Chapter 2
Evidence for the Indefinite

2.1 Chance in Theoretical Models

We begin by examining the type of theoretical indefiniteness in physics which is due to our partial, subjective lack of knowledge of the properties of individual particles within large systems. Historically, this problem was first treated in the latter part of the 19th century. At that time, thermodynamics was enjoying great success; it had developed empirically from the treatment of heat engines and the study of chemical reactions. It could successfully describe macroscopic phenomena, for example the rise in pressure in a steam boiler when its temperature is increased. Nevertheless, Ludwig Boltzmann developed his own theory—*statistical mechanics*—which refers to microscopic particles (atoms or molecules) that had not been directly observed at the time and whose dynamics were unknown. In one mole of oxygen, i.e. 16 grams, there are more than 10^{23} molecules; 10^{23} is a *very* large number, a 1 followed by 23 zeroes. Each molecule has three spatial coordinates and velocities in each of the three spatial dimensions. The amount of data required to specify the positions and velocities of all of those molecules makes the task of determining them seem hopeless.

In order to attack this problem, Boltzmann introduced chance into physics. He started with microstates whose details are random, determined by chance; they form a statistical ensemble, which is determined by only a few macroscopic state variables. Microstates are chance elements of a larger whole, the macrostate, which is well defined. The macrostate is described by physical quantities such as temperature, volume and the number of particles, which lie in the realm of classical measurement methods. Boltzmann applied the concept of "entropy", originally introduced by Robert Clausius and Nicolas Léonard Sadi Carnot. It is a measure of our lack of knowledge of a system—the indefiniteness of the state, so to speak. The less we know about the microstates, i.e. the less information we have about the system, the greater its entropy.

H.J. Pirner, *The Unknown as an Engine for Science*,
The Frontiers Collection, DOI 10.1007/978-3-319-18509-5_2

Boltzmann's opponents, the so-called "energeticists", maintained that all of the laws of thermodynamics could be derived from energy conservation alone. Boltzmann countered with the statement that the energy available to perform work is determined by the entropy. Energy is conserved in all processes, but the *quality* of the energy decreases in technical processes as the entropy increases. For example, we have little use for the exhaust gases of an automobile; their entropy is too high, so that they cannot be used to power engines or drive chemical processes. When the entropy of a system is high, its energy is worthless.

Mathematically, statistical mechanics is based on probability theory. Just as in a casino or a card game, chance "determines" the dynamics of the atoms or molecules in a boiler. At a given temperature, the energy of a given atom can be specified only in terms of a probability. The distribution of energies has its maximum at the average energy. Variations or fluctuations around this average do occur, but they are unimportant for the increase of the pressure in the boiler with increasing temperature. In equilibrium, each microstate has a well-defined probability which does not change over time.

The approach to equilibrium presumes systems whose equations of motion are not integrable. Integrable systems usually consist of only one or two particles. For example, the one-dimensional motions of a single point mass are determined solely by energy conservation, and the system is integrable. The orbital motion of a planet around the sun can be completely and analytically described by applying the conservation of energy and of angular momentum. In non-integrable, chaotic systems there are no additional conserved quantities besides energy and momentum which could predetermine the system's evolution. The ensemble of all the positions and velocities of the individual particles moves over a hypersurface which is determined by energy conservation. Such a system begins with a spatial distribution of the velocities and position coordinates that is sharply localized. If we wait long enough, this cell spreads out like a drop of blue ink in a glass of clear water. Such systems are termed "ergodic".

The question arises as to whether a statistical description can be applied only to simple objects such as atoms—can it also be applied to complex time-dependent phenomena such as biological or economic processes? In the year 1827, the English botanist Robert Brown used a microscope to observe a pollen grain suspended in a liquid. He noted that the grain was moving, as though it were alive. It took almost 80 years until Albert Einstein was able to explain this process in terms of a random walk.

