

Chapter 2

Background Physics and Biogeochemistry

2.1 Conservation Principles and Governing Equations

Conservation equations can be formulated for most aquatic properties (ϕ). For transient three-dimensional problems, the general differential equation is:

$$\frac{\partial}{\partial t} \phi + U_i \frac{\partial \phi}{\partial x_i} = S_\phi \quad \text{where } i = x, y, z \quad (2.1)$$

where ϕ could be, for example, momentum, temperature, salinity, or oxygen, U_i is the velocity, where the index indicates velocity components in the horizontal and vertical directions, and S_ϕ is the source/sink term related to the properties considered. The coordinates in space are denoted x , y , and z , while t is the coordinate in time. As geophysical flows are not only typically turbulent, but also include waves, their properties can be divided into mean, wave, and fluctuation or turbulent parts and simplifications can then be made based on scale analysis. If we neglect the wave motion, division can be formulated as:

$$\begin{aligned} U_i &= \overline{U}_i + U'_i \\ \phi_i &= \overline{\phi}_i + \phi'_i \end{aligned} \quad (2.2)$$

where $\overline{U}_i$ and $\overline{\phi}_i$ represent the mean velocity and mean property, and U'_i and ϕ'_i represent the corresponding fluctuating parts. Equations derived from the conservation principles using the Reynolds method of averaging and the eddy viscosity concept read:

$$\begin{aligned} \frac{\partial}{\partial t} \overline{\phi} + \overline{U}_i \frac{\partial \overline{\phi}}{\partial x_i} &= \frac{\partial}{\partial x_i} \left(\Gamma_\phi \frac{\partial \overline{\phi}}{\partial x_i} \right) + \overline{S}_\phi \\ \Gamma_\phi &= \frac{\mu}{\rho \sigma_\phi} + \frac{\mu_T}{\rho \sigma_{\phi T}} \end{aligned} \quad (2.3)$$

where Γ_ϕ includes the sum of molecular and turbulent diffusion processes; μ denotes the dynamic viscosity, μ_T the turbulent viscosity, ρ density, σ the Prandtl/Schmidt number, and $\sigma_{\phi T}$ the turbulent Prandtl/Schmidt number. The terms of the conservation equation, from left to right, represent property changes: in time, due to advection, due to turbulent diffusion, and due to sources/sinks. The geophysical flow equations are described in detail by Cushman-Roisin and Becker (2011).

The source/sink terms in the momentum equation are pressure, gravity, and the Coriolis term; the last term is part of acceleration but can also be treated as a source term. The source term in the temperature equation is the sun radiation that penetrates the water body.

An important aspect of the general conservation equation (Eq. 2.1) is that it can form the basis for developing general equation solvers. Based on this equation, one can derive the corresponding discretized equation through integration over a control volume. The general discretization equation can then be formulated as:

$$a_P \theta_P = a_W \theta_W + a_E \theta_E + a_S \theta_S + a_N \theta_N + a_B \theta_B + a_T \theta_T + S_\theta \quad (2.4)$$

where we now use θ to represent the finite volume form of ϕ . The indices represent the calculations in position P determined from information from the west (W), east (E), south (S), north (N), bottom (B), and top (T) nodal points. This type of equation forms the basis of several general equation solvers (Versteeg and Malalasekera 1995).

Exercise 2.1

The mean depth of the Baltic Sea is 54 m and its surface area is $3.9 \times 10^5 \text{ km}^2$. How much would the level of the Baltic Sea increase over a year with river water inflow of $15,000 \text{ m}^3 \text{ s}^{-1}$ and no outflows? If the outflowing volume flow were $30,000 \text{ m}^3 \text{ s}^{-1}$, how large would the inflowing volume flow need to be to keep the sea level unchanged? If the salinity of the inflowing water were 17 salinity units, what would the salinity be in the basin?

2.2 Physical Aspects

A description of a fluid should relate the fluid's density to its state variables, a relationship called the equation of state. For natural waters, this equation is generally a function of temperature, salinity, and pressure (Gill 1982, App. 3); for coastal seas, however, one can often neglect pressure and use the following approximation:

$$\begin{aligned}\rho &= \rho_0 \left(1 - \alpha_1 (T - T_{\rho m})^2 + \alpha_2 (S - S_{ref}) \right) \\ T_{\rho m} &= 3.98 - 0.22S\end{aligned}\quad (2.5)$$

where T and S are temperature and salinity, ρ_0 the reference density, $T_{\rho m}$ the temperature of maximum density, S_{ref} reference salinity, and α_1 and α_2 the thermal and salinity expansion/contraction coefficients, respectively. Typical values for brackish water are $\rho_0 = 1000 \text{ (kg m}^{-3}\text{)}$, $S_{ref} = 0$, $\alpha_1 = 5.10^{-6} \text{ (}^\circ\text{C}^{-2}\text{)}$, and $\alpha_2 = 8.10^{-4}$.

The freezing temperature of seawater is:

$$T_f = -0.0575S + 0.0017S^{1.5} - 0.0002S^2 - 0.00753P_w \quad (2.6)$$

where P_w is the water pressure in bars (we can neglect the pressure term for shallow lakes and seas). Brackish water is often defined as saline water that has a freezing point below the temperature of maximum density. The upper salinity limit is thus 24.7 and the lower limit is estimated to be 0.5 (Leppäranta and Myrberg 2009).

In the case of freezing water, ice needs to be considered. Ice forms a thin, rigid but fragile layer over the water body that dramatically changes the heat, momentum, and gas exchanges between atmosphere and water. In nature, ice forms in two ways: (1) under calm conditions when the water surface is slightly supercooled and (2) after atmospheric seeding, when ice crystals start growing horizontally into large crystals. When the horizontal space is occupied, the crystals then grow vertically (in seawater, ice crystals form from pure water, the salinity leaking out along the crystal boundaries). These ice crystals grow, forming columnar ice that, together with snow, is often seen in sheltered lakes and inland waters. Under windy conditions accompanied by supercooling, small ice crystals get mixed into the water column and form frazil ice. Under open water conditions, all the heat that escapes the cold water surface is used for ice production, resulting in huge amounts of frazil ice. Frazil ice is later transformed into grease ice and pancake ice, which often constitutes the base for sea ice formation.

When a thin ice layer has formed, winds and currents cause it to drift. During free ice drift, the ice moves at approximately 2–3 % of the 10-m wind velocity and 20–30° to the right (in the Northern Hemisphere) of the wind direction, due to the Coriolis effect. Under the influence of onshore winds or when the ice concentration is high, the plastic behavior of ice starts to influence the drift. This reduces the ice velocity, but at a certain ice pressure, the ice breaks and starts forming ridged ice.

Exercise 2.2

Investigate the equation of state by plotting Eq. 2.5 for different temperatures and salinities. What are the typical densities in the Baltic and Mediterranean seas? What are the dominant factors that control density in coastal seas? Compare Eq. 2.5 with the full equation of state given by Gill (1982, App. 3).

2.3 Simplifications

The conservation equations are nonlinear, and it is often impossible to find analytical solutions except in simple flow cases. Instead, we must rely on numerical methods to solve the equations. Before making any calculations, we must systematically reduce the problem complexity by making proper simplifications. We begin a process-based approach by carefully identifying the problem. The first step is to identify the water volume and divide it, guided by observation, into dynamically relevant processes. Hypsographic curves yield important information about the water volumes available at various depths, while bathymetric charts tell us where narrow straits, channels, and canyons are and where natural sub-basins occur (Fig. 2.1). Knowledge of forcing functions (i.e., meteorological, hydrological, and ocean conditions outside the coastal sea) also provides a good basis for modeling design. As water circulation is often crucial, hydrodynamic equations could be simplified by identifying the scales of motion.

The use of dimensionless numbers can help with the scaling. Important dimensionless numbers in geophysical flow dynamics are the temporal Rossby number, $Ro_t = \frac{1}{\Omega T}$, and the Rossby number, $Ro = \frac{U}{\Omega L}$, where $\Omega = \frac{2\pi}{\text{time of one revolution}}$ is Earth's

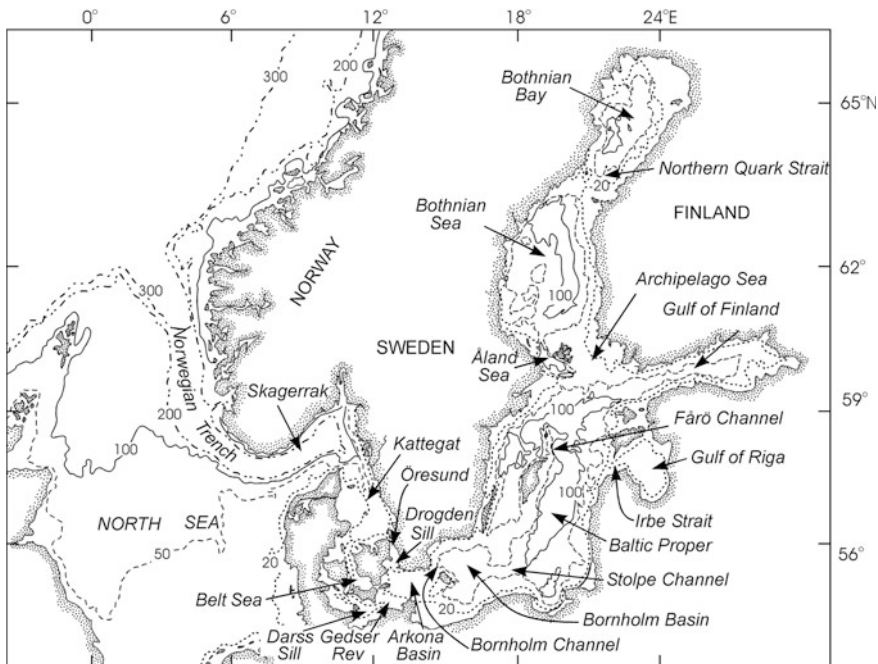


Fig. 2.1 The Baltic Sea–North Sea region with depth contours indicated (from Omstedt et al. 2004)

rotation frequency, equal to $7.29 \times 10^{-5} \text{ (s}^{-1}\text{)}$, and L , T , and U represent the typical scales of length, time, and speed, respectively. In large-scale flows, these dimensionless numbers are often small, implying that the acceleration terms are small.

