e(1-e)}{2\left(\frac{\eta}{6}+\mu\right)} + \frac{\left(\frac{1}{2}-e\right)\{(1+2\lambda)e(1-e)-\eta\}}{\frac{\eta}{2}(1+2\lambda+4\mu)+\mu(1+2\lambda)} \tag{2.37}$$

where ρ_1 and ρ_2 are the distribution coefficients for longitudinal bending moments in girders 1 and 2, respectively; and $\rho_1{}^*$ and $\rho_2{}^*$ are the distribution coefficients for longitudinal torsional moments. It is noted that the left girder is referred to as girder 1, the right girder as girder 2, and that the quantity e is positive when the load is to the right hand side of girder 1.

In the above equations, η, λ and μ are the dimensionless characterizing parameters of the bridge defined by:

$$\eta = \frac{12}{\pi^4}\left\{\frac{L}{S}\right\}^3 \frac{LD_y}{EI} \tag{2.38}$$

$$\lambda = \frac{1}{\pi^2}\left\{\frac{L}{S}\right\}^2 \frac{SD_{yx}}{EI} \tag{2.39}$$

$$\mu = \frac{1}{\pi^2}\left\{\frac{L}{S}\right\}^2 \frac{GJ}{EI} \tag{2.40}$$

in which L = bridge span; S = girder spacing; EI = the combined flexural rigidity of one girder and the associated portion of the deck slab; GJ = the combined torsional rigidity of one girder and the associated portion of the deck slab; D_y = the transverse flexural rigidity per unit length of the deck slab; and D_{yx} = the transverse torsional rigidity per unit length of the deck slab. For a deck slab of thickness t, modulus of elasticity E_c and shear modulus G_c, the values of D_y and D_{yx} are given by:

$$D_y = \frac{E_c t^3}{12} \tag{2.41}$$

$$D_{yx} = \frac{G_c t^3}{6} \tag{2.42}$$

It is recalled that for analysis by the semi-continuum method, the applied loading is represented by harmonic series, and that Eqs. (2.38), (2.39), and (2.40) are applicable for only the first term of the loading series. The characterizing parameters for higher terms, which are not utilized for the development of the proposed method, are obtained by replacing π in these equations by $m\pi$, where m is the harmonic number.

A slab-on-girder bridge carrying a single concentrated load is considered. For such a case, the ratio of the moment induced in a girder to the total moment at the transverse section under consideration varies from section to section; furthermore, this ratio of girder moments is not the same as the ratio for girder shears (Jaeger and Bakht 1989). For these reasons, the girder moments and shears cannot be directly derived simply by multiplying the total moments and shears by ρ_1 and ρ_2.

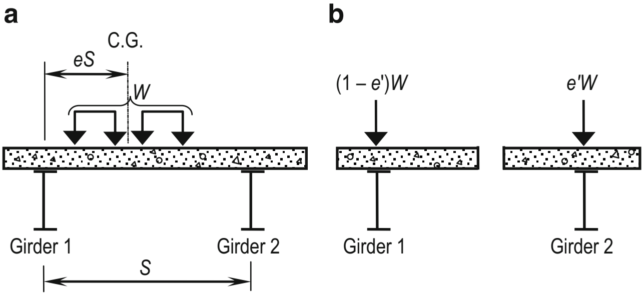


Fig. 2.19 Notation used in conjunction with the proposed method (**a**) four loads between two girders, (**b**) static apportioning of load by considering torsional rigidities

Variation of the transverse distribution patterns of longitudinal responses along the span is caused mainly because some girders are directly loaded while others are not. In a two-girder bridge, most of the applied loads are located transversely between the two girders, as a result of which the situation of only some girders carrying the load directly is eliminated. It can be shown that in a two-girder bridge which carries several concentrated loads, the coefficients ρ_1 and ρ_2, which are strictly true only for the first harmonic, nevertheless represent with very good accuracy the fractions of loads transformed to the girders.