Einstein made the assumption that the light molecules of the liquid transfer momentum to the heavier pollen grain by colliding with it, and cause it to recoil in a random way. The forces which act during these collisions are not determined in detail; we do not know their directions and strengths. On the whole, these forces average out to zero. They have values within a zone between $-F$ and $+F$ around zero. Nevertheless, the pollen grain moves. The reason for this is that within a certain time interval, the forces are correlated, i.e. they do not change abruptly from large negative values to large positive ones. Physicists parametrize these correlations. Einstein succeeded in solving the equation of motion of the heavy particle in the presence of the random external forces. The solution leads to a mean square velocity which at long times is proportional to the strength of the correlation function. If the particle comes into equilibrium with the liquid after some long time, then there is a simple relationship between the strength of the internal friction and the mean square random force.

Similar differential equations with random external terms can be used to describe complex biological or economic systems. The random "forces" might be political crises or climatic catastrophes which influence the target parameter, e.g. the price of a stock or of a commodity. In these generally very large systems of equations, a great number of interactions are parametrized, insofar as their parametrization is possible. The remaining influences which cannot be analyzed are random perturbations, of which one knows only mean values and correlations. With the goal of learning as much as possible about the world around us, physics has dealt with objects about which we will never obtain information with certainty. Nevertheless, using the combination of probability calculus and differential equations, it has been possible to reach a better understanding of such systems. The statistical treatment of medium-sized quantum systems raises new questions. Here, the indefiniteness in the description of individual systems combines with the intrinsic indeterminacy of quantum mechanics, which I will describe separately later. The number of particles in such systems is small compared to thermodynamic systems; we are dealing here with perhaps 100–200 particles. The systems are themselves small in size, i.e. they are subject to the laws governing quantum objects (cf. Sect. 2.3). The spacing of their energy levels is comparable with the resolution with which one can determine the levels. In contrast, in thermodynamic systems, the energy levels are so closely spaced that it would be hopeless to try to compute or measure them individually. In the mesoscopic (medium-sized) systems considered here, the mean level density and the behavior of the energy-level spacing are accessible to experiments. Examples of mesoscopic systems are atomic nuclei at excitation energies of a few million electron volts, or quantum billiards, i.e. two-dimensional racetracks of electrons on solid surfaces or at interfaces. The theoretical treatment of these systems models their indefiniteness in a unique way and can thus describe certain aspects of the energy levels.

> The treatment of systems of this kind was initiated by Niels Bohr, who coined the term "compound nucleus" for the combined nucleus which is formed during the scattering of a nuclear projectile from an atomic 'target' nucleus. In low-energy collisions of neutrons from a reactor with atomic nuclei, the projectile chooses one of the many possible final states only after "circling" around the target nucleus for some time. This long contact time leads to a quasi-equilibrium state which permits a statistical description of the system. Since the work of Werner Karl Heisenberg, it has been known that wave mechanics gives an equivalent representation of every quantum-mechanical problem with a well-defined matrix (in Heisenberg's "matrix mechanics"). Modern methods are based on a theory which models a class of energy matrices, determined by symmetry alone, instead of a single quantum-mechanical energy matrix with sharply defined energy levels.

Physical theory cannot parametrize and solve the complex interactions of even these in principle relatively few particles. However, it can successfully model a statistical distribution of matrices which reflects the important symmetry properties of the system.