The importance of friction can be estimated from the horizontal, $Ek_h = \frac{\mu_r}{\rho\Omega L^2}$, and vertical, $Ek_v = \frac{\mu_r}{\rho\Omega H^2}$, Ekman numbers. Outside boundary layers, the Ekman numbers are small and friction effects can be neglected. The implications of these numbers can be easily understood if we examine the governing geophysical equations (Cushman-Roisin and Becker 2011), considering the x and y dimensions first and then estimating scales under the various terms:

$$\begin{aligned} \frac{\partial}{\partial t} \bar{U} + \bar{U} \frac{\partial \bar{U}}{\partial x} + \bar{V} \frac{\partial \bar{U}}{\partial y} + \bar{W} \frac{\partial \bar{U}}{\partial z} - f \bar{V} = & -\frac{1}{\rho_0} \frac{\partial P_w}{\partial x} + \frac{\partial}{\partial x} \left(\Gamma_h \frac{\partial \bar{U}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\Gamma_h \frac{\partial \bar{U}}{\partial y} \right) + \frac{\partial}{\partial z} \left(\Gamma_z \frac{\partial \bar{U}}{\partial z} \right) \\ \frac{U}{T} + \frac{U^2}{L} + \frac{U^2}{L} + \frac{WU}{L} - \Omega U & \quad \frac{P_w}{\rho_0 L} \quad \frac{\mu_h U}{\rho_0 L^2} \quad \frac{\mu_h U}{\rho_0 L^2} \quad \frac{\mu_v U}{\rho_0 H^2} \end{aligned} \quad (2.7)$$

$$\begin{aligned} \frac{\partial}{\partial t} \bar{V} + \bar{U} \frac{\partial \bar{V}}{\partial x} + \bar{V} \frac{\partial \bar{V}}{\partial y} + \bar{W} \frac{\partial \bar{V}}{\partial z} + f \bar{U} = & -\frac{1}{\rho_0} \frac{\partial P_w}{\partial y} + \frac{\partial}{\partial x} \left(\Gamma_h \frac{\partial \bar{V}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\Gamma_h \frac{\partial \bar{V}}{\partial y} \right) + \frac{\partial}{\partial z} \left(\Gamma_z \frac{\partial \bar{V}}{\partial z} \right) \\ \frac{U}{T} + \frac{U^2}{L} + \frac{U^2}{L} + \frac{WU}{L} - \Omega U & \quad \frac{P_w}{\rho_0 L} \quad \frac{\mu_h U}{\rho_0 L^2} \quad \frac{\mu_h U}{\rho_0 L^2} \quad \frac{\mu_v U}{\rho_0 H^2} \end{aligned} \quad (2.8)$$

where $f = 2\Omega \sin \varphi$ is the Coriolis parameter and φ the latitude.

By dividing the estimated sizes beneath the equation by ΩU , we can easily identify the various dimensionless numbers given above. For example, if $Ro_t \ll 1$, the transient term can be neglected relative to the rotation part.

The incompressible approximation for water leads to the implication of divergence-free motion (represented by the continuity equation, Eq. 2.9). That is, if the density fluctuations are small relative to the density itself, any imbalance in horizontal motion will be reflected in the vertical motion:

$$\frac{\partial \bar{U}}{\partial x} + \frac{\partial \bar{V}}{\partial y} + \frac{\partial \bar{W}}{\partial z} = 0 \quad (2.9)$$

By scaling the continuity equation, we learn that $\frac{U}{L}$ must be of the same order as $\frac{W}{H}$, and as the horizontal dimension is often much larger than the depth (i.e., $\frac{H}{L} \ll 1$), the vertical velocity must be much less than the horizontal velocity (i.e., $\frac{W}{U} \ll 1$). In addition, as geophysical flows often have time scales larger than the rotation scale (i.e., $T \geq \frac{1}{\Omega}$ and $\frac{U}{L} \leq \Omega$), we can simplify the vertical equation by analyzing the various terms one by one. The equation for the vertical velocity component with estimated scales reads:

$$\frac{\partial}{\partial t} \bar{W} + \bar{U} \frac{\partial \bar{W}}{\partial x} + \bar{V} \frac{\partial \bar{W}}{\partial y} + \bar{W} \frac{\partial \bar{W}}{\partial z} - f_* \bar{U} = -\frac{1}{\rho_0} \frac{\partial P_w}{\partial z} - \frac{g\rho}{\rho_0} + \frac{\partial}{\partial x} \left(\Gamma_h \frac{\partial \bar{W}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\Gamma_h \frac{\partial \bar{W}}{\partial y} \right) + \frac{\partial}{\partial z} \left(\Gamma_z \frac{\partial \bar{W}}{\partial z} \right)$$

$$\frac{W}{T} \quad \frac{UW}{L} \quad \frac{UW}{L} \quad \frac{W^2}{H} \quad \Omega U \quad \frac{P_w}{\rho_0 H} \quad \frac{g\Delta\rho}{\rho_0} \quad \frac{\mu_h W}{\rho_0 L^2} \quad \frac{\mu_h W}{\rho_0 L^2} \quad \frac{\mu_v W}{\rho_0 H^2}$$

where $f_* = 2\Omega \cos \varphi$ is the reciprocal Coriolis parameter.

From the scaling conditions given, we conclude that the first term in this equation can be neglected. The next three terms are also much less than ΩU and could therefore be neglected. If we now compare the fifth term with the first pressure term on the left side of the equation, one finds that $\frac{\rho_0 \Omega H U}{P} \sim \frac{H}{L}$, so even the fifth term can be neglected. Finally, we realize that the last three terms are small and can be neglected. The scaling thus illustrates that in geophysical flows we may often only need to consider hydrostatic balance or:

$$0 = -\frac{1}{\rho_0} \frac{\partial P_w}{\partial z} - \frac{g\rho}{\rho_0} \quad (2.10)$$

Other important simplifications are that large-scale circulation is often in geostrophic and hydrostatic balance, implying a balance in the horizontal dimensions between Earth's rotation and the horizontal pressure gradients and in the vertical dimension between gravity and the vertical pressure gradient. In this case, the dimensionless numbers Ro_t , Ro , Ek_h , and Ek_v are all much less than one.

At the surface and bottom boundary layers (i.e., $Ek_v \sim 1$), one can find analytical solutions to the boundary equations. For the surface boundary layer, the net mass transport is perpendicular to the wind direction and to the right in the Northern Hemisphere. The analytical solution for the mass transport in the bottom boundary layer indicates transport to the left of the geostrophic flow direction.

If Ro_t , Ro , and $Ek_v \ll 1$, several important analytical aspects of the fluid flow can be derived. For example, the flow is geostrophic. The Taylor–Proudman theorem states that the horizontal velocity field has no vertical shear and that the flow cannot proceed across changes in bottom topography; instead, all motions follow depth contours. Bathymetric charts then give us important information about circulation, information that has implications for how to divide the water body into dynamic regions.

For time-dependent frictionless geostrophic flow, one can derive useful expressions for the vorticity dynamics; with Ro and $Ek_v \ll 1$ and $Ro_t \leq 1$, the change in relative vorticity needs to be conserved, as follows:

$$\frac{d}{dt} \left(\frac{f + \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}}{H} \right) = 0 \quad (2.11)$$

For a change in the bottom topography, this equation means that the fluid flow will change its vorticity.

In stratified fluids, the Froude number, $Fr = \frac{U}{NH}$, where N is the stratification frequency or the Brunt–Väisälä frequency (i.e., $N^2 = -\frac{g}{\rho_0} \frac{d\rho}{dz}$), also yields useful information. The rule is that if $Fr \leq 1$, then the stratification effects are important. The length scale at which stratification and rotation become equally important is called the internal Rossby radius of deformation, i.e., $L_{Ro} = \frac{NH}{\Omega}$.

When fluids are stratified (or non-stratified), we can simplify the equations by assuming wave solutions. Assuming a wave solution with no space limitation, i.e., $W = W_0 e^{i(Lx + my + nz - \omega t)}$ (note that $e^{i\phi} = \cos \phi + i \sin \phi$), one can demonstrate that the internal wave frequency is limited by the Coriolis frequency and the stratification frequency (i.e., $f < \omega < N$). Such waves, called inertia–gravity waves, are oscillating in nature and typical of open sea conditions. The wave number and frequency are denoted by $n = \frac{2\pi}{L_x}$, $m = \frac{2\pi}{L_y}$, $n = \frac{2\pi}{L_z}$, and $\omega = \frac{2\pi}{T}$, respectively.

If we instead assume a wave solution (i.e., $w = w_0 e^{i(my + nz - \omega t) - Lx}$) with one coast along $x = 0$ and a normal velocity of zero at the coast, the results indicate that the waves have no lower frequency limit (i.e., $\omega < N$). These waves, called coastal-trapped waves, move along the coast with the coast to the right of the wave propagation and often at a low frequency. In semi-enclosed basins, one may thus expect to have coastal-trapped waves along coastlines without lower wave frequency limits; in the open parts of such basins, one may encounter oscillating currents bounded by the stratification and Coriolis frequencies.

Analytical solutions have important implications for any model design. From analytical solutions, we learn the basic processes involved, and solutions can be used to test the accuracy of numerical modeling. However, we must eventually face the fact that we must solve equations using numerical methods. Numerical models are based on grids in which one can solve equations at a limited resolution. Dynamic features larger than the grid domain must then be prescribed, while features smaller than the grid size must be parameterized. If we were to construct a numerical model of, for example, coastal upwelling (with typical spatial and time scales of $L = 10$ km, $H = 100$ m, and $T = 10$ days), we would need a domain at least 10 times larger than the upwelling in both space and time. To resolve the motions, we often need a grid size less than 1/10 of the typical spatial and time scales we wish to resolve. Given these considerations, we need to prescribe motion on a horizontal scale of 100 km and a vertical scale of 1000 m, as well as computer time of 100 days. Processes such as geostrophic eddies, thermohaline circulation, tides, and climate effects (Fig. 2.2) must therefore be prescribed. The resolved motion includes front dynamics, but we should be aware that errors in numerical models could be of several types, such as discretization, iteration, and rounding errors. Numerical tests, conservation checks, and validation using observations are therefore needed. Parameterized motions are those at grid scales less than the resolved one, in this case, at a horizontal scale of 1 km, a vertical scale of 10 m, and a time interval of 1 day. All motions at these spatial and time scales must therefore be parameterized, which in our example includes turbulence, surface waves, Langmuir circulation, swell, shipping effects, and breaking waves.

