

Chapter 2

Mathematical Modelling as a Strategy for Building-Up Systems of Knowledge in Different Cultural Environments

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Abstract Knowledge is a cumulative succession of strategies developed by humans living in different natural and cultural environments in response to the pulsions of survival and transcendence. The objective of knowledge is to understand, to explain and to cope with selected facts and phenomena of reality, ideally reality as a whole. Mathematical modelling is such a strategy that deals with facts and phenomena. In this chapter, how knowledge is generated (cognition), how it is individually and socially organised (epistemology) and how it is expropriated by power structure, institutionalised and given back to the people who generated it through filters (politics) is discussed. These steps are treated in an integrated and holistic way.

2.1 Introduction

In the text, I use, many times, the terms *artifacts* and *mentifacts*. These words, together with *sociofacts*, were introduced by biologist Julian Huxley (1887–1975) as the bases for a theory of culture (Huxley 1955). These terms have also been used in cultural semiotics. Whereas artifacts are the elements of the material culture, the mentifacts can be understood as the elements of the mental culture. Mentifacts include the symbols and codes of a culture, the signifier and the artifacts are related to the users of signs, the signified. Mental culture can, therefore, be regarded as a set of symbols and codes. This is studied by several authors and I mention Umberto Eco as a good reference. Particularly relevant are folkloristic studies in which models, that is, artifacts, come from mentifacts (Hale 2013).

I base my arguments in a behavioural hierarchy that leads to individual behaviour, which includes learning, the acquisition of knowledge and strategies for action, and to social behaviour, which results from the encounter of an individual

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and another individual. These behaviours, individual and social, generate the context of cultural behaviour, including the processes of cultural transmission and mutual exposure of diverse cultures, are objects of study of the dynamics of cultural encounters. The transfer of knowledge, particularly of technology, is a crucial issue in the analysis of the process of development, fundamental to understanding the process of globalisation.

Preliminarily, I am interested in understanding the process of learning, acquisition of knowledge and strategies for action, that constitute a hierarchy of behaviours.

2.2 The Generation of Knowledge

Initially, it is the individual behaviour, which implicitly includes the processes of learning and, in particular, of the acquisition of language. Following this, we have social behaviour that develops and evolves within the so-called educational process.

Therefore, social behavior becomes more complex and generates a cultural phenomenon. It is vital to understand how arts and techniques, which incorporate artifacts to reality, develop into ideas, such as religion, values, philosophies, ideologies and sciences, as mentifacts, which are also incorporated to reality in a broad sense, that is artifacts + mentifacts + natural facts and phenomena. Once incorporated to reality, artifacts and mentifacts change it. Thus I conceptualize technology as the synthesis of artifacts (instruments) and mentifacts. That is, technology represents a merger of doing with the knowledge, contributing to the ways that man deals with reality and copes with situations and problems. Not only material instruments, such as tools and practices are responsible for action, but also the substratum of mentifacts, mainly religion and ideology. A very pertinent example is the agricultural use of transgenic. In history, the emergence of the gothic is an example. Instruments, both material and intellectual, such as counting, are responsible for ad hoc solutions.

We may understand the construction of knowledge as a three-step process:

1. How are ad hoc practices and solution of problems developed into methods?
2. How are methods developed into theories?
3. How are theories developed into scientific invention?

While methods are essentially a rational and coordinated use of techniques, theories are impregnated modes of explanation and understanding, based on myths, on spirituality and even religions, on science and mathematics and in ideology, which are all mentifacts.

Let's examine in more detail each step and the dynamics of their evolution. We discuss the learning process as something that creates a context which is the interaction of a genetic program and the environment. This is the subject of an important line of research, usually identified as 'nature versus nurture'.