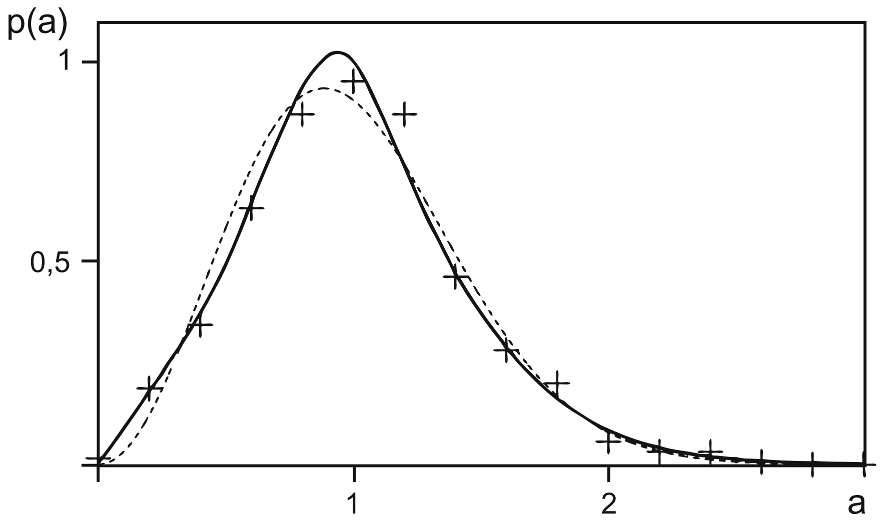

Fig. 2.1 The scaled distribution of the spacing of parked automobiles is plotted as a function of their bumper-to-bumper distance *a* for the automobiles on the right-hand side of the street; after Hradec Králové. The value $a = 1$ on the horizontal axis corresponds to the average distance. The *dashed curve* shows the distribution given by a Gaussian unitary random-matrix ensemble. The *solid curve* was calculated from a detailed model (Šeba [2]; see also earlier references by Abul-Magd [3]) which distinguishes between the left-hand and the right-hand sides of the street

One refers to these models as *random-matrix theories*.[1] Note how theoretical physicists introduce structure into the limiting cases that make up the twilight zone. Symmetry is necessary in order to limit the number of possible cases and to correlate the classes of matrices with classes of phenomena. This approach has proved successful in many areas of low-energy nuclear physics, quantum chaos, number theory, and the physics of disordered solids.

Random matrices can also successfully describe the spacing between parked cars. Gaussian unitary or orthogonal random matrices lead to a distribution of the distances from bumper to bumper which has a simple form (cf. Fig. 2.1). One finds a characteristic distribution which begins at zero, passes through a maximum and then drops slowly again towards zero. There are only a few virtuoso drivers who manage to park very close to the car ahead. I have to add that the empirical studies were not carried out in Paris, where it is well known that the cars behind or ahead of a space are often shoved together by desperate motorists, leading to a spacing which makes it impossible for pedestrians to cross the street.

The statistical method has even attracted proponents who assert that the fundamental form and the coupling constants in the Standard Model of elementary-particle physics are the result of a stochastic averaging over generically known interactions.

[1]Bohigas and Weidenmüller [1]. The authors give an introduction to this relatively new area of physics.

This would mean that the question of a deeper understanding of the 40 unknown couplings of all the elementary particles and the form of their interactions could be attributed to a random model. Holger Bech Nielsen,[2] an exponent of this random dynamics, starts with the assumption that the truely fundamental elementary-particle theory is extremely complicated. He presumes that, independently of this complex fundamental theory, at the highest energies effective laws have emerged whose form is determined randomly.[3] This yields an approach to elementary-particle physics which has not attracted many supporters. The majority of physicists are in fact convinced that the fundamental laws of physics become simpler and simpler as one investigates smaller and smaller elementary objects. This assumption seems well justified if one considers how physics has evolved towards smaller and smaller size scales. From the physics of molecules, atoms, atomic nuclei, and nucleons to the physics of quarks, one cannot deny a certain trend towards simplification.

There is an almost metaphysical debate between the proponents of a rationally-planned world and the supporters of the idea that chance played a significant role even in the origin of the universe. Many of the exponents of these random cosmologies believe that only the anthropic principle can be made responsible for choosing our universe out of the multitude of possible existing universes ("the multiverse"). This principle states that our universe has precisely those properties that are necessary so that we can live in it; the theory thus derives the laws governing the origin of the world from observations of its current state.

> In string theory, the indefiniteness is based not on our lack of knowledge of the elementary excitations, but rather on the uncertainty about the background environment in which the strings move. There are presumed to be 10^{500} possible realizations ("landscapes") of this background, which give rise to a similar number of vacua. One can imagine the background to be like a potential landscape (a contour map), with hills and valleys in which the fields or strings seek their lowest energy states like balls rolling on a hilly surface. Among these enormously many possible states is the one state which corresponds to our universe, in which the elementary electric charge, the velocity of light and the mass of the proton have precisely their known values in our world. In cosmology, our universe plays the role of just one variant within a 'multi'verse. The other universes cannot be observed, and thus their existence is rather controversial in modern theoretical physics.