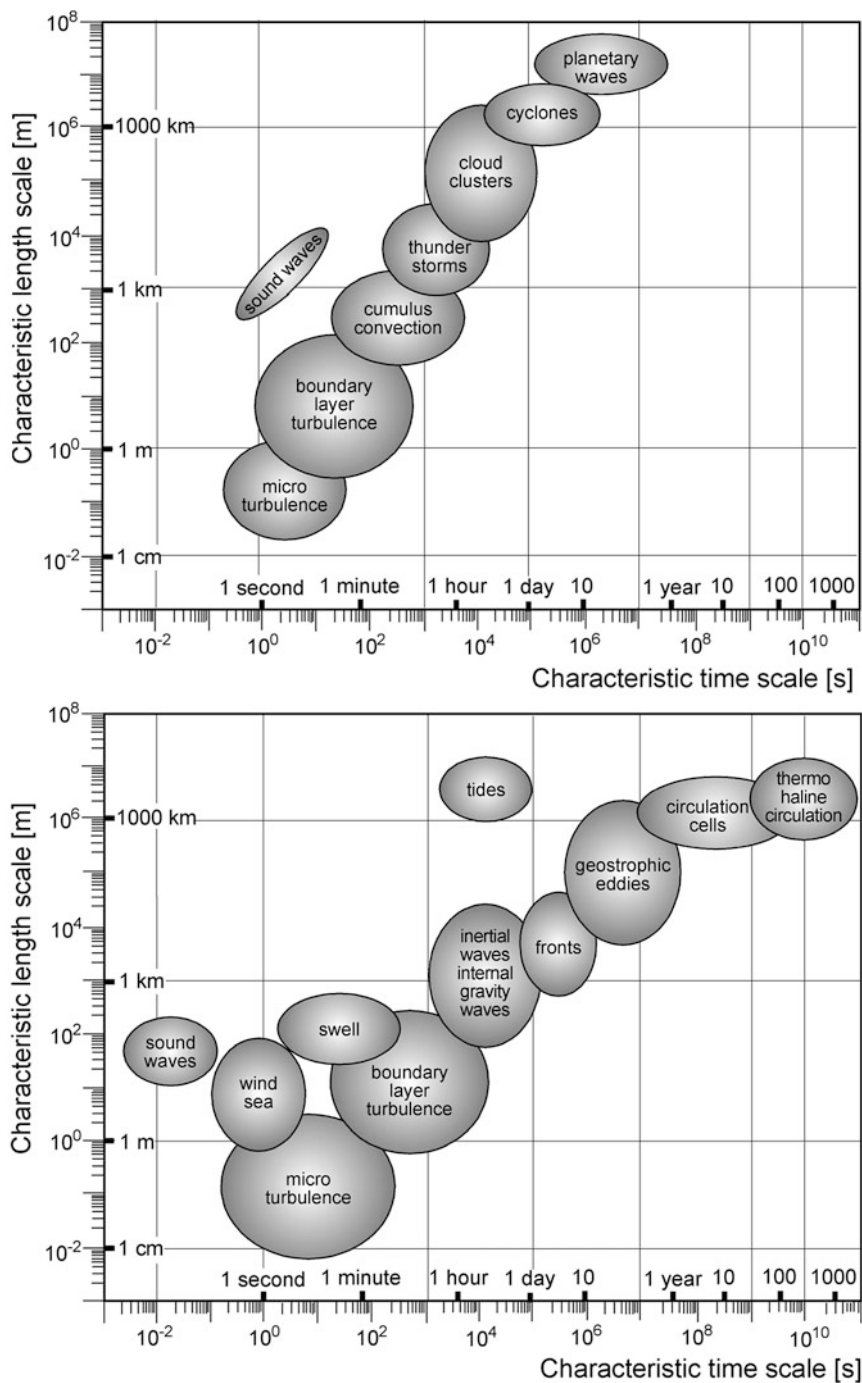


Fig. 2.2 Spatial and temporal scales of some atmospheric and ocean processes (courtesy of Hans von Storch)

Exercise 2.3

Some oceanographers imagine studying Earth's rotation by sitting in a bathtub and letting the water drain while they are passing over the Equator. Would Earth's rotation significantly affect the water flow when emptying a bathtub? Assume a horizontal scale of 1 m, a drainage rate in the order of 0.01 m s^{-1} , a motion time scale of 1000 s, and an ambient rotation rate of 7.3×10^{-5} .

Exercise 2.4

A 60-m-deep surface layer with a salinity of approximately 7 characterizes the central Baltic Sea. Below the halocline, salinity is approximately 10. Using a value of 8×10^{-4} for the coefficient of salinity expansion, calculate the stratification or Brunt–Väisälä frequency. What is the horizontal scale at which rotation and stratification play comparable roles? *Hint:* Use the equation of state, i.e., $\rho = \rho_0(1 + \alpha_2 S)$, and assume that the density change takes place over 60 m.

Exercise 2.5

Examine vorticity dynamics by assuming that the outflow from the Baltic Sea into the Kattegat conserves potential vorticity. What happens to the flow when the outflow enters the much deeper Skagerrak? Demonstrate how the relative vorticity might change.

2.4 Water Masses and Water Pools

Water mass and tracer analysis are the two basic methods for determining ocean circulation (e.g., Aken 2007). Based on temperature (T) and salinity (S) measurements and on plotting these variables in a T–S diagram, we can obtain important information about coastal sea circulation and mixing processes. Figure 2.3 depicts the T–S structure of some sub-basins of the Baltic Sea. Starting from the deep Northwestern Gotland Basin, the T–S structure indicates three different water masses: (1) the surface layer, with large temperature and small salinity variations, (2) the halocline layer, with quite small temperature and salinity variations, and (3) the deep water layer, with small temperature but larger salinity variations. In the other sub-basins, the surface water is diluted by river runoff and the deep water masses are influenced by inflow from surrounding basins and the presence of sills.

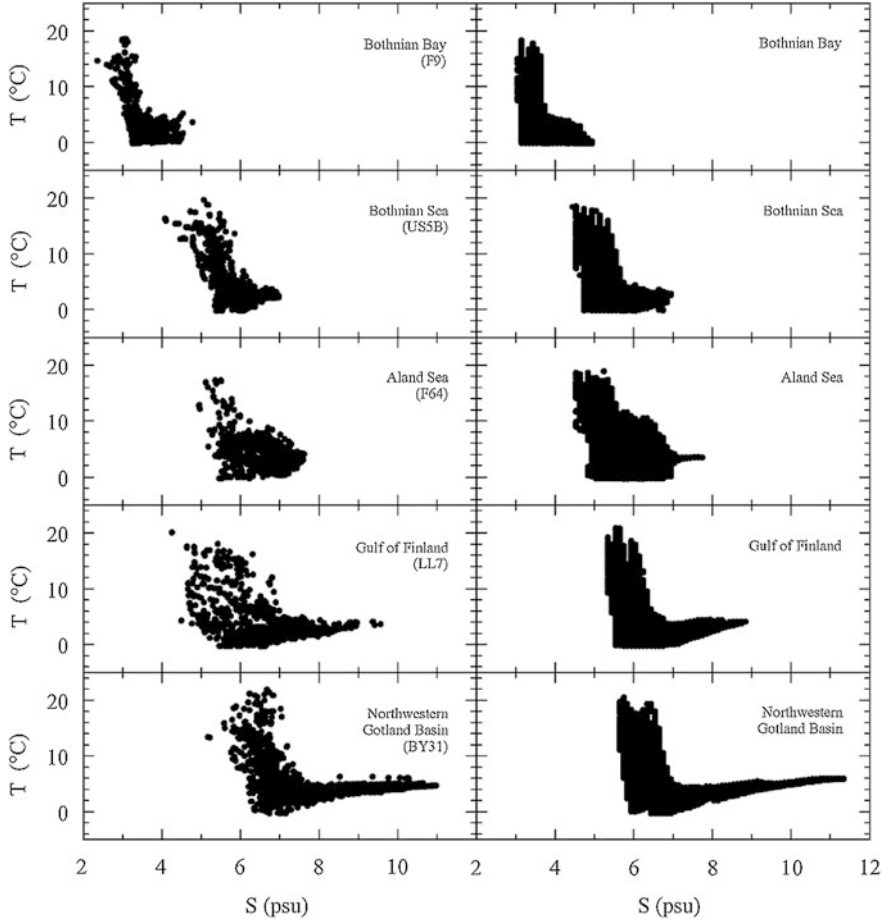


Fig. 2.3 Observed long-term T–S structure in the Northern Baltic Sea (from Omstedt and Axell 2003)

Water masses often form pools of water under specific dynamic control mechanisms; for example, we can speak of the Arctic Ocean surface water pool or the Bornholm Basin deep water pool. Analyzing the dynamics underlying the filling and emptying of such pools can often lead to new insights and interesting model simplifications (Stigebrandt 2001).

When water leaves a water mass, it may form surface brackish layers, as in the Kattegat, or dense bottom layers, as in the Baltic Proper (Fig. 2.4). These surface or bottom water masses are filled with currents transporting dense or light water and influenced by mixing processes. Sometimes the dynamics are very transient, with the formation of pulses and eddies indicating transient dynamics and strong mixing (Piechura and Beszczynska-Möller 2004).

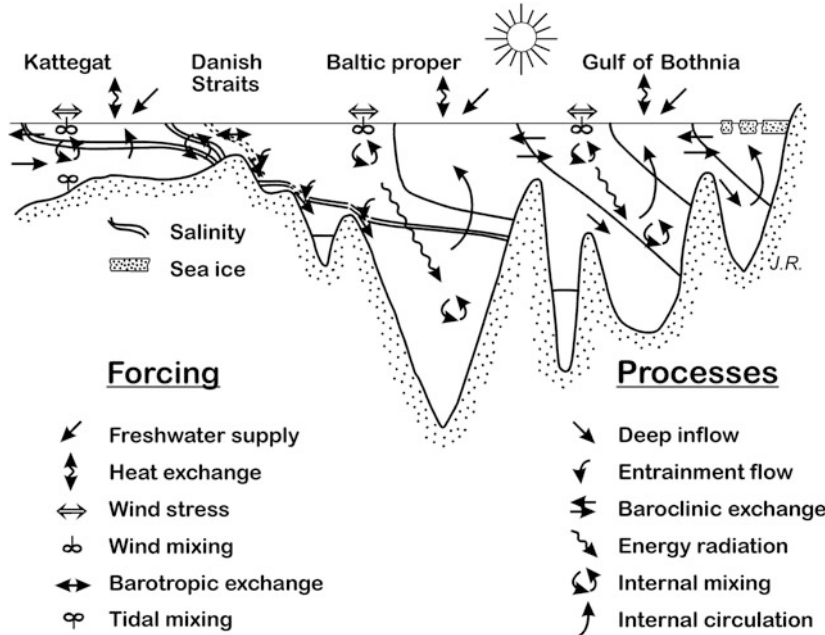


Fig. 2.4 Schematic of processes and forcing mechanisms in the Baltic Sea (redrawn from Winsor et al. 2001, 2003)

The level at which water is drawn when passing over a sill depends on the dynamics. When the water is drawn from a layer in a stratified fluid, several processes may take place, such as fluid acceleration, wave generation, and turbulence. Selected withdrawal dynamics may therefore need to be considered when examining outflow levels. One such example is the Sound, for which Mattsson (1996) estimated that the water flowing into the Baltic Sea comes from levels below the Drogden Sill depth.

2.5 Strait Flows

Inshore–offshore water exchange and exchange between sub-basins are often controlled by geometric constrictions such as sills and straits (Fig. 2.5). Straits are often narrow, shallow regions, and a basic understanding of strait flow dynamics (including mixing) is needed to facilitate modeling. In this context, we generally speak of barotropic and baroclinic flows being associated with sea level changes and density changes, respectively.

