

Contents

Part I Preliminaries

1	Issues and Problems in Decision Making Under Uncertainty	3
1.1	Introduction	3
1.1.1	Decision Making as Constrained Optimization Problems	3
1.1.2	Facing Uncertainty	4
1.1.3	The Role of Information in the Presence of Uncertainty	5
1.2	Problem Formulations and Information Structures	7
1.2.1	Stochastic Optimal Control (SOC)	7
1.2.2	Stochastic Programming (SP)	10
1.3	Examples	12
1.3.1	A Basic Example in Static Information	12
1.3.2	The Communication Channel	13
1.3.3	Witsenhausen’s Celebrated Counterexample	16
1.4	Discretization Issues	16
1.4.1	Problems with Static Information Structure (SIS)	17
1.4.2	Working Out an Example	18
1.5	Conclusion	25
2	Open-Loop Control: The Stochastic Gradient Method	27
2.1	Introduction	27
2.2	Open-Loop Optimization Problems	28
2.2.1	Problem Statement	28
2.2.2	Sample Approximation in Stochastic Optimization	30
2.3	Stochastic Gradient Method Overview	31
2.3.1	Stochastic Gradient Algorithm	31
2.3.2	Connection with Stochastic Approximation	34

2.4	Convergence Analysis	39
2.4.1	Auxiliary Problem Principle	40
2.4.2	Stochastic Auxiliary Problem Principle Algorithm	41
2.4.3	Convergence Theorem	42
2.4.4	Conclusions	45
2.5	Efficiency and Averaging.	45
2.5.1	Stochastic Newton Algorithm	45
2.5.2	Stochastic Gradient Algorithm with Averaging	48
2.5.3	Sample Average Approximation	49
2.6	Practical Considerations	50
2.6.1	Stopping Criterion	51
2.6.2	Tuning the Standard Algorithm	51
2.6.3	Robustness of the Averaged Algorithm	54
2.7	Conclusion.	56
2.8	Appendix	57
2.8.1	Robbins-Siegmund Theorem	57
2.8.2	A Technical Lemma	58
2.8.3	Proof of Theorem 2.17.	59

Part II Decision Under Uncertainty and the Role of Information

3	Tools for Information Handling.	65
3.1	Introduction	65
3.2	Basic Facts on Binary Relations and on Lattices	66
3.2.1	Binary Relations	66
3.2.2	Lattices	70
3.3	Partitions and Fields Approach	71
3.3.1	The Lattice of Partitions/Equivalence Relations	71
3.3.2	The Lattice of π -Fields (Partition Fields)	74
3.3.3	The Lattice of σ -Fields.	78
3.4	Mapping Measurability Approach	80
3.4.1	Measurability of Mappings w.r.t. Partitions	80
3.4.2	Measurability of Mappings w.r.t. π -Fields	81
3.4.3	Measurability of Mappings w.r.t. σ -Fields	86
3.5	Conditional Expectation and Optimization	87
3.5.1	Conditional Expectation w.r.t. a Partition	87
3.5.2	Interchanging Minimization and Conditional Expectation.	90
3.5.3	Conditional Expectation as an Optimal Value of a Minimization Problem.	92
3.6	Conclusion.	93

4	Information and Stochastic Optimization Problems	95
4.1	Introduction	95
4.2	The Witsenhausen Counterexample	96
4.2.1	A Simple Linear Quadratic Control Problem	96
4.2.2	Problem Transformation Exploiting Sequentiality	98
4.2.3	The Dual Effect of the Initial Decision	100
4.3	Other Information Patterns	101
4.3.1	Full Noise Observation	101
4.3.2	Classical Information Pattern	102
4.3.3	Markovian Information Pattern	103
4.3.4	Past Control Observation	103
4.3.5	The Witsenhausen Counterexample	104
4.4	State Model and Dynamic Programming (DP)	104
4.4.1	State Model	105
4.4.2	State Feedbacks, Decisions, State and Control Maps	106
4.4.3	Criterion	108
4.4.4	Stochastic Optimization Problem	109
4.4.5	Stochastic Dynamic Programming	109
4.5	Sequential Optimization Problems	111
4.5.1	Sequential Optimal Stochastic Control Problem	112
4.5.2	Optimal Stochastic Control Problem in Standard Form	115
4.5.3	What Is a State?	119
4.5.4	Dynamic Programming Equations	119
4.6	Conclusion	132
5	Optimality Conditions for Stochastic Optimal Control (SOC) Problems	133
5.1	Introduction	133
5.2	SOC Problems, Formulation and Assumptions	134
5.2.1	Dynamics	135
5.2.2	Cost Function	135
5.2.3	Constraints	136
5.2.4	The Stochastic Programming (SP) Version	137
5.3	Optimality Conditions for the SP Formulation	138
5.3.1	Projection on the Feasible Set	138
5.3.2	Stationary Conditions	140
5.4	Optimality Conditions for the SOC Formulation	141
5.4.1	Computation of the Cost Gradient	141
5.4.2	Optimality Conditions with Non-adapted Co-States	144
5.4.3	Optimality Conditions with Adapted Co-States	145

5.5	The Markovian Case	146
5.5.1	Markovian Setting and Assumptions	146
5.5.2	Optimality Conditions with Non-adapted Co-States . . .	147
5.5.3	Optimality Conditions with Adapted Co-States	149
5.5.4	Optimality Conditions from a Functional Point of View	150
5.6	Conclusions	151

Part III Discretization and Numerical Methods

6	Discretization Methodology for Problems with Static Information Structure (SIS).	155
6.1	Quantization.	156
6.1.1	Set-Theoretic Quantization	156
6.1.2	Optimal Quantization in Normed Vector Spaces	157
6.2	A Systematic Approach to Discretization	160
6.2.1	The Problematics of Discretization.	160
6.2.2	The Approach Inspired by Pennanen's Work	161
6.2.3	A Constructive Proposal.	168
6.3	A Handicap of the Scenario Tree Approach	174
6.3.1	How to Sample Noises to Get Scenario Trees.	174
6.3.2	Variance Analysis	175
6.4	Conclusion.	179
7	Numerical Algorithms.	181
7.1	Introduction	181
7.2	A Simple Benchmark Problem	183
7.2.1	Formulation	183
7.2.2	Numerical and Functional Data	186
7.3	Manipulating Functions with a Computer and Implementation in Dynamic Programming (DP)	187
7.3.1	The DP Equation.	187
7.3.2	Discrete Representation of a Function	188
7.3.3	The Discrete DP Equation	190
7.3.4	Application to the Benchmark Problem	192
7.4	Resolution by the Scenario Tree Technique	192
7.4.1	General Considerations.	193
7.4.2	Formulation of the Problem over a Scenario Tree	194
7.4.3	Optimality Conditions and Resolution	196
7.4.4	About Feedback Synthesis	197
7.4.5	Results Obtained for the Benchmark Problem	198

7.5	The Particle Method	200
7.5.1	Algorithm.	201
7.5.2	Results Obtained for the Benchmark Problem and Comments	204
7.6	Conclusion.	206

Part IV Convergence Analysis

8	Convergence Issues in Stochastic Optimization	211
8.1	Introduction	211
8.2	Convergence Notions	213
8.2.1	Epi-Convergence and Mosco Convergence	213
8.2.2	Convergence of Subfields.	217
8.3	Operations on Integrands	219
8.3.1	Multifunctions.	219
8.3.2	Integrands	220
8.3.3	Upper Integral.	222
8.3.4	Conditional Expectation of a Normal Integrand.	223
8.3.5	Interchange of Minimization and Integration.	224
8.4	Application to Open-Loop Optimization Problems.	227
8.5	Application to Closed-Loop Optimization Problems.	228
8.5.1	Main Convergence Theorem.	228
8.5.2	Revisiting a Basic Example in Static Information	231
8.5.3	Discussion About Related Works	233
8.5.4	Revisiting the Example of Sect. 1.4.2.	235
8.5.5	Companion Propositions to Theorem 8.42	245
8.6	Conclusion.	252

Part V Multi-Agent Systems

9	Multi-Agent Decision Problems	255
9.1	Introduction	255
9.2	Witsenhausen Intrinsic Model.	256
9.2.1	The Extensive Space of Decisions and States of Nature	256
9.2.2	Information Fields and Policies	259
9.3	Causality and Solvability for Stochastic Control Systems	262
9.3.1	Solvability and Solvability/Measurability	262
9.3.2	Causality	264
9.3.3	Solvability, Causality and “State”	264

9.4	Four Binary Relations Between Agents	265
9.4.1	The Precedence Relation \mathfrak{P}	265
9.4.2	The Subsystem Relation \mathfrak{S}	267
9.4.3	The Information-Memory Relation \mathfrak{M}	270
9.4.4	The Decision-Memory Relation \mathfrak{D}	272
9.5	A Typology of Stochastic Control Systems	274
9.5.1	A Typology of Systems	274
9.5.2	Examples of Systems with Two Agents	277
9.5.3	Partially Nested and Sequential Systems.	282
9.5.4	Summary Table.	285
9.6	Policy Independence of Conditional Expectations and Dynamic Programming	286
9.6.1	Policy Independence of Conditional Expectations	286
9.6.2	Application to Decomposition by Dynamic Programming	288
9.7	Conclusion.	292
10	Dual Effect for Multi-Agent Stochastic Input-Output Systems. . . .	293
10.1	Introduction	293
10.2	Multi-Agent Stochastic Input-Output Systems (MASIOS). . . .	294
10.2.1	Definition of Multi-Agent Stochastic Input-Output Systems.	294
10.2.2	Control Laws	295
10.2.3	Precedence and Memory-Communication Relations . . .	297
10.2.4	A Typology of MASIOS	298
10.3	No Open-Loop Dual Effect and No Dual Effect Control Laws	300
10.3.1	No Open-Loop Dual Effect (NOLDE)	301
10.3.2	No Dual Effect Control Laws	301
10.3.3	Characterization of No Dual Effect Control Laws	302
10.4	Conclusion.	307
	Appendix A: Basics in Analysis and Optimization.	309
	Appendix B: Basics in Probability	327
	References.	349
	Index	357

Stochastic Multi-Stage Optimization

At the Crossroads between Discrete Time Stochastic
Control and Stochastic Programming

Carpentier, P.; Chancelier, J.-P.; Cohen, G.; De Lara, M.

2015, XVII, 362 p. 45 illus., 14 illus. in color., Hardcover

ISBN: 978-3-319-18137-0