A longitudinal line load P at a distance eS from girder 1 is considered as shown in Fig. 2.18a. In light of the above discussion, it can be appreciated that this load is effectively transferred as $\rho_1 P$ and $\rho_2 P$ on girders 1 and 2 respectively. By denoting $e^{*} = \rho_2$, it follows that these loads can be written $(1 - e')\,P$ and $e'P$, which are the loads that would be obtained if the external load were located at a distance $e'S$ from girder 1, and were then apportioned statically in the usual manner. Using Eq. (2.34), the equation for e' can be written as:

$$e' = \frac{\frac{\eta}{2}(e + \lambda + 2\mu) + \mu\left(\lambda + 3_e^{2} - 2_e^{3}\right)}{\frac{\eta}{2}(1 + 2\lambda + 4\mu) + \mu(1 + 2\lambda)} \tag{2.43}$$

In the case of a single concentrated load, eS is the distance of the load measured in the transverse direction from the left girder. When there are two or more concentrated loads on a transverse line, eS becomes the distance of the centre of gravity of the loads from the left girder as illustrated in Fig. 2.19a. The approach of apportioning loads to girders by using e' as defined by Eq. (2.43) can also be used in the case of multiple loads on a transverse line. It is noted that both e and e' are measured positive on the right hand side of the left hand girder, i.e. girder number 1.

The use of the proposed method can be illustrated with the help of the example shown in Fig. 2.19a. As shown in this figure, there are four concentrated loads on a transverse line with a total weight W. The centre of gravity of these loads is a distance eS from girder 1. Using the values of η, λ and μ obtained from Eqs. (2.38), (2.39), and (2.40), respectively, and e, the value of e' is obtained from Eq. (2.43). As shown in Fig. 2.18b, the four loads can be transformed as single concentrated loads

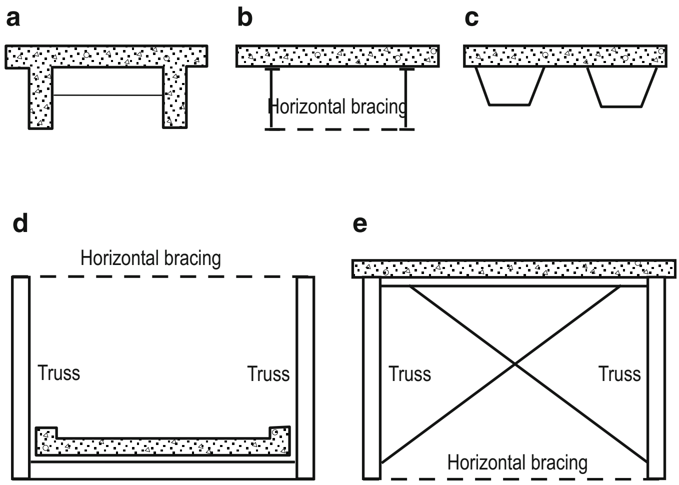


Fig. 2.20 Bridges with two main longitudinal members: (**a**) girder bridge without *horizontal* bracing, (**b**) girder bridge with *horizontal* bracing, (**c**) box girder bridge with two spines, (**d**) through truss bridge, (**e**) deck truss bridge

of weights $(1 - e')W$ and $e'W$ on girders 1 and 2, respectively, both acting on the same section which contains the four applied loads. Thereafter, each girder can be analysed in isolation under the action of the transformed loads.

For the case shown in Fig. 2.18b, equilibrium can be maintained only if the two girders have torsional couples T_1T_2. While it is usual to ignore the effect of these couples in design, they can be derived from the distribution coefficients ρ_1^* and ρ_2^*.

2.5.2 Calculation of Stiffnesses

It will be appreciated that the simplified method proposed above is applicable to a variety of bridges including (a) two-girder bridges without horizontal bracing; (b) two-girder bridges with horizontal bracing; (c) box girder bridges with two spines; (d) through truss bridges; and (e) deck truss bridges. The cross-sections of these bridges are shown in Fig. 2.20a–e, respectively. It is noted that the semi-continuum method can also be applied for the analysis of single cell box girders, as discussed later.

While the proposed method is simple enough to be applied without any difficulty, the calculation of the stiffnesses needed for obtaining the characterizing parameters needs care, and is not always self-evident. The procedures for calculating these stiffnesses for certain kinds of bridges are given below, mainly because not all of them are available in readily-available references.