The flow dynamics through the Baltic Sea entrance area are transient, with volume flows changing from 0 to $100,000 \text{ m}^3 \text{ s}^{-1}$ in both directions, so direct

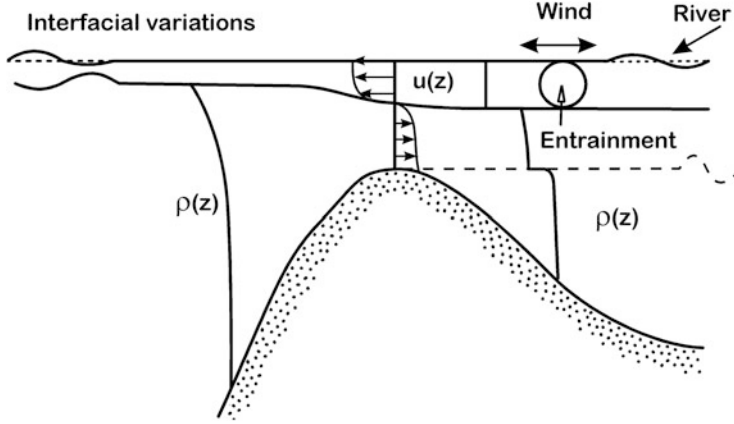
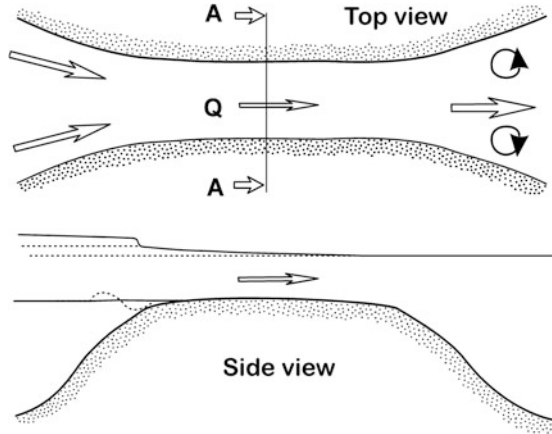


Fig. 2.5 Exchange in an inshore-offshore region with a sill (redrawn from Green 2004)

Fig. 2.6 Barotropic exchange through a shallow channel (redrawn from Green 2004)



measurements are quite difficult to make. As these strait flows are driven mainly by the differences in sea level across the entrance area, inflows and outflows have so far mainly been calculated from sea level observations or from models (Fig. 2.6). This complex exchange can be modeled according to the volume conservation principle and a simple barotropic strait model (Stigebrandt 2001):

$$A_s \frac{dz_s}{dt} = Q_b + (P - E)A_s + Q_r \quad (2.12)$$

$$Q_b^2 = \frac{1}{c_s} \Delta z$$

where A_s is the surface area of the Baltic Sea, z_s the water level of the Baltic Sea, Δz the sea level difference between the two basins, Q_b the barotropic inflows and

outflows through the Baltic Sea entrance area, c_s a strait-specific constant, P and E the precipitation and evaporation rates, and Q_r river runoff.

Freshwater exchange from land often results in surface layers that mix with the surrounding coastal waters and denser, deeper waters that flow inshore. In straits wider than the internal Rossby radius of deformation, we often assume a geostrophic baroclinic flow formulated as follows (Stigebrandt 2001):

$$Q_g = \frac{g(\frac{\rho_2 - \rho_1}{\rho_0})H_1^2}{2f} \quad (2.13)$$

where H_1 represents a mixed surface layer of density ρ_1 overlying a deeper layer of density ρ_2 .

In narrow straits, the concept of baroclinic controls can be used. If freshwater runoff from land causes the surface layer to be thicker inshore than offshore, then its thickness must be adjusted as it approaches the offshore basin. A two-layer flow can then become hydraulically controlled, establishing a well-defined condition for the properties of the flow at the mouth (Stommel and Farmer 1953). Of special interest is the concept of maximum baroclinic transport capability applied in the Northern Kvarf Strait, connecting Bothnian Bay to the Bothnian Sea, as suggested by Stigebrandt (2001) and later validated through model simulations by Omstedt and Axell (2003) and studied in the field by Green et al. (2006).

Exercise 2.6

Calculate mean sea level variation in the Baltic Sea by examining the barotropic strait model given in Eq. 2.12. Assume that river runoff and net precipitation are constant and equal $15,000$ and $1000 \text{ m}^3 \text{ s}^{-1}$, respectively. In addition, assume that the surface area is $3.9 \times 10^5 \text{ km}^2$ and that the strait-specific constant, c_s , is typically $0.3 \times 10^{-5} (\text{s}^2 \text{ m}^{-5})$. Use sea level data from the Kattegat to force the model and compare the mean sea level with sea level variation in Stockholm (for the data needed, see Appendix C).

2.6 Turbulence

All flows become unstable above a certain Reynolds number (i.e., $\text{Re} = \frac{UL}{\nu}$). Flow is laminar at low Reynolds numbers but turbulent at high numbers. Turbulent flow is chaotic and random, including a whole spectrum of eddies of various sizes. From the energy-containing part of this spectrum, the kinetic energy of eddies is transformed into smaller eddies, through the inertial sub-range and into the dissipative range in which the kinetic energy is dissipated into heat. The energy to feed the turbulence comes from the shear currents or from convection, and the size of these energy-containing eddies is often determined by flow geometry. In our

mathematical formulation, Eq. 2.3, we represent the turbulent processes using effective eddy momentum diffusion as follows:

$$\Gamma_{\rho u} = \frac{\mu}{\rho} + \frac{\mu_T}{\rho} \quad (2.14)$$

where μ and $\frac{\mu}{\rho}$ represent the dynamic and kinematic viscosities, respectively, and μ_T is the turbulent dynamic viscosity. The unit of kinematic viscosity is square meters per second, which incorporates the velocity dimension times length. Solving eddy viscosity therefore requires two equations for turbulent quantities. Here we take one equation for the turbulent velocity scale and another for the turbulent length scale. Two-equation turbulent models describe processes that both generate and damp these two scales. Typical processes that generate turbulence are current shear (due to winds and tides), buoyancy (due to the cooling of surface water above the temperature of maximum density), evaporation, ice formation, and the breaking of surface or internal waves. Damping effects include stable stratification, precipitation or melting ice, interfaces (the air–water, ice–water, and water–bottom interfaces), and dissipation.

2.7 Water and Salt Balances

The water and heat cycles are the heart of the climate system. It is impossible to understand the expected processes of change in the climate system without understanding these cycles and the interconnections between them. For example, we cannot understand changes in ocean salinity and sea level if we do not understand changes in the water balance. Likewise, we cannot understand changes in sea ice cover and water temperature if we do not understand changes in the heat balance. Estimating the various components of the water and heat cycles is therefore crucial (Fig. 2.7) and is the main objective of research programs such as BALTEX (the Baltic Sea Experiment) and HYMEX (the HYdrological cycle in the Mediterranean EXperiment).

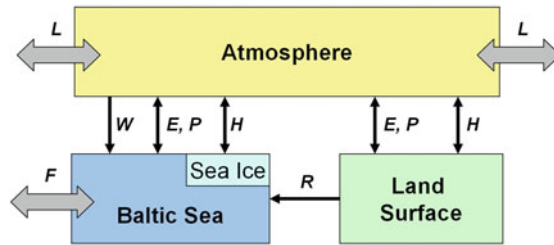


Fig. 2.7 The BALTEX box during Phase I (1993–2002) when water and heat balances were included: L denotes lateral exchange with the atmosphere outside the region, W wind stress, E evaporation rate, P precipitation rate, H heat and energy fluxes, R river runoff, and F inflows and outflows through the entrance area (*courtesy* of Marcus Reckermann)

Starting from the volume conservation principle, we can formulate the water balance equation as follows:

$$A_s \frac{dz_s}{dt} = Q_{in} - Q_{out} + (P - E)A_s + Q_r + Q_g \quad (2.15)$$

where A_s is the surface area of the sea, z_s the water level of the sea, Q_{in} and Q_{out} the inflows and outflows through the entrance area, P and E the precipitation and evaporation rates (the difference is called net precipitation), Q_r river runoff, and Q_g groundwater inflow. The left term in Eq. 2.15 is the change in water storage, which can be important for short-term estimations of the water balance. This term also includes volume changes due to thermal expansion and salt contraction. The major water balance components of the Baltic Sea are inflows and outflows at the entrance area, river runoff, and net precipitation (Omstedt et al. 2004) (Fig. 2.8).

If we know the water balance, we can calculate the salt balance, which, based on the conservation principles, reads as follows:

$$\frac{dV_0 S}{dt} = S_{in} Q_{in} - S Q_{out} - S((P - E)A_s + Q_r) \quad (2.16)$$

where V_0 and S are the water volume and salinity of the semi-enclosed basin and S_{in} is the salinity of the inflowing water. Salinity is thus closely linked to the water balance and forms the basis of many budget calculations. A useful integral property is the mean salinity (vertically and horizontally integrated) or freshwater content. The annual mean salinity of the Baltic Sea has varied around 7.6 over the last century, with a variability of approximately ± 0.5 (Winsor et al. 2001, 2003).

2.8 Heat Balance

From conservation principles, we can formulate the heat balance equation for a semi-enclosed sea area, according to Omstedt and Rutgersson (2000), as follows:

$$\frac{dH}{dt} = (F_i - F_o - F_{loss})A_s \quad (2.17)$$

where $H = \int \int \rho c_p T dz dA$ is the total heat content of the sea, F_i and F_o the heat fluxes associated with in- and outflows, and F_{loss} the total heat loss to the atmosphere (note that the fluxes are positive when going from the water to the atmosphere). F_{loss} is formulated as follows:

$$F_{loss} = (1 - A_i)(F_n + F_s^w) + A_i(F_w^i + F_s^i) - F_{ice} + F_r + F_g \quad (2.18)$$

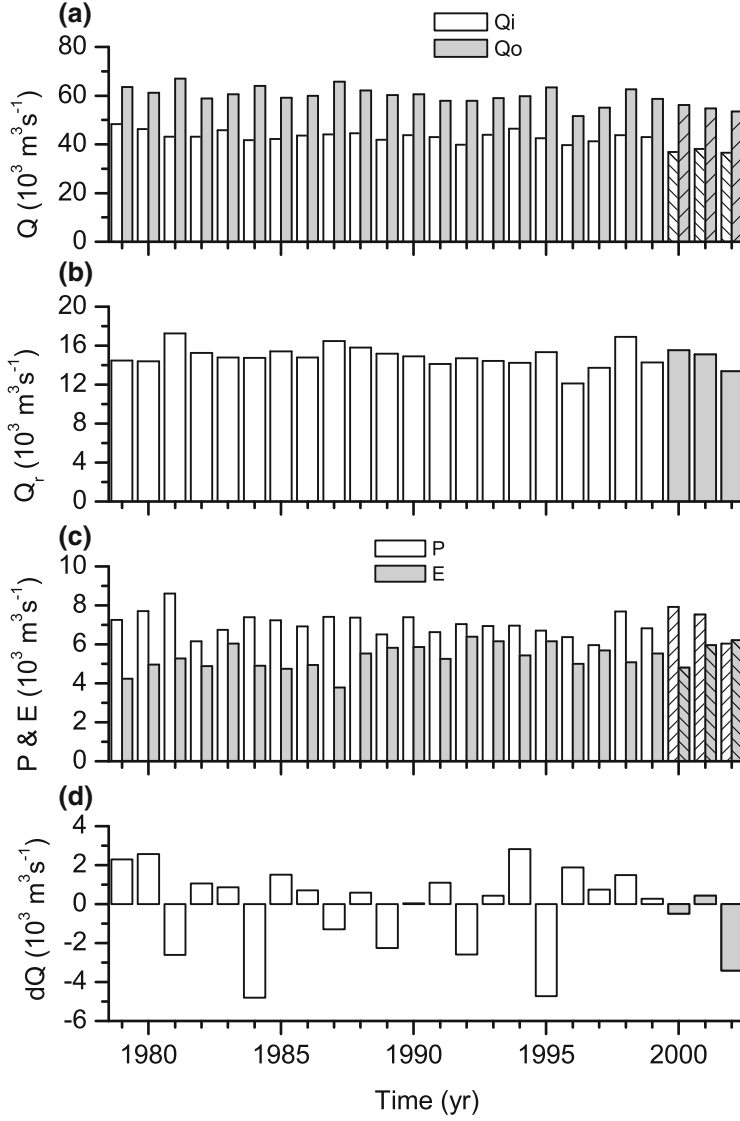


Fig. 2.8 Baltic Sea (excluding the Kattegat and the Belt Sea) annual mean **a** inflows and outflows, **b** river runoff, **c** net precipitation, and **d** net volume change (Omstedt and Nohr 2004)

where

$$F_n = F_h + F_e + F_l + F_{prec} \quad (2.19)$$

The various terms of the equations are denoted as follows: A_i is ice concentration, F_h sensible heat flux, F_e latent heat flux, F_l net long-wave radiation, F_{prec} heat

fluxes associated with precipitation in the form of rain and snow, F_s^w sun radiation to the open water surface, F_w^i heat flux from water to ice, F_s^i sun radiation through the ice, and F_r and F_g heat flows associated with river runoff and groundwater flow, respectively. In terms of long-term mean, the Baltic Sea is almost in thermodynamic balance with the atmosphere and only 1 W m^{-2} is needed to compensate for the heat loss due to a net outflow from the Baltic Sea (Omstedt and Nohr 2004). Dominant fluxes, in terms of annual means, are sensible heat, latent heat, net long-wave radiation, solar radiation to the open water, and the heat flux between water and ice (Fig. 2.9).

A useful integral property is the heat content of water (or the vertically and horizontally integrated water temperature), in particular, the change of heat content directly connected to heat fluxes (Eq. 2.17). Surface temperatures are often a poor measure of heat content and may even be unable to measure the accumulation of heat in the water, as other fluxes may compensate for the net effect (Omstedt and Nohr 2004). Pielke (2003) therefore suggested that stored heat content and its change over time should be the focus of international climate-monitoring programs, as change in heat content is a much better measure of climate change than is surface temperature.

Exercise 2.7

Consider the Baltic Sea and its surface area of $3.9 \times 10^5 \text{ km}^2$. Assume that the volume and heat content of the Baltic Sea do not change over time and that the exchange through the entrance area occurs via a two-layer flow. Assume that inflowing water and river water both equal $15,000 \text{ m}^3 \text{ s}^{-1}$ and that the inflowing and outflowing water temperatures both equal 8°C . If the river runoff temperature is 1°C colder/warmer than the Baltic Sea surface temperature, what is the estimated heat exchange with the atmosphere?

2.9 Nutrient Balance and Primary Production

Water and energy balances form the basis of the physical state of lakes or coastal seas. The balances of nutrients—including nitrogen, phosphorus, and silicon, together with the carbon balance—form the basis for the chemical and biological state of the water body under study. These balances actively interact and involve many interesting modeling considerations, such as knowledge of human activity (Fig. 2.10). In addition, reliable datasets incorporating marine and atmospheric data, biochemical variables, atmospheric deposition, and river load are needed in modeling efforts (Omstedt et al. 2014).

Biogeochemical modeling often starts from simple stoichiometric considerations and links the water, salinity, and nutrient budgets in which, for example, carbon (C), nitrogen (N), and phosphorus (P) are considered. The first step is often to establish

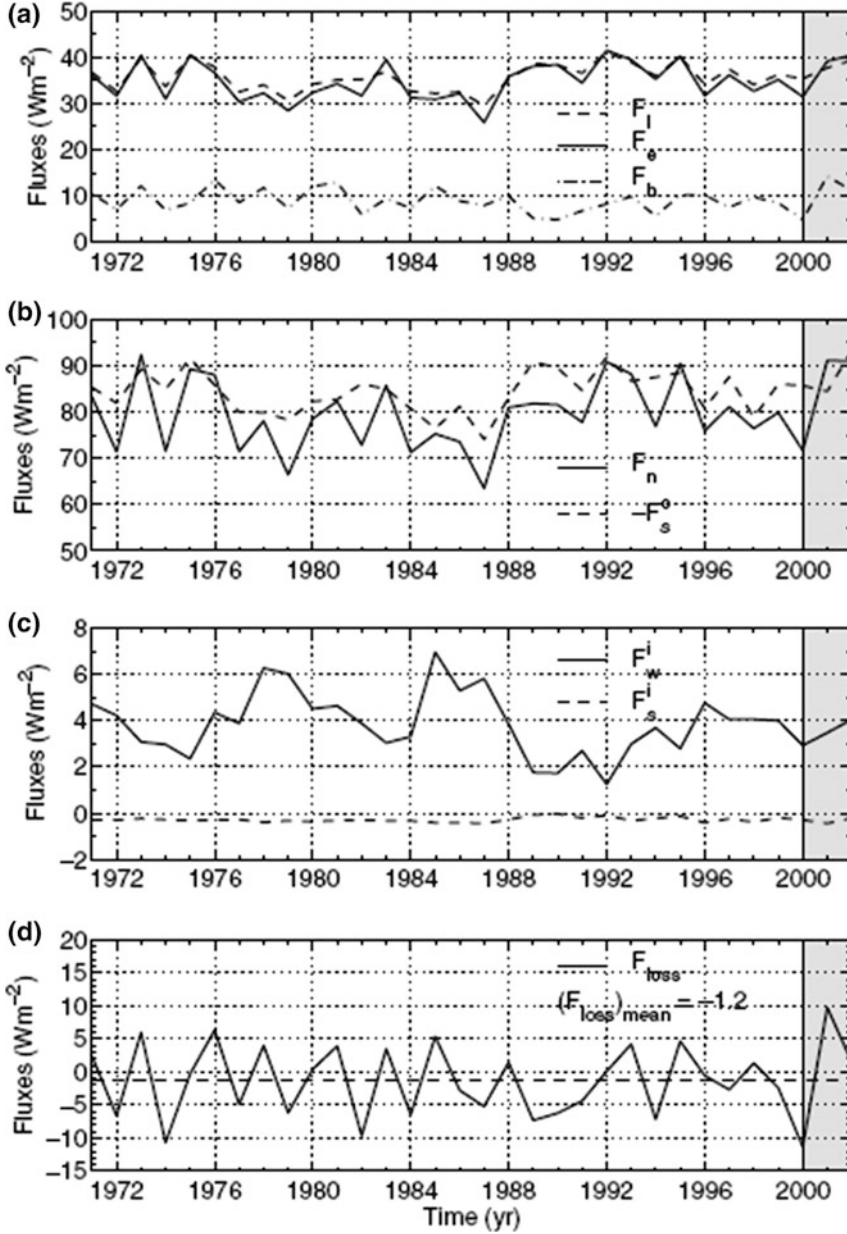


Fig. 2.9 Annual means of: sensible heat (F_h), latent heat (F_e), net long-wave radiation (F_l), net heat flux ($F_n = F_h + F_e + F_l$), sun radiation to the open water surface (F_s^o), sun radiation through ice (F_s^i), heat flow from water to ice (F_w^i), and net Baltic Sea heat loss, i.e., $F_{\text{loss}} = (1 - A_i)(F_s^o + F_h + F_e + F_l) + A_i(F_s^i + F_w^i)$, where A_i is the ice concentration (Omstedt and Nohr 2004)

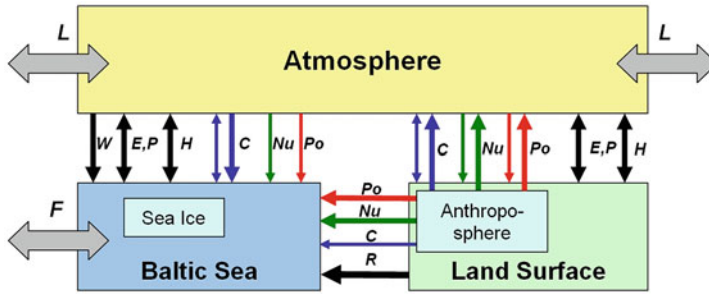


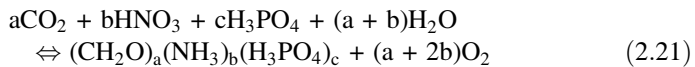
Fig. 2.10 The BALTEX box during Phase II (2013–2012) when water, heat balances, nutrients, and carbon cycles were included: L denotes lateral exchange with the atmosphere outside the region, W wind stress, E evaporation rate, P precipitation rate, H heat and energy fluxes, R river runoff, F inflows and outflows through the entrance area, C carbon fluxes, Nu nutrient fluxes, and Po pollutant fluxes (courtesy of Marcus Reckermann)

the steady state of the water and salt balances; based on these two equations, the various water flows can then be calculated. Salinity is a conservative property suitable for establishing the exchange of water between the system and adjacent seas. Total alkalinity can sometimes also be used as a conservative property and supplies additional information on the total alkalinity of river water entering the system and water circulation (Hjalmarsson et al. 2008). The next step is to establish budgets for non-conservative materials (Gordon et al. 1996), as follows:

$$\frac{dV_0^\phi C}{dt} = {}^\phi C_{in} Q_{in} - {}^\phi C Q_{out} + \Delta^\phi C \quad (2.20)$$

where ${}^\phi C$ is the concentration of the non-conservative material (mol kg^{-1}) and $\Delta^\phi C$ the sum of all non-conservative fluxes. The results of Eq. 2.20 let us quantify the net non-conservative reaction of the material in the system.

Stoichiometric relationships and chemical reaction formulas are used to describe the proportions of the compounds involved in chemical reactions. The expression to the left of the arrow represents the elements before the reaction. The products of the reaction are described by the expression to the right of the arrow. For example, the carbon cycle can be related to organic material due to primary production, as follows:



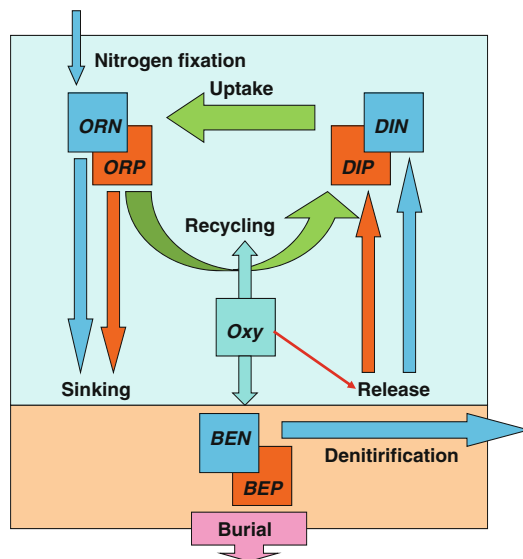
where $(a:b:c) = (106:16:1)$ are the standard Redfield values. The expression illustrates how carbon dioxide, water, and nutrients may form plankton in its simplest form (i.e., $(\text{CH}_2)_a(\text{NH}_3)_b(\text{H}_3\text{PO}_4)_c$) and oxygen. For a discussion of the

Redfield concept and marine biogeochemistry, the reader should consult Sarmiento and Gruber (2006).

The presence of many strongly nonlinear relationships and the frequent absence of clear basic laws make marine ecosystems difficult to describe (i.e., measure and model) quantitatively. Included in these variables are many kinds of marine organisms that transform nutrients and inorganic carbon into organic material. The primary production of the world's oceans is based on diatoms, dinoflagellates, coccolithophorids, silicoflagellates, and blue-green and other bacteria, all of which need to be understood quantitatively. Regional and species distributions vary considerably, as do light and nutrient limitations.

Figure 2.11 depicts some processes related to modeling Baltic Sea eutrophication, based on the approach of Savchuk and Wulff (2007). The importance of the oxygen dynamics for the phosphate dynamics is depicted in Fig. 2.12. At low oxygen concentrations, the sediments start to leak phosphate. From observations, Conley et al. (2002) demonstrated that the annual change in dissolved inorganic phosphate was positively correlated with the area of sea bottom depleted of oxygen. The figure also indicates that, in oxic water, the sediments may bind phosphate. However, the nitrate dynamics are also influenced and, if the water volume becomes anoxic, this will result in a nitrogen sink, as the NO_3 in the water is reduced to N_2 . Oxygen depletion may also increase the total alkalinity (Edman and Omstedt 2013). The modeling of eutrophication must therefore consider how low oxygen concentrations influence the nutrient and carbon dynamics.

Fig. 2.11 Processes related to modeling the eutrophication in the Baltic Sea based on the modeling approach of Savchuk and Wulff (2007). The organic nitrogen and phosphate parts are denoted by ORN and ORP, the inorganic parts by DIN and DIP, and the benthic parts by BEN and BEP, respectively



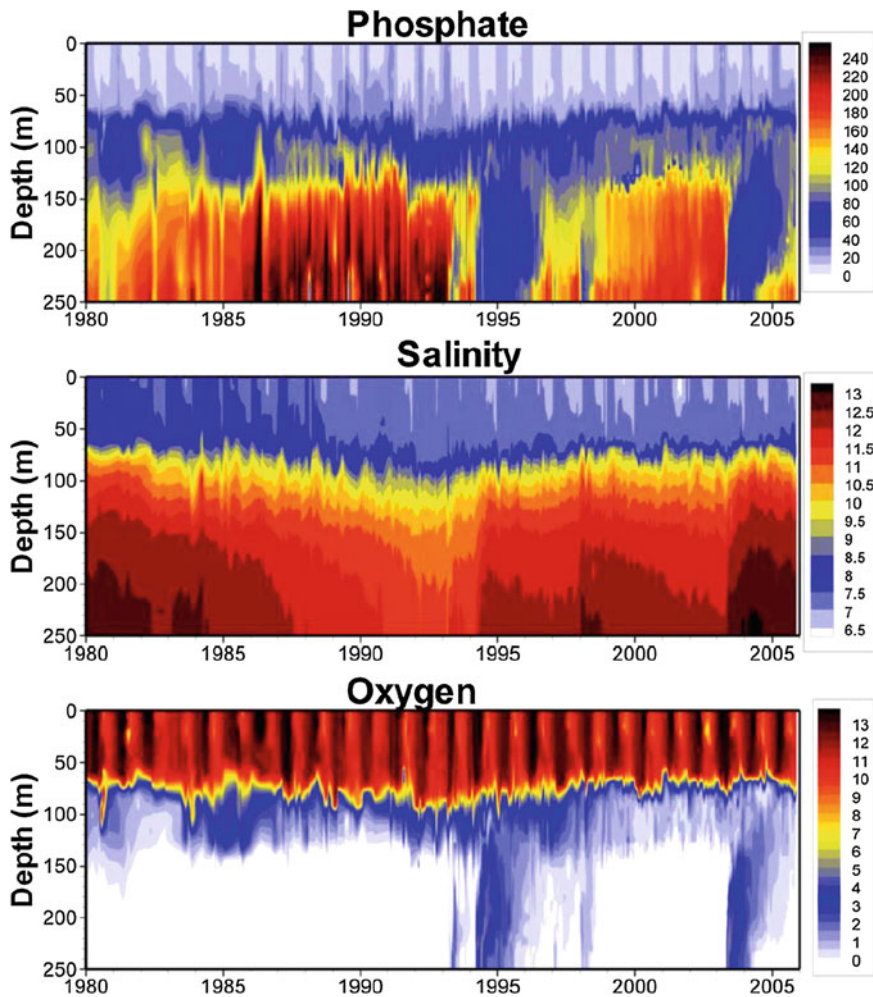


Fig. 2.12 Measurements from the Gotland Deep in the Baltic Sea (Stigebrandt and Gustafsson 2007)

Exercise 2.8

Use P and N observations from the Baltic Sea and plot the surface properties of PO_4 and NO_3 for the last five years. Discuss the dynamics and discover what is controlling the primary production (for the data needed, see Appendix C).

2.10 Acid–Base (pH) Balances

Water and heat balances are at the heart of climate research and the carbon cycle lies at the heart of biogeochemical modeling. Great efforts are being made to link climate and biogeochemical models and to include them in Earth System Modeling. For anthropogenic climate change due to increased carbon dioxide, the connection between eutrophication and climate change, including ocean acidification, relies on the correct modeling of the CO_2 – O_2 marine system, which requires a full understanding of the carbon system.

The seawater acid–base (pH) balance is characterized by the total alkalinity, A_T , which is defined as the excess of proton acceptors (anions of weak bases) over proton donors (strong acids). The major proton acceptors in seawater are hydrogen carbonate (HCO_3^- , 95.5 %), carbonate (CO_3^{2-} , 4 %), and borate (B(OH)_4^- , 0.5 %) ions, whereas hydrogen ions (H^+) and hydrogen sulfate ions (HSO_4^-) act as proton donors. A_T is influenced by, for example, limestone dissolution through the increase in carbonate ions. This addition of carbonate ions strengthens the buffering capacity (A_T) and increases the pH. The addition of weak acids such as CO_2 lowers the pH. However, the effect is small because most of the added CO_2 reacts with CO_3^{2-} and only a small fraction of the CO_2 adds hydrogen ions to the system by dissociation of H_2CO_3 to HCO_3^- and H^+ . The problem is that when enough carbonic acid has been added, not enough carbonate and bicarbonate ions remain, so pH decreases more rapidly with further additions (Fig. 2.13). This has been observed in freshwater systems exposed to airborne sulfuric acid, though freshwater systems generally have much weaker buffering capacities than do marine systems. In addition, primary production (i.e., carbon dioxide uptake) and mineralization (i.e., carbon dioxide release) greatly influence the acid–base (pH) balance with daily, seasonal, interannual, and regional variations being observed in coastal regions (Wesslander et al. 2010; Wotton et al. 2008).

To estimate the pH of the water, one must consider the concentration of dissolved inorganic carbon, which includes the following components: carbon dioxide (CO_2), carbonic acid ($\text{H}_2\text{CO}_{3\text{aq}}$), bicarbonate ($\text{HCO}_{3\text{aq}}^-$), and carbonate ($\text{CO}_{3\text{aq}}^{2-}$), the sum of which is referred to as the total dissolved inorganic carbon (C_T). The state variables for the dissolved inorganic carbon system are C_T and A_T , defined as follows:

$$\begin{aligned} C_T &= [\text{CO}_2] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}] \\ A_T &\approx [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}] + [\text{B(OH)}_4^-] + [\text{OH}^-] - [\text{H}^+] \end{aligned} \quad (2.22)$$

If the total inorganic carbon and total alkalinity are known, we can derive a simplified relationship for the pH by ignoring the presence of boric acid. This simplified analytical relationship is as follows:

$$[\text{H}^+] \approx \frac{K_2(2C_T - A_T)}{A_T - C_T} \quad (2.23)$$

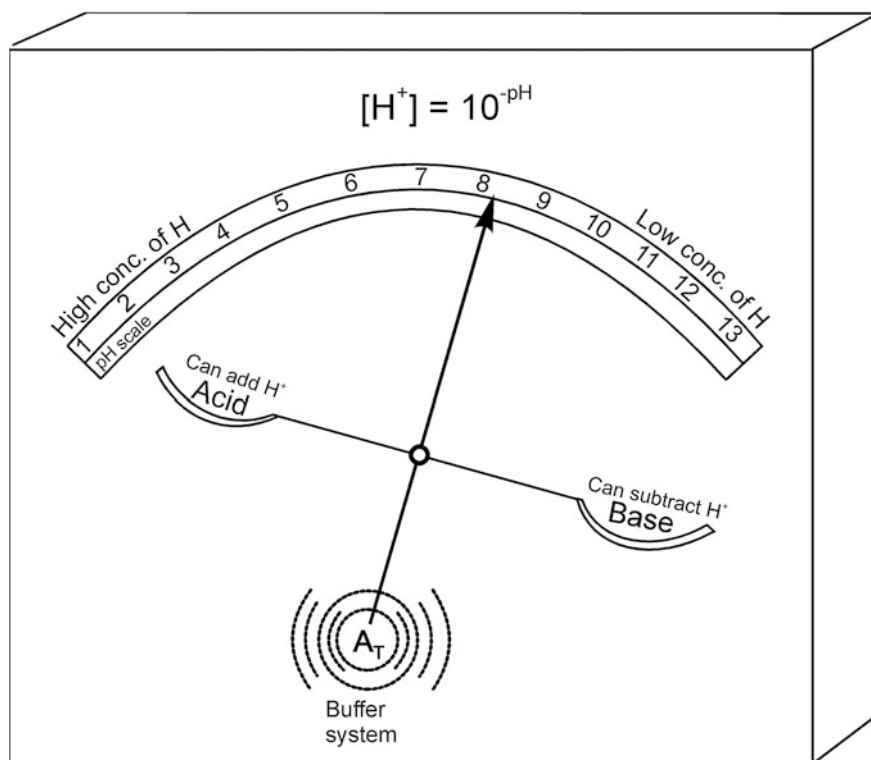


Fig. 2.13 The acid–base (pH) balance depicted as a balance between the concentrations of proton donors and proton acceptors, damped by the buffer system represented by the total alkalinity (A_T)

where K_2 is a solubility constant, which is temperature and salinity dependent. An important aspect of this simplified equation is that the pH ($[H^+] = 10^{-pH}$) of water is dependent on the difference between total alkalinity and total inorganic carbon. For seawater with a temperature of 10 °C and salinity of 10, K_2 is 2.94×10^{-10} . With a total alkalinity of $1600 \mu\text{mol kg}^{-1}$ and total inorganic carbon of $1500 \mu\text{mol kg}^{-1}$, the pH equals 8.4. The mathematical behavior of Eq. 2.23 indicates that pH is mainly dependent on the difference between total alkalinity and total inorganic carbon. In addition, small differences between these parameters can drastically reduce the pH.

Using calculations based on the marine carbon system, the sensitivity of Baltic Sea surface pH was examined by Omstedt et al. (2010). Figure 2.14 illustrates how pH varies with changes in A_T and in the partial pressure of CO_2 in water, $p\text{CO}_2^W$. pH sensitivity to change in A_T varies throughout the Baltic Sea, the greatest impact being anticipated in the north where A_T is the lowest. The buffer effect is illustrated by pH curves becoming less steep at high A_T values.

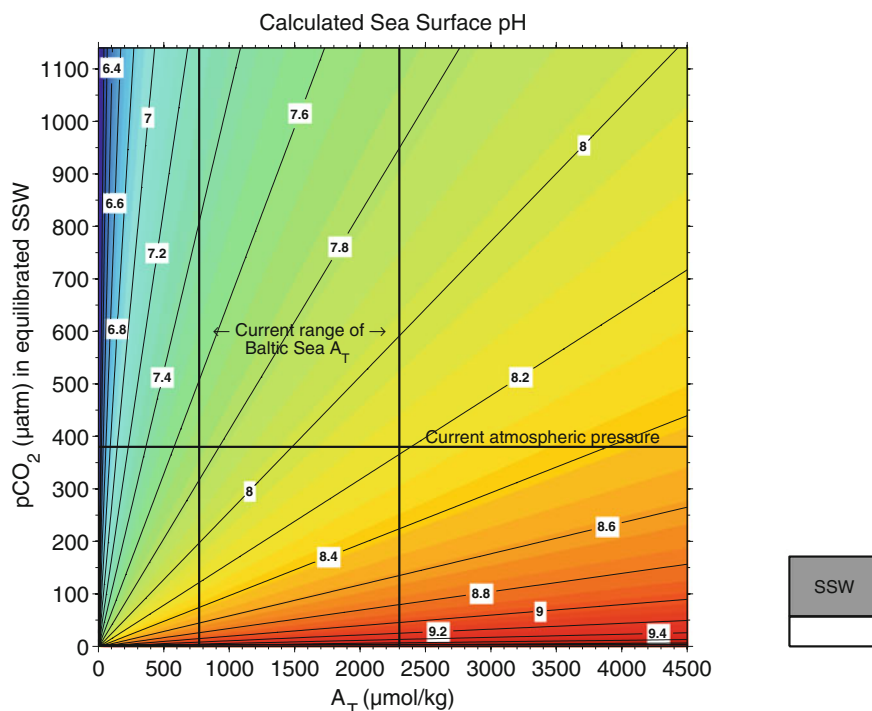


Fig. 2.14 Change of pH with variation in water carbon dioxide pressure and A_T . Salinity is kept at 8 and temperature at 0 °C throughout the calculations. Indicator lines show the current status of the area with regard to A_T (Hjalmarsson et al. 2008) and atmospheric carbon dioxide pressure (Omstedt et al. 2010)

Exercise 2.9

Use pH observations from the Baltic Sea and plot the surface values. Discuss what controls seasonal and long-term variations of the acid–base balance (for the data needed, see Appendix C).

2.11 Some Comments Related to Climate Change

Observations of climate parameters constitute the basis of our understanding of climate and climate change. However, what we subjectively see from data depends greatly on what we are expecting and on our scientific training (BACC I Author Team 2008). Figure 2.15 presents a time series as both original and normalized data. In the figure, the same time series is interpreted in three ways. The original

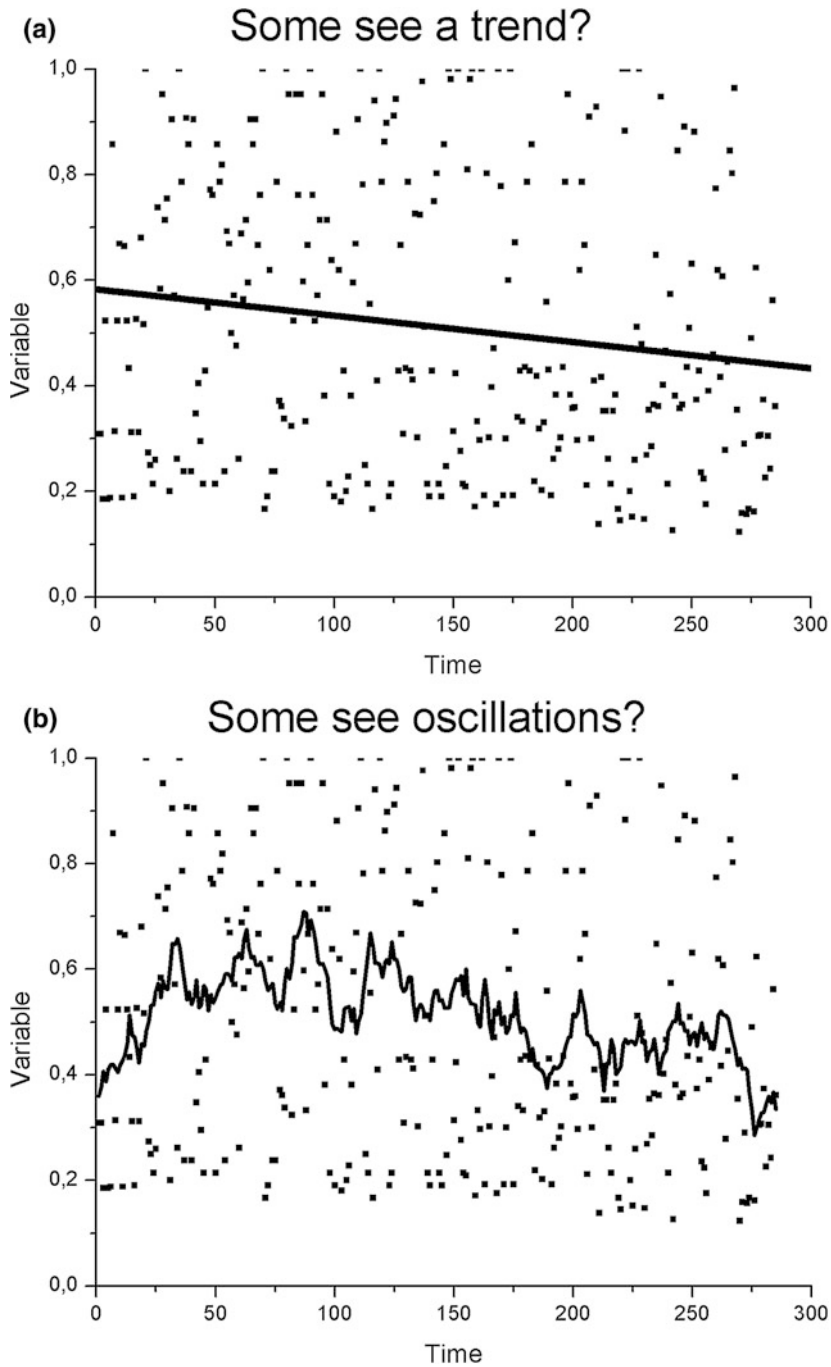
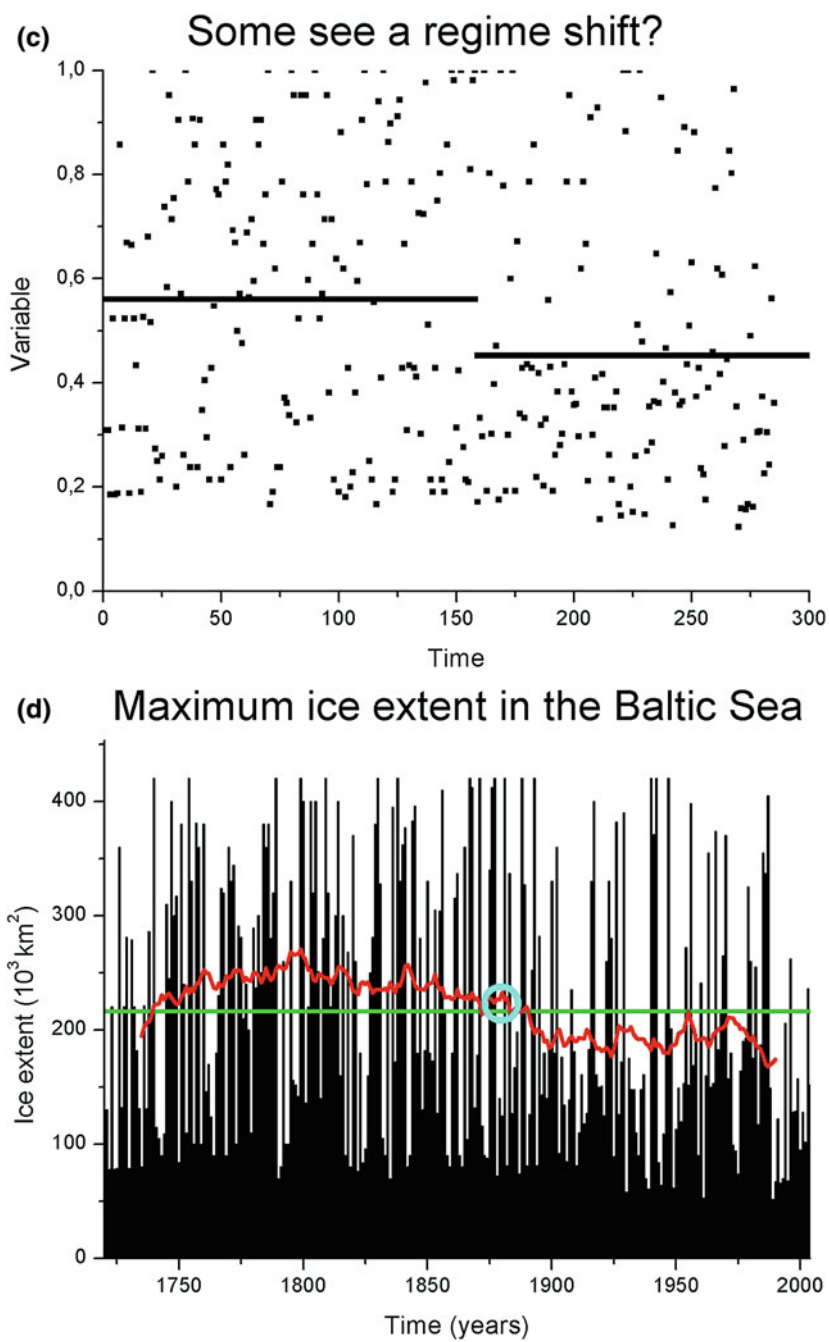


Fig. 2.15 Climate change can be detected in terms of **a** trends, **b** oscillations, and **c** jumps or regime shifts. In this figure, the same data are used and normalized (**a-c**); the original dataset is presented in (**d**) (redrawn from BACC Author Team 2008)

**Fig. 2.15** (continued)

dataset captures how the maximum annual ice extent in the Baltic Sea (MIB) has varied from 1720 up to now. Obviously, the data are very noisy. When scientists communicate their results to decision makers or the public, the message is often given political implications. Presenting a trend, the message implies that society must do something. When we present oscillation, the message is that society does not need to take any action. Finally, when we are speaking about regime shifts, the message to the public is that we have found something that will not be able to change back to earlier conditions. How scientists address their messages to the public sends strong implicit signals regarding various potential actions; thus, there is a need for scientists to improve the way in which they convey science in lay terms (e.g., Pielke 2007). Often we need to define terms such as climate variability, climate change, and anthropogenic climate change, as the public understands these terms differently from the way scientists do (BACC I Author Team 2008).

The MIB time series is a key data source for understanding the Baltic Sea climate, and has recently been extended back to AD 1500 (Hansson and Omstedt 2008). While MIB is only an estimated number for each year, it represents an integrated measure of the severity of winter ice. Proxy information about climate change based on MIB, the North Atlantic Oscillation (NAO) index, and Stockholm sea level data gives us a preliminary simplified view of climate change and variability. Using such proxy information together with other long-term datasets, Eriksson et al. (2007) have characterized the last 500 years of climate in northern Europe, illustrating event-type conceptual behavior in 15 different periods.

Climate and weather display stochastic and periodic behavior; to understand such behavior, we must look far into the past. A rule of thumb is that we must understand at least ten 100-year periods in the past, taking us back to AD 1000, to understand the coming 100 years. From the record of the past 500 years, it seems as though the change from the warm 1730s to the extremely cold year 1740 spans the climate variability range in the Baltic Sea region for the last 500 years (Jones and Briffa 2006). Another interesting consideration comes from studies of the Greenland ice and Baltic Sea sediment records: Kotov and Harff (2006) identify periodicities of 900, 500, and 400 years, which they claim reflect climate processes relevant at least to the North Atlantic region. We must therefore consider both the periodic (non-stochastic) and stochastic aspects of our time series when drawing conclusions regarding climate change.

Figure 2.16 presents air temperature in Stockholm, indicating a warmer climate since the end of the nineteenth century, which has been identified as marking the end of the “Little Ice Age” in the Baltic Sea (Omstedt and Chen 2001).

Baltic Sea mean salinity exhibits large variations over long time scales (Winsor et al. 2001, 2003), so systematic increases or decreases may be discerned when limited segments of many years each are considered (see Fig. 2.17). However, the time series extending across the entire twentieth century indicates that thinking in terms of long-term trends makes little sense. Hansson et al. (2010) have reconstructed river runoff to the Baltic Sea, finding no long-term trends over the past 500 years; their study also demonstrated that total river runoff decreased with warmer air temperatures. Omstedt and Hansson (2006a, b) analyzed the Baltic Sea

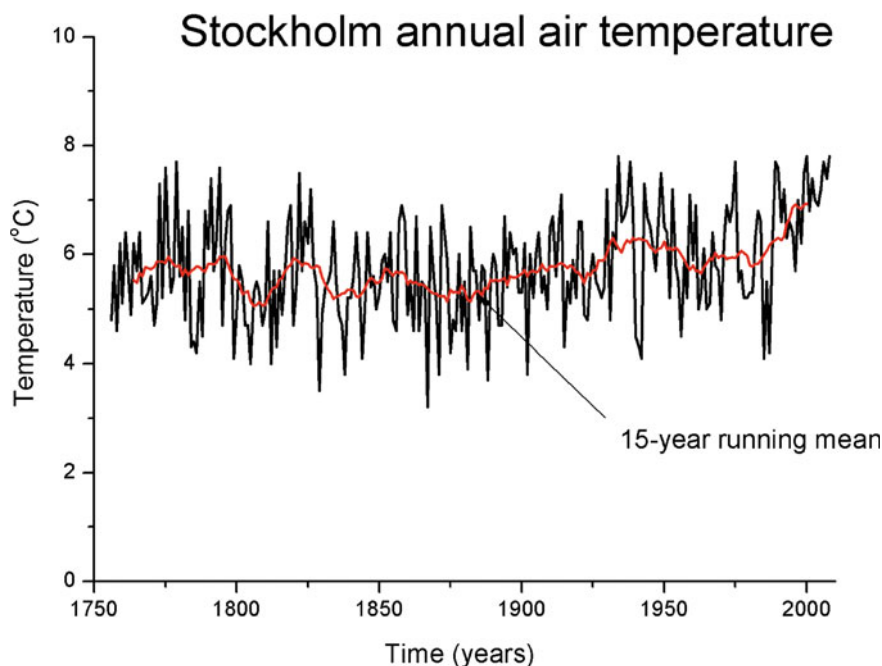


Fig. 2.16 Stockholm annual air temperature; data adjusted for the city effect according to Moberg et al. (2002)

climate system memory and demonstrated that two important time scales should be considered: one is associated with the water balance and has an e -folding time of approximately 30 years; the other is associated with the heat balance and has an e -folding time of approximately one year. These studies indicate that, on an annual time scale, the Baltic Sea is almost in thermal balance with the atmosphere and that atmospheric changes will rapidly influence the sea ice extent and water temperatures. We therefore expect that the effects of climate warming will first be observed in parameters related to the heat balance, such as water temperature and sea ice extent, and that it will take longer before they can be detected in parameters related to the water balance, such as salinity. This is also in good agreement with BACC assessments (BACC I Author Team 2008; BACC II Author Team 2015), which conclude that the present warming is limited to temperature and directly related variables, such as ice and snow conditions, and that water cycle changes will likely become evident later.

Two very basic climate change questions spring to mind. First, how sensitive is the climate to changes in solar irradiance, atmospheric aerosols, greenhouse gases (including water vapor), and other climate forcing? Second, how large is natural climate variability? Based on proxy data, climate forcing factors have been estimated by several authors (e.g., Crowley 2000). Figure 2.18 depicts the change in radiative forcing over the last 1000 years. The forcing sensitivity between surface

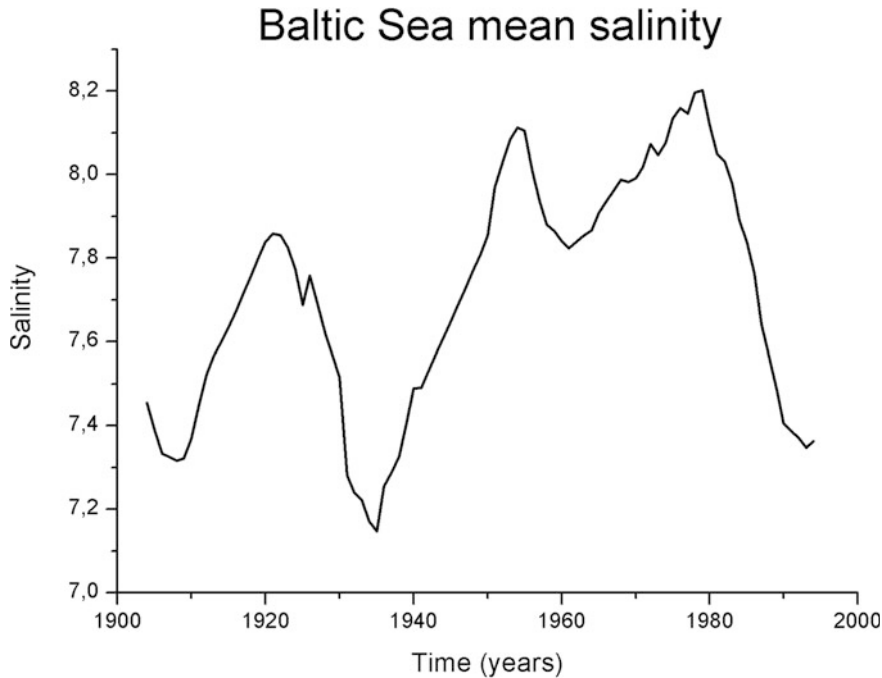


Fig. 2.17 Baltic Sea mean salinity, annually, horizontally, and vertically averaged (from Winsor 2001, 2003)

temperature (ΔT_s) and radiation changes (ΔF) can be studied by considering the net irradiance at the tropopause, which is believed to be a good indicator of global mean temperature (globally and annual averaged), defined as follows (IPCC 2001, p. 354):

$$\Delta T_s / \Delta F = \lambda \quad (2.24)$$

where λ is a constant that, in simplified radiation–convection models, is typically approximately 0.5 K/Wm^{-2} . A change in the radiation balance of 2 Wm^{-2} thus corresponds to a global temperature increase of 1 K.

Currently, the dominant concern is the effects of increased human-produced greenhouse gases and aerosols (IPCC 2013). The increased CO_2 content of the atmosphere is believed to increase the greenhouse effect and therefore warming (the first climate change aspect). How strongly atmospheric CO_2 will increase the atmospheric temperature is under discussion; however, it is generally believed that atmospheric CO_2 first influences the global radiation balance, which may in turn influence the temperature and cloud dynamics. The second aspect of increasing atmospheric CO_2 is its effects on the acid–base (pH) balance in the ocean. Increased atmospheric CO_2 will reduce the ocean pH, which may threaten marine ecosystems.

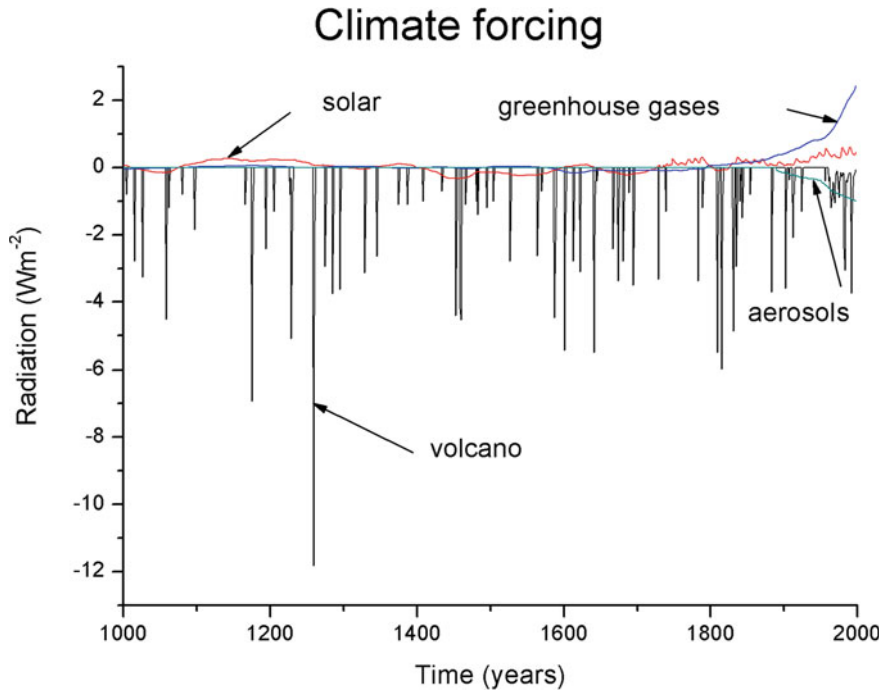


Fig. 2.18 Climate forcing estimates (redrawn from Crowley 2000)

A third aspect of increasing atmospheric CO_2 is its effects on land vegetation and increased biological production on land where water is not a limiting factor. Increased biological production may lead to increasing levels of dissolved organic carbon entering the coastal seas via rivers. Through mineralization processes, this could become an additional source of acid material in the seas.

Few long time series extend to before the eighteenth century, so we should accept that we will probably underestimate variability in both space and time (Storch et al. 2004) by going back only a few hundred years when modeling. Various feedback mechanisms may increase and/or decrease climate change and cause regional differences. Unsurprisingly, it is not easy to attribute present climate change to a single factor, and it is even more difficult to determine the causes of climate change on the regional scale (BACC I Author Team 2008; BACC II Author Team 2015). Determining the various anthropogenic climate effects in combination with natural variability is currently a major research area.

Exercise 2.10

Investigate the climate variability and trend in Stockholm air temperature observations (for the data needed, see Appendix C). What determines the trend? What are the causes of the trend? Can trends tell us anything about the future?

Exercise 2.11

Compare the Stockholm air temperature observations with the long-term variations in sea surface temperatures at Christiansö, near Bornholm Island in the southern Baltic Sea (for the data needed, see Appendix C). Examine the trend of the 15-year running mean data for the period since 1900.

Exercise 2.12

Investigate Stockholm sea level variations relative to climate change (for the data needed, see Appendix C). Assume, as Ekman (2003) does, that the land uplift can be determined from the trend from 1774 to 1864.

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