The opponents of string theory and the multiverse insist that most of the possible string theories are not consistent with the observations of type IIa supernovas,

[2]Nielsen and Brene [4].

[3]Nielsen and Brene, *ibid*.: "In the search for the most fundamental theory of physics, one usually looks for a simplest possible model, but could it not be that the fundamental "World Machinery" (or theory) could be extremely complicated? We see that we have some very beautiful and simple laws of nature such as Newton's laws, Hooke's law, the Standard Model and so on—how could such transparency and simplicity arise from a very complex world? The Random Dynamics project is based on the idea that all known laws of nature can, in a similar way as Hooke's law, be derived in some limit(s), practically independent of the underlying theory of the World Machinery. The limit which could suggestively be the relevant limit for many laws would be that the fundamental energy scale is very large compared to the energies of the elementary particles, even in very high energy experiments. A likely fundamental energy scale would be the Planck energy."

which document an accelerating expansion of the universe.[4] Those theories predict with a high probability exactly the converse behavior. A simplified account of recent understanding of a small positive cosmological constant is given by Raphael Bousso,[5] and Bousso together with Joseph Polchinski.[6] They argue that such stable vacuum solutions are possible in string theory and are selected in the cosmic evolution by us as observers (anthropic principle).

Pierre Duhem, in his “Theory of Physics”, described such a confused situation as a preliminary state of affairs: “In any case, a state of indecision never persists for long. The day arrives when common sense points clearly to a particular theory, and the opposing fractions give up their resistance, although there is no purely logical reason to do so”.[7] The experimental physicists are presently hoping that the new LHC (Large Hadron Collider) will put an end to the confusion. Whether it will in fact support the speculations of string theory or will contradict them remains to be decided. Results of the first experimental run of the LHC have not given any evidence for physics beyond the standard model.

String theory has proved to be such a fruitful branch of mathematical physics that it will continue to be investigated, possibly in complete disregard of the experimental results. Duhem’s assessment of theoretical physics may prove not to be correct.

Indefinite in the sense of *random*

Examples

- The positions and velocities of individual atoms in a well-defined gas;
- The forces on a pollen grain suspended in a liquid;
- The matrices and the spacing of parked automobiles;
- Our universe within the multiverse.

2.2 The Unpredictable Future

Dante Alighieri[8] in the twentieth canto of his “Divine Comedy” describes how he meets up with the fortune-tellers in Hell: “Their faces were turned backwards, and they had to move their feet in a backwards direction; they no longer had any way of

[4]Ellis and Smolin [5].

[5]Bousso [6].

[6]Bousso and Polchinski [7].

[7]Duhem [8].

[8]Dante Alighieri: *The Divine Comedy*; see for example http://www.gutenberg.org/files/8800/8800-h/8800-h.htm.

seeing ahead". Dante lets the fortune-tellers walk with slow-moving steps through the twisted valley in Hell. Just as the fortune-tellers are punished for their false prognoses, incorrect predictions can decisively influence the lives of individuals or of society. In order to elucidate the uncertain future, one must analyze the past and investigate the possible future with the aid of models. We shall ask in the following sections just how uncertain predictions of the future based on such models really are.

A physical model represents a simplified picture. We cannot see light waves, but nonetheless we use a wave concept to describe light, since it exhibits phenomena similar to water waves. Models are thus heuristic representations which describe new phenomena. A theory is more detailed than a model and therefore has a higher status. It evolves on the basis of many phenomena which it sums up with the aid of physical laws, such as energy conservation; these are concepts that provide relationships between the fundamental quantities. Physicists have laws at their disposal in classical mechanics which predict the position and velocity of an object at a later time from knowledge of its position and velocity at one particular time, presuming that they know the forces which act upon it. A point like object (a "point mass") follows a path that satisfies Newton's equation, which predicts precisely its position and velocity at a later time $t > t_0$ from their known values at the earlier time t_0. This is also possible for several point masses with complex interactions. The French mathematician and astronomer Pierre-Simon Laplace (1749–1827) was the first to prove the constancy of the mean orbits of the planets in celestial mechanics. The solar system is thereby so stable that it will "remain intact" up to the final inflation of the sun some 10^9 years from now. Classical mechanics is a deterministic theory of the world; it computes the future unambiguously from knowledge of the past.