The longitudinal flexural rigidity *EI* of girder bridges having uniform section along the span can be obtained in the usual manner and needs no explanation. When

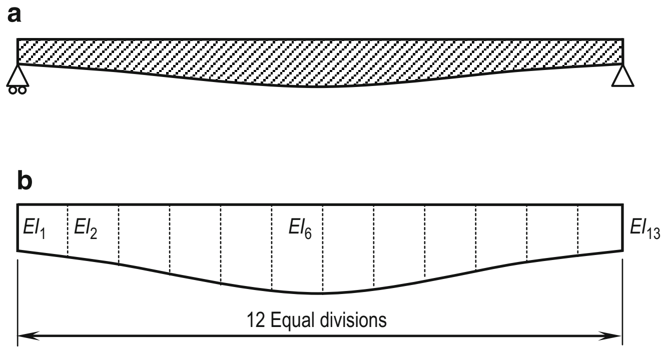


Fig. 2.21 A girder with variable flexural rigidity: (**a**) elevation, (**b**) flexural rigidity

the flexural rigidity of a girder varies along the span, the following expression proposed by Jaeger and Bakht (1989) can be used to obtain the equivalent uniform flexural rigidity EI_e:

$$EI_e = \left(\frac{\pi}{72}\right)\left\{\begin{array}{l} 0.0000EI_1 + 1.0352(EI_2 + EI_{12}) \\ +1.0000(EI_3 + EI_{11}) + 2.8284(EI_4 + EI_{10}) \\ +1.7320(EI_5 + EI_9) + 3.8636(EI_6 + EI_8) \\ +2.0000EI_7 \end{array}\right\} \tag{2.44}$$

where EI_1, EI_2,. . ., EI_{12} are the flexural rigidities of the girder at longitudinal locations identified in Fig. 2.21.

For truss bridges, the equivalent EI can be obtained by seeking equivalence between the maximum truss and equivalent beam deflections under uniformly distributed loads.

In the absence of transverse diaphragms, or transverse floor beams, the transverse flexural rigidity per unit length, D_y is obtained from Eq. (2.41). However, when these transverse members are present, their contribution to D_y may be significant, especially when they are closely spaced. When the deck slab is supported on both longitudinal main members and transverse beams, and the spacing of transverse beams is less than about $0.75S$, D_y can be calculated from:

$$D_y = \frac{EI_t}{S_t} \tag{2.45}$$

where EI_t is the flexural rigidity of a transverse beam and the associated portion of the deck slab, and S_t is the spacing of these beams.

When the deck slab is supported only by transverse beams, as it usually is in the floor systems of truss bridges, Eq. (2.41) can be used for calculating D_y even when S_t is greater than S.

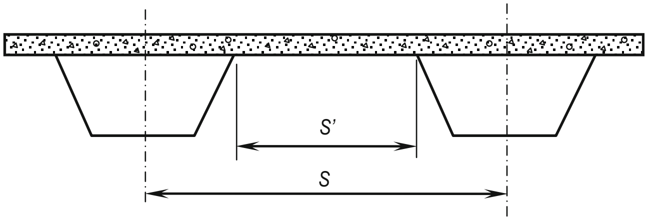


Fig. 2.22 Cross-section of a bridge with two box girders

For twin-cell box girder bridges, the simplified analysis can be performed by idealizing each box girder by a one-dimensional longitudinal beam having the same *EI* and *GJ* as the box girder and the associated portion of the deck slab (taken from centre to centre of the individual cells). However, this will underestimate its ability to transfer loads laterally; to correct for this inconsistency, the value of D_y can be enhanced as follows:

$$D_y = \frac{E_c t^3}{12}\left(\frac{S}{S'}\right)^3 \tag{2.46}$$

where, as shown in Fig. 2.22, S is the centre to centre spacing of the two box girders and S' is the clear transverse span of the deck slab.

For a two-girder bridge without horizontal bracing at the bottom flange level, the longitudinal torsional rigidity of a girder and the associated portion of the deck slab is estimated simply as the sum of the torsional rigidities of the girder and the deck slab, so that:

$$GJ = G_g J_g + G_c b \frac{t^3}{6} \tag{2.47}$$

where G_g and J_g are the shear modulus of the girder material and the torsional inertia of the girder, respectively; G_c is the shear modulus of the deck slab material; b is half the width of the bridge; and t is the slab thickness.