Since the early philosophers this discussion is central. The psychologists have joined the philosophers over the relative importance of the environment, that is, upbringing, experience, and learning ('nurture'), and heredity, that is, genetic inheritance ('nature'), in determining the make-up of human personality and of intelligence as an organism, as related to behaviour and knowledge. The implications of these discussions for eugenics are obvious, in which differences in the capacities of individuals (and hence their behaviour) can be attributed to inherited differences in their genetic make-up.

Moving into these discussions leads to the theme religion versus science. Recent research leads to what has been called the epic of creation versus the epic of evolution. This is an area that gains in importance.¹

I will try to avoid going into this discussion by just assuming life as an observable fact and recognizing that body and mind follow parallel and interconnected paths in this process of interaction of the genetic program with the environment. In the process, reality is recognized and analysed, thus originating intentional actions, concepts of meaning, which are response to will and need. Space, time, causality, imitation, the ludic and other categories play important roles in the process of interaction of the genetic program with the environment.

This is well illustrated by child behaviour. The concept of reality changes step by step, and the child, initially reacting only to instincts of survival (breathing, eating \approx need) incorporate decision-making (\approx will) and go from individual behaviour to social behaviour. The action of a child which initially results purely from their perception of situations and objects in their self-centred universe, changes upon reflection on the consequences of the action. Thus proceeds a modification of the action, considering all the information resulting from the complex of the senses, the emotions and memory combined. This action changes reality by adding facts, both artifacts and mentifacts (i.e., objects, things, ideas, and values), to that reality. Such change of reality by the action of the individual immediately provokes new thinking, new behaviour, interaction with new information already stored and newly acquired information. As a result, new action is initiated, with immediate effect in reality and, as a consequence, the addition of new facts. It is the individual as a maker of the reality by the addition of facts produced by the individual.

Man assumes the role of a creator, generating knowledge (mentifacts) and its *thingification* (in the sense of becoming an artifact). I use the term thingification to emphasize the material aspect of the action. Many authors, for example Marx, have used the word as well as *reification*. Both knowledge and their thingification are in the form of arts, sounds, objects, things in general, ideas, images, fantasy, concepts, theories, values and interpretations, in order to cope with, to understand, and to explain reality. They are added to the existing reality, enlarging and remaking it, to best fit the individual needs and will. These remarks are appropriate to discuss knowledge of different cultural systems, as it occurred in the conquests after

¹ See, for example, *ZYGON: Journal of Religion & Science*, Volume 44 Number 1, March 2009, which is entirely devoted to the theme.

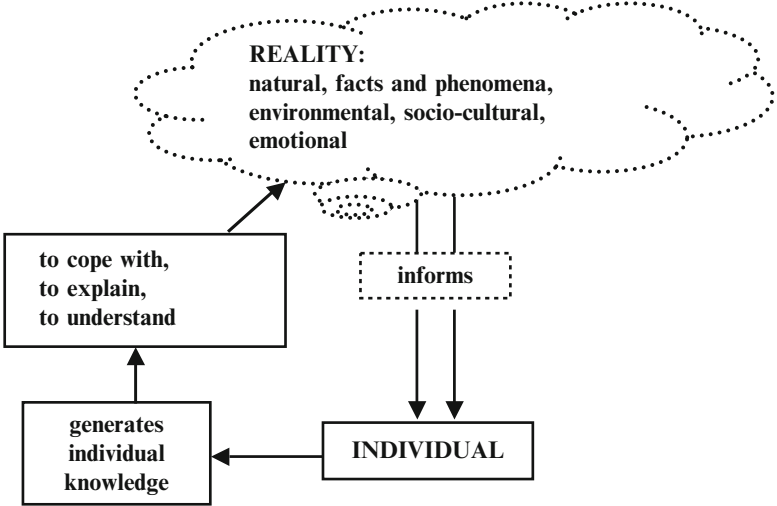


Fig. 2.1 The cycle of individual generated knowledge (After D’Ambrosio 2009, p. 90)

