

# Preface

By systematically building an optimal theory, this monograph develops and explores several approaches to Hardy spaces ( $H^p$  spaces) in the setting of  $d$ -dimensional Ahlfors-regular quasi-metric spaces. The text is broadly divided into two main parts. The first part debuts by revisiting a number of basic analytical tools in quasi-metric space analysis, for which new versions are produced in the nature of best possible. These results, themselves of independent interest, include a sharp Lebesgue differentiation theorem, a maximally smooth approximation to the identity, and a Calderón-Zygmund decomposition for a brand of distributions suitably adapted to our general setting. Such tools are then used to obtain atomic, molecular, and grand maximal function characterizations of  $H^p$  spaces for an optimal range of  $p$ 's. This builds on and extends the work of many authors, ultimately creating a versatile theory of  $H^p$  spaces in the context of Ahlfors-regular quasi-metric spaces for a sharp range of  $p$ 's.

The second part of the monograph establishes very general criteria guaranteeing that a linear operator  $T$  acts continuously from a Hardy space  $H^p$  into some topological vector space  $\mathcal{L}$ , emphasizing the role of the action of the operator  $T$  on  $H^p$ -atoms. Applications include the solvability of the Dirichlet problem for elliptic systems in the upper-half space with boundary data from  $H^p$  spaces. The tools originating in the first part are also used to develop a sharp theory of Besov and Triebel-Lizorkin spaces in Ahlfors-regular quasi-metric spaces.

The monograph is largely self-contained and is intended for an audience of mathematicians, graduate students, and professionals with a mathematical background who are interested in the interplay between analysis and geometry.

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A Sharp Theory

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