The longitudinal torsional rigidity of a thin walled box girder having a closed section can be obtained from the following equation:

$$GJ = G_c\left\{\frac{4A^2}{\oint \frac{ds}{n_s t'}}\right\} \tag{2.48}$$

where A is the area enclosed by the median line passing through the walls of the closed section; and t' is the thickness of the steel girder; $\oint \frac{ds}{t'}$ refers to the contour

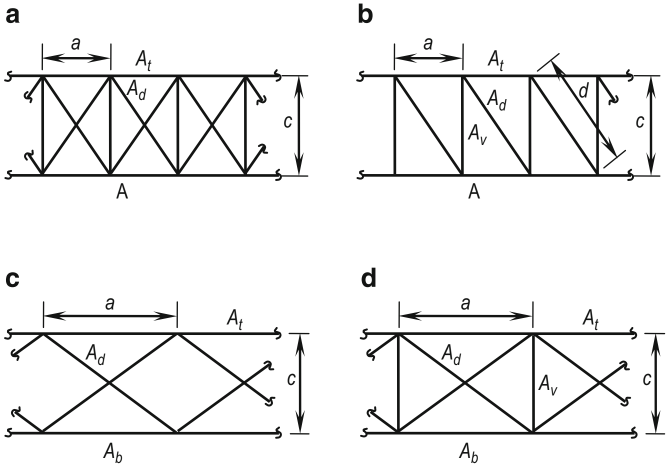


Fig. 2.23 Various frame configurations (**a**) K-type, (**b**) N-type, (**c**) X-type without transverse members, and (**d**) X-type with transverse members

integral along the median line of the reciprocal of the wall thickness; and n_s is the ratio of the shear modulus of the deck slab material and the material of the wall under consideration.

The bottom flanges of steel girder bridges are usually connected by horizontal bracing, which is also referred to as wind bracing. This bracing has the effect of closing the cross-section and thus enhancing considerably the longitudinal torsional rigidity of the bridge. This enhancement of the longitudinal torsional rigidity of the bridge by closing the section also takes place in through-truss bridges and deck-truss bridges. As shown in Fig. 2.20d, the horizontal bracing at the top closes the section in through-truss bridges, whilst in deck truss bridges the section is closed by the horizontal bracing at the bottom, as shown in Fig. 2.20e.

The case is now considered of a closed-section box member in which one or more walls of the member are composed of a framework such as a truss or a system of horizontal bracing. In such a case, as noted by Kollbrunner and Basler (1969), the framework can be idealized as a plate whose thickness t depends upon the configuration of the framework and the cross-sectional areas of the various chord members. For the K-type of framework shown in Fig. 2.23a, the equivalent thickness t is given by:

$$t = \frac{E_s}{G_s} \frac{ac}{\frac{d^3}{A_d} + \frac{a^3}{3}\left\{\frac{1}{A_t} + \frac{1}{A_b}\right\}} \tag{2.49}$$

For the N-type of framework shown in Fig. 2.23b, t is given by:

$$t = \frac{E_s}{G_s} \frac{ac}{\frac{d^3}{A_d} + \frac{c^3}{A_v} + \frac{a^3}{12}\left\{\frac{1}{A_t} + \frac{1}{A_b}\right\}} \tag{2.50}$$

For the X-type of framework shown in Fig. 2.23c, t is given by:

$$t = \frac{E}{G_s} \frac{ac}{\frac{d^3}{2A_d} + \frac{a^3}{12}\left\{\frac{1}{A_t} + \frac{1}{A_b}\right\}} \tag{2.51}$$

When the X-type of framework incorporates members perpendicular to the main members as shown in Fig. 2.23d, the idealized thickness t can be found either by ignoring these transverse members and thus using Eq. (2.50), or by ignoring the diagonal members in compression in which case the system becomes identical to the one shown in Fig. 2.23b. For this latter case t can be obtained from Eq. (2.49).

In the above equations, E_s and G_s are respectively the elastic and shear modulus of the material of the framework; a, c and d are the chord lengths identified in Fig. 2.23; and A_t, A_b A_d and A_v are the cross-sectional areas of the chord members, also identified in Fig. 2.23. For horizontal bracing in girder bridges, A_t and A_b can be taken as the areas of cross-section of the bottom flanges of the relevant girders.

For all the bridges discussed so far, it is conservative to assume that only the deck slab provides the transverse torsional rigidity, so that D_{yx} can always be obtained from Eq. (2.41).

2.5.3 Numerical Example

As an illustration, the proposed simplified method is used to analyse a two-girder bridge under two eccentrically-placed vehicles. Various details of the bridge are given in Fig. 2.24 including the areas of cross-sections of the various members of the horizontal bracing system; other relevant details are given below.

Span, L	= 18.0 m
Girder spacing, S	= 8.75 m
Flexural rigidity, EI, of a girder	$= 5.886 \times 10^6$ kN•m^2
Modular ratio, m_s	= 10
Floor beam spacing, S_t	= 4.5 m
Flexural rigidity, EI_t, of floor beam	$= 1.320 \times 10^6$ kN•m^2

The bracing system is the same type as is shown in Fig. 2.23d. The equivalent thickness for this system can be obtained either from Eqs. (2.50) or (2.51), which give $t = 0.42$ mm and 0.36 mm, respectively. Both of these values are conservative estimates of the effective thickness. The smaller thickness would give an even safer side estimate of the load distribution characteristics of the structure. Accordingly, this value was chosen.

Using $t = 0.36$ mm and other relevant properties of the bridge cross-section, the total longitudinal torsional stiffness of the bridge is found to be 3.480×10^6 kN·m^2. By assigning half of this torsional rigidity to each girder, the effective girder GJ becomes 1.74×10^6 kN·m^2.

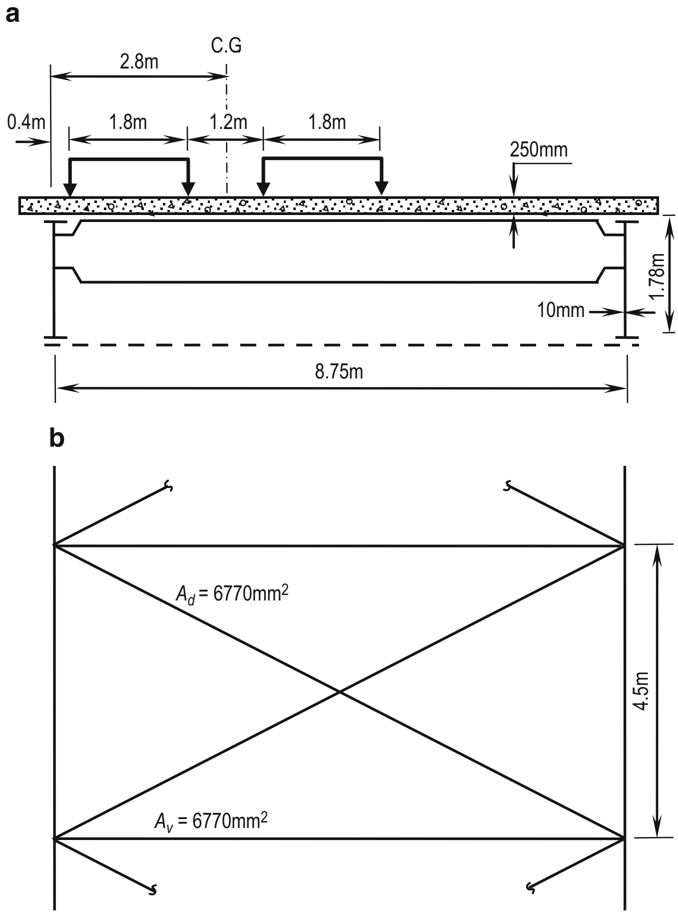


Fig. 2.24 Details of a two-girder bridge: (**a**) cross-section, (**b**) plan

Using Eq. (2.45) $D_y = 386{,}667$ kN·m, and from Eq. (2.42) $D_{yx} = 22{,}100$ kN·m. By using the various stiffnesses and other properties of the bridge, the values of the non-dimensional parameters η, λ and μ are found to be 0.962, 0.014 and 0.127, respectively.

As shown in Fig. 2.24a, the centre of gravity of the four axle loads is 2.8 m from the left girder. Hence $e = 2.8/8.75 = 0.32$. Substituting this value of e and the characterizing parameters calculated above into Eq. (2.43), one obtains $e' = 0.372$.

If the loads were apportioned to the two girders in the usual static manner, girder 1 would have been called upon to sustain 0.68 times the total load. By contrast, load distribution analysis by the proposed method shows that this fraction is 0.628. This reduction represents a 7.6 % reduction in maximum live load effect. Such a reduction in maximum live load effects may not be of great consequence for the design of new bridges, but it can prove to be very useful in the evaluation of the load carrying capacity of existing ones.

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