Our predictive powers are to be sure rather limited when we consider systems with many degrees of freedom. The solutions of the equations of motion of many-body systems depend so sensitively on their initial conditions that even a minimal deviation in those initial values leads to a large variation in the predicted results after a finite time. One terms such systems "chaotic", since the predictions may change erratically due to small variations in the initial conditions. Since we can never determine the initial values with absolute precision, our ability to predict events precisely diminishes rapidly with increasing time. If one improves the precision of the initial values for a chaotic system by a factor of 100, then the length of time for which predictions hold within a given margin of error is increased by only a factor of 4. The components of the system and their interactions are themselves well-defined and determined, but their future evolution is uncertain, since the exact initial state of the system can never be precisely known.

Similar problems occur when we model clouds, winds and air currents in order to predict our weather and climate. The meteorologist Edward Norton Lorenz[9] was the first to point out the chaotic solutions of the hydrodynamic equations, which are functions of time and space. He distinguishes two types of predictions: Predictions

[9]Lorenz [9].

of the first kind refer to the time evolution of a system as a function of its initial conditions in time with given boundary conditions in space. Predictions of the second kind refer to the solution of the equations with fixed initial conditions but variable spatial boundary conditions. Weather predictions are of the first type, since a finite number of satellites and weather stations cannot completely and precisely determine the initial weather conditions. They depend upon the local boundary conditions in the atmosphere and in the oceans.

It is a simple truth that long-term predictions, e.g. over a time period of 20 years, would have to contain the technological and political changes that will occur during those years. As we well know, it is not sufficient simply to extrapolate observed trends from the past, since here real discontinuities and qualitative improvements or setbacks can occur. Climate predictions refer to long periods of time, while weather prognoses apply to the short term. For the weather, a time frame of 5 days is a considerable challenge; for the climate, one has to consider time periods of several years or even decades. The preferred method in the latter case is to model the atmospheric circulation only in large time steps and then to relate it to the dynamics of the oceans, the biosphere, the polar regions and other slowly-changing components of the climate. Relatively good results have been obtained for the El Niño events, a cold circulation in the Pacific which appears every three to six years.[10] At the beginning of the year 2009, the National Weather Service in the USA predicted a temperature increase of 2 ± 0.5 °C for December 2009, which could affect the yield of fisheries.[11] Of most concern is the possibility that instabilities could arise in the long-term climate as a result of human activities, for example an interruption of the flow of the Gulf Stream in the North Atlantic,[12] or global warming due to CO_2 emissions.

Model calculations in the form of equations of motion or partial differential hydrodynamic equations are imprecise due to chaotic dynamics, amplified by the insufficiently-known initial conditions. Statistical predictions, which become necessary when we know too little about the system itself, should be distinguished from such calculations. There, we make use of probabilities to characterize the system. If within a statistical description we assume that the parameters of the system do not change within the time period covered by the forecasts, then the path is clear for authoritative predictions. We will give two examples of the application and the limitations of statistics.

In Germany, the average height of the whole population is 178 cm, with a variance of 7 cm. This means that within a probability of 70 %, the height of a German citizen lies between 171 and 185 cm (178 ± 7 cm). The height distributions of adult men and women in Germany are shown in Fig. 2.2. The bell-shaped curves show that women on the average are somewhat shorter than men.

[10]http://www.cpc.noaa.gov/products/.

[11]This prediction was rather well fulfilled at the time of writing. For the week centered on December 23, 2009, the temperature anomaly reached 1.94 °C, as quoted in https://bobtisdale.wordpress.com/2009/12/.

[12]Hasselmann [10].