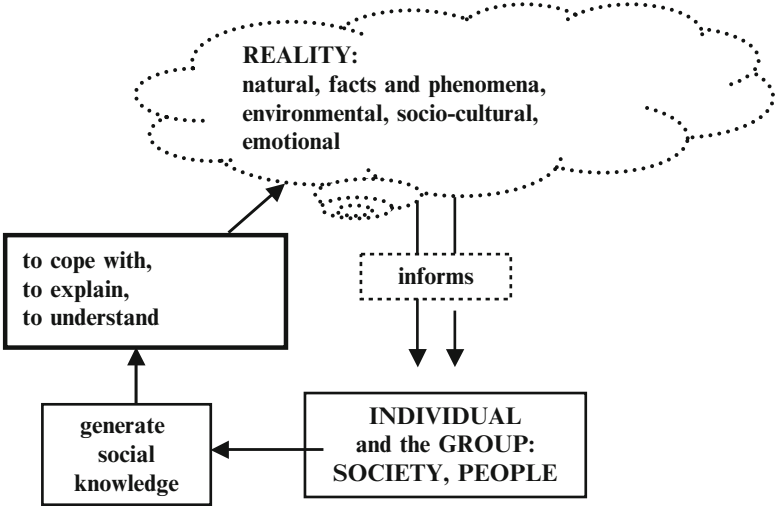


Fig. 2.2 The cycle of social knowledge

Columbus and the emergence of other knowledge systems as a result of the dynamics of cultural encounters (D’Ambrosio 1992). The knowledge cycle introduced in this chapter, served as the basis for elaborating Figs. 2.1, 2.2, and 2.3 used here.

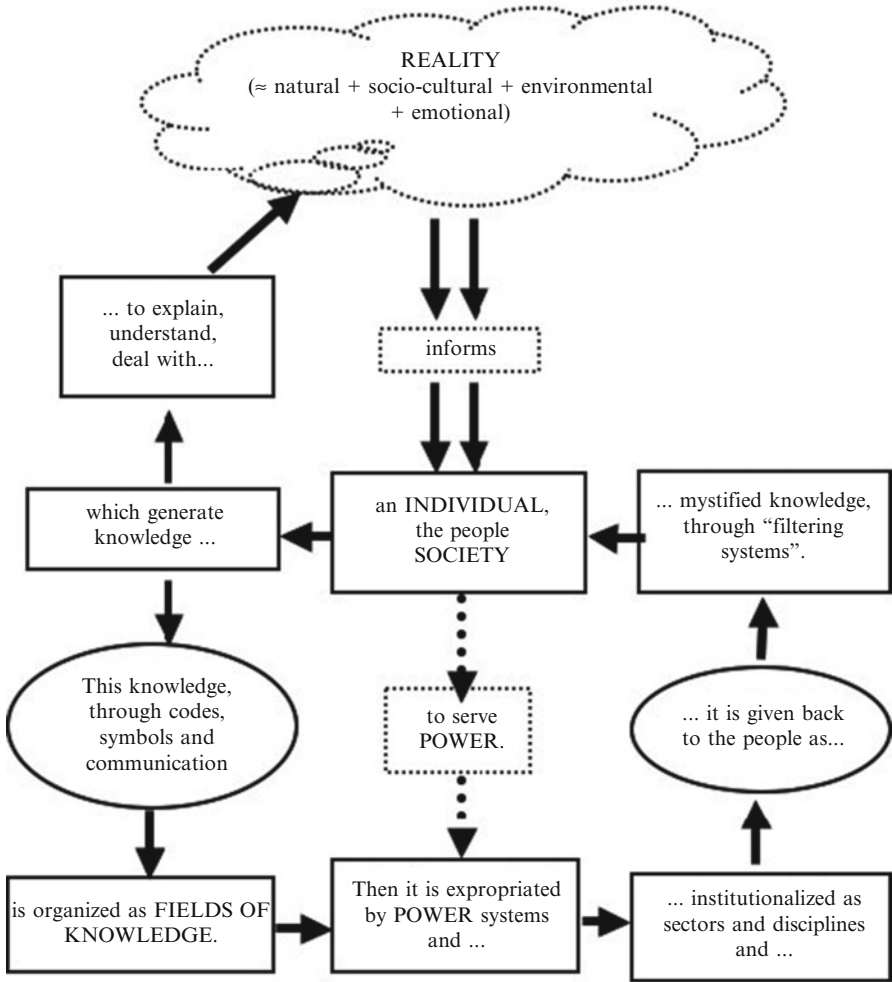


Fig. 2.3 The full cycle of knowledge

The individual is not alone. Gregariousness is a characteristic of animal species. How do the individual and the other interact? Communication plays a fundamental role in the interaction.

Particularly important is the emergence of language. When did utterance of humans become a word (Kenneally 2007)? Language is greatly advantageous in conveying to others individual will and needs. I will simply admit that through communication, even before the emergence of language, individuals interacted with others to produce knowledge and to making their behaviours compatible.

Similarly with ‘nature versus nurture’, there is a controversy about the ‘individual versus social’ in building up knowledge. The main question is how social structures impact the cognitive structures of the individual and how structures of

individual consciousness and cognition can and do impact the structures of society. Essentially, the question becomes: how do individuals and society interact in cognitive actions, as well as in socio-political actions? How do ideologies, for example, languages, arts, religions, styles of knowing, become established, and how are social actions coordinated, for example as political movements? (See Wu 2007)

Thus, through encounters and interaction of individuals, there is mutual exposure and exchange of ad hoc practices and solutions of problems organized by each individual as knowledge. These are in general different practices and solutions. Through neurophysiological processes, as yet not well understood, which certainly include language and mimicry, the ad hoc practices and solutions for common problems, organized as individual knowledge, are shared and transformed, and result in socially organized knowledge. Thus, the cycle of knowledge is represented as in Fig. 2.2.

These two figures for the cycle of knowledge are understood as: (1) the cycle in which REALITY informs the INDIVIDUAL, who processes the information and exerts an ACTION (bodily and mental \approx individual knowledge) which affects and *modifies* REALITY, which, once modified, informs (now incorporating newer elements) the INDIVIDUAL, who realizes a different ACTION (bodily and mental \approx *modified* individual knowledge), which again modifies REALITY, and so on; (2) the cycle in which REALITY informs the GROUP (individual, society, people), who processes the information and exerts an ACTION (bodily and mental \approx social knowledge) which affects and modifies REALITY, which, once modified, informs (now incorporating newer elements) the GROUP, who realizes a different ACTION, which again modifies REALITY, and so on. Hence, we may consider the individual cycle of knowledge, which is active as far as there is life:

$$\dots \rightarrow \text{reality} \rightarrow \text{individual/group} \rightarrow \text{action} \rightarrow \text{reality} \rightarrow \dots$$

This cycle synthesizes life as a dynamic process, to which every animal is subjected. Action manifests in several ways. Action may be the result of instinct and leads to the satisfaction of the *pulsion* of survival, meaning the permanent drive towards the survival of the individual and of the species, in other words nourishment and mating, which are subordinated to physiology, sociobiology and ecology, as a common characteristic of all living species. I will clarify the use of the word *pulsion*, rarely used in English. It is widely recognized that the English translation of the texts of Sigmund Freud is problematic, particularly the concept of *trieb*, which has been translated as “drive” and “instinct”. Both are not faithful to the concept. Instead the French, Spanish and Portuguese translations use the word “pulsion”. This word, which in the English language is used in different contexts, rarely in psychoanalysis, represents very well my understanding of survival.

In the human species, survival is a *pulsion*, which is loaded with emotions and intensions. Like every animal species, humans satisfy the *pulsion* of survival developing strategies to work with the most immediate environment, which supplies air, water, food, and with the other of the same species, necessary for

procreation. This means, everything that is necessary for the survival of the individuals and of the species. These strategies are modes of behaviour and individual and collective knowledge, which include communication. Through interaction, there is an action of learning how to cope with the pulsion of survival. This gives origin to a form of communication between individuals which results in sharing strategies for individual survival. There is a form of learning, which involves mimicry and other sophisticated forms of interaction. Recent work in primatology shows some rudimentary form of instruction in chimpanzees.

Humans differ from other animal species. The species *homo* subordinates the strategies developed by other animal species for survival, a drive towards satisfying needs, which is usually called instinct, to will. In other words, the *homo* species go beyond survival and the continuation of species. Will leads to choices, preferences and desires, thus to emotions. As a consequence, another pulsion, which I call the *pulsion of transcendence*, is intrinsic to the *homo* species. The pulsion of transcendence is responsible for the needs of explaining, of understanding and of creating or, in other words, for transcending our own existence and projecting ourselves into the past and into the future. This is responsible for the development of instruments and techniques, for codes and a sophisticated communication system which has a cognitive dimension and developed into language. The use of instruments and techniques, of codes and communication, is organized as labor and power. For a more detailed discussion see D'Ambrosio (2012).

In satisfying the pulsion of transcendence, the species *homo* develops the perception of past, present and future, and their linkage, and the explanations of facts and phenomena encountered in their natural and imaginary environment. These are incorporated to the memory, individual and collective, and organized as arts and techniques, which evolve as representations of the real (models), as elaborations about these representations which result in organised systems of explanations of the origins and the creation of myths and mysteries (mentifacts). Some of the representations materialize as objects, concrete representations and sophisticated instruments (artifacts).

All this behaviour encounters support in the memory, where myths, mysteries, history and the traditions are organised, generally as religions and value systems. Explanations of the origins and the creation and of myths and mysteries generate curiosity and will to know the future, and give rise to divination organised and theorised as divinatory arts.

Probably the most basic of all systems of knowledge, present in every culture, are the divinatory arts. The human species, different from any other animal species, developed the concepts of *past*, *present* and *future*, and how they are enchainned. Nothing is more characteristic of the human species than the desire to know the future. Thus, divinatory arts, such as astrology, the oracles, logic, the I Ching, numerology and the sciences, in general, through which we may know what will happen, are exemplary systems of knowledge. All these divinatory arts are all based on observing, comparing, classifying, ordering, measuring, quantifying, inferring, which are the quintessence of mathematical ideas.

The cycle of knowledge leads to the explanation of individual behaviour, social behaviour and cultural behaviour as the result of the incessant change of reality, as expressed in the cycle $\dots \rightarrow \text{reality} \rightarrow \text{individual/group} \rightarrow \text{action} \rightarrow \text{reality} \rightarrow \dots$

Systems of knowledge reveal not only their convenience to explain reality, facts and phenomena, but also they are important strategies to cope with daily situations and problems not only for the individual, but for society and the people in general.

Societies are organised subjected to different forms of power structure. The power structures recognise the advantage of mastering these strategies for their benefit, hence they proceed in expropriating and controlling these strategies, and consequently the system of knowledge in which they are based. Thus, the knowledge shared by the group is detained and controlled by the power structure and is institutionalized as clergy, as norms and laws, as disciplines, as academies, indeed in many ways, which are controlled by classes subordinated to the power structure. They are given back to the people as mystified systems of knowledge, subjected by filters. The mystification and filters guarantee that the systems of knowledge and the strategies associated with them do not challenge the power structure.

The full cycle of knowledge includes its generation, individually and socially, its organisation, its expropriation, institutionalization, transmission and diffusion, through systems of education and different forms of filters (such as examinations, degrees, certifications). Thus we are led to the full cycle of knowledge, as in Fig. 2.3.

These steps shown in Figs. 2.1, 2.2, and 2.3 are commonly treated as disciplines, respectively cognition, epistemology and politics. A serious limitation to understanding knowledge, as an intrinsic characteristic of the human species in response to the pulsions of survival and transcendence, is to treat it in separate steps, through the academic disciplines just mentioned.

2.3 How About Modelling?

Consider again the cycle $\dots \rightarrow \text{reality} \rightarrow \text{individual/group} \rightarrow \text{action} \rightarrow \text{reality} \rightarrow \dots$. In it, selected facts and phenomena of reality inform individuals and groups. Obviously, no one has full access, awareness and knowledge of reality; no one is omniscient. Our natural limitations give us access to selected facts and phenomena. The reason and the form of selection are extremely complex. They go from an uneven capability of individuals and groups to receive information, in some cases related to sensorial qualities or deficiencies, in other cases to the interest in the information received. The interest may be because of needs, or preference or merely by chance. Anyway, the information received is processed, in a way not yet well understood. The individual or group exerts an action of generating artifacts and mentifacts from the selected part of reality. They are incorporated into reality as representations, which inform the individuals or groups and the cycle goes on. The main question is then, how individuals and groups deal with the representations of selected facts and phenomena.

Fig. 2.4 Amazonian *pariko*

In a representation, reality is restricted to selected facts and phenomena and the result is a sort of “isolated individualized reality”. To deal with the “isolated individualized reality”, individuals attribute codes or parameters to the selected facts and phenomena. These parameters may be of a mathematical nature, such as mathematical forms and mathematical symbols. The isolated individualized reality, with the mathematical symbols attributed to the selected facts and phenomena, is a *mathematical model* of it. As an example, consider a *pariko*, typical of an Amazonian culture (Fig. 2.4). This artifact is a model of a complex social reality. It serves as a form of identity of its owner: indicates age, affiliation, origin and many other components of social life. However, it is impossible to attribute to this model parameters of a mathematical nature. Other examples may be drawn from urbanization.

Through models, humans try to give explanations of myths and mysteries, and these explanations are organized as arts, techniques, theories, as strategies to explain and deal with facts and phenomena. These strategies, have been historically organized, in different groups, in different spatial and temporal contexts, which are the support of cultures, as systems of knowledge.

The result is a sort of “isolated individualised reality”, restricted to the representation of selected facts and phenomena. The “isolated individualised reality”, dealt with the resource of parameters, is a model. Individuals are informed and elaborate on the model analysing the parameters associated with it.

Intellectual resources allow the individual to deal with the model and the parameters created by the individual are representations of facts and phenomena of the reality in the broad sense. The most common intellectual resources are based on observing, comparing, classifying, ordering, measuring, quantifying, and

Fig. 2.5 The practice of mathematical modelling



inferring. As mentioned earlier, these intellectual resources are the basic pillars on which mathematics is based.

These parameters may be in terms of formal mathematics. I call *mathematical modelling* the process of dealing with a model in which the parameters associated with it, which is the objective of coping with and explaining selected facts and phenomena of reality, are in terms of formal mathematics.

The practice of mathematical modelling is an iterative method starting with reality, with which we started by selecting parameters, constructing a model, proceeding to its mathematical analysis, verifying results through control procedures and reformulating the model, repeating the analyses and control until we reach a satisfactory perception of the selected facts and phenomena. This is illustrated by the diagram in Fig. 2.5.

In each step, the practitioner reformulates the choice of parameters and resumes the process, which eventually allows a better understanding of the selected facts and phenomena of reality, which is the goal that justifies our practices as scientists.

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Mathematical Modelling in Education Research and
Practice

Cultural, Social and Cognitive Influences

Stillman, G.A.; Blum, W.; Salett Biembengut, M. (Eds.)

2015, XI, 613 p. 148 illus., Hardcover

ISBN: 978-3-319-18271-1