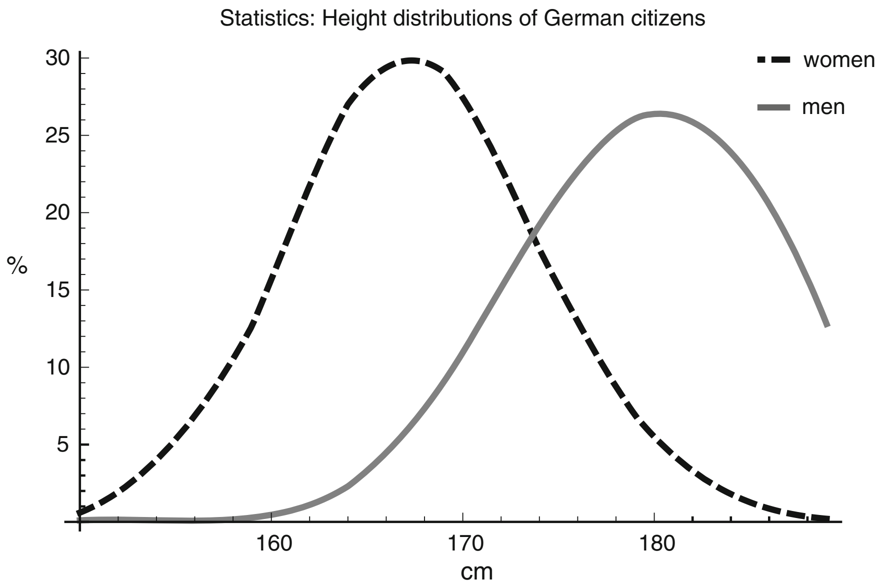

Fig. 2.2 The probability distribution [Figure from http://de.wikipedia.org/wiki/K%C3%B6rpergr%C3%B6%C3%9Fe, dated 02.06.2010; *Source* statista.org, Socio-economic Panel (SOEP).] of the heights of men and women in Germany. They resemble bell curves, whose maxima coincide with the average height of men and of women

Physicists refer to the corresponding probability distributions as *normal distributions*. Nassim Nicholas Taleb attributes them to a fictitious country called "Mediocristan", the land of mediocrity, where everything occurs "normally".

If I have an appointment with a business partner whom I have not met previously, I can say with a high probability that he or she will be between 171 and 185 cm in height. If I know whether the partner is a man or a woman, I can make this prediction more precise, since the average heights and their variances are different for men and women. The problem becomes more difficult if I attempt to estimate the economic status of my unknown partner, as manifested in his/her income or net worth. In practice, this estimate will have to be made from the social context; it makes a big difference whether one is meeting with a sales representative or with the CEO of a corporation.

Let us now assume that we have an appointment with a CEO, and thus with a presumably wealthy person. Taleb places a group of such people in another country, which he calls "Extremistan". In Extremistan, different laws hold from those of Mediocristan, i.e. the probability distributions there do not have the bell-curve or normal-curve shape that we saw for the height distribution within the general population; instead, they exhibit power laws like shown in the income distribution of the wealthiest (Fig. 2.3). The Italian economist Vilfredo Pareto[13] discovered such

[13]Pareto [11].

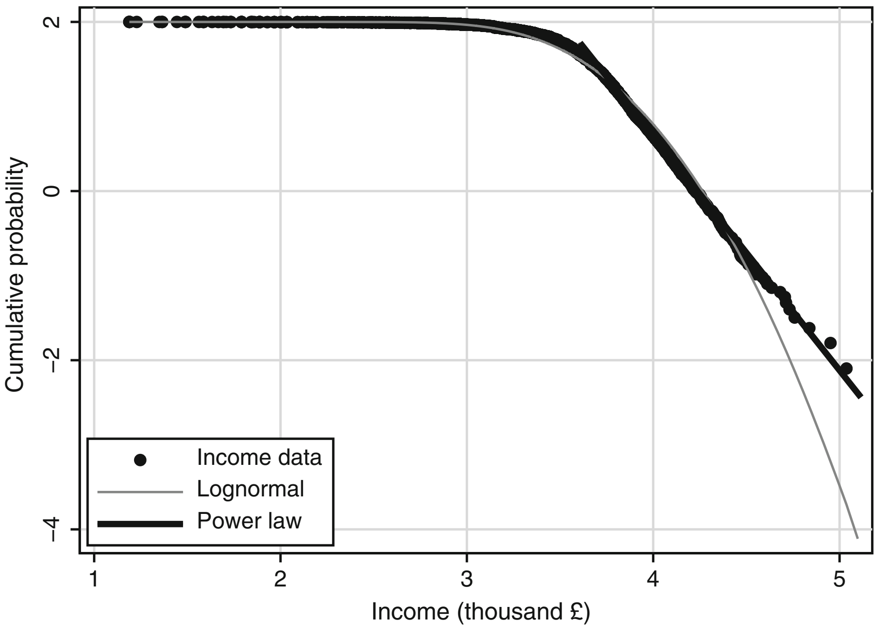

Fig. 2.3 The cumulative, i.e. integrated income distribution (Clementi und Gallegati, *ibid.*) of Italian households in the year 1998. The *curve* shows which percentage of the population (the number 2 corresponds to $10^2 = 100\ \%$) has an income higher than the income on the horizontal scale (the number 3 corresponds to $10^{3+3} = 1{,}000{,}000$ Lire). Up to very high incomes, the data are fit well by a log-normal distribution (*thin line*). For the "very wealthy", they follow a power distribution, which is higher in comparison to the log-normal distribution (*full line*, "fat tail")

distributions empirically at the end of the 19th century. The probability of encountering great wealth is given by the distribution $P(w) = ak^a/w^{1+a}$ with the index $a = 1.4$ and the minimum parameter k, if one accepts the analyses of Moshe Levy and Sorin Solomon,[14] or of Fabio Clementi and Mauro Gallegati.[15] The distributions of price variations, the sizes of corporations, fluctuations of stock and bond prices and their returns also follow similar power laws. In particular, investment returns have the same index as the distribution of wealth, which led the above authors to the hypothesis that the richest segment of the population invests large amounts of money and then profits the most from the returns on those investments.

Physicists are fascinated by scaling laws of the above type, because they occur in many systems near critical points. The critical point of a magnetic material is defined as the temperature at which it loses its macroscopic magnetization. Below this point, long-range spatial correlations between the elementary magnets or magnetic moments occur, which obey power laws. This process is to be sure quite different from economic processes.

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[14] Levy and Solomon [12].

[15] Clementi and Gallegati [13].

In economic terms, success has no natural upper limit. As in the game of chance "Winner takes all", or in the Biblical saying, "He that hath, to him shall be given", wealth can increase enormously; it grows without limits, in contrast to the height of individuals, which is subject to a natural upper limit.

What does all this have to do with the reliability of predictions of future events? For the following discussion, we shall assume a power-law distribution and ignore the deviations of the distribution of wealth from the power law at low and middle asset values (they follow a log-normal distribution there). Then the mean value of wealth is indeed well defined by the power-law distribution, but in the case of an index of $a < 2$, the variance from the mean is not. We found the value $a = 1.4$; it is thus difficult owing to the broad fluctuations to determine the limits of the amount of assets of our business partner. Likewise, it may prove impossible to predict the future behavior of other quantities which obey scaling laws, when their variances are undefined.

The question "What is meant by 'wealthy'?" is thus not vague in the sense that there is no sharp boundary between rich and poor, as is asserted by some philosophers. The answer to this question is adequately provided by the empirical distribution of wealth. There is a well-defined point at which the log-normal distribution that applies to average citizens departs from the power law that describes the assets of the wealthy.

The indefiniteness in the case of power laws results from their undefined variances. In Extremistan, we would have difficulties in making predictions about the future. The economic crises and collapses of 1987, 1997, 2003, and 2008 would seem to verify this statement. As recently as 2002, Harry Eugene Stanley and Rosario Nunzio Mantegna[16] optimistically discussed the successes of *econophysics*: "In fact, power law distributions lack a typical scale, which is reflected by the property that the variance is infinite for $a < 2$. One important accomplishment is the almost complete consensus concerning the finiteness of the variance of price changes (on the stock market)". They were convinced that the fluctuations of stock-market prices are calculable, and on the basis of this hypothesis, they defended the efficiency of the market, which supposedly adjusts prices quickly and rationally to their optimum values. The term "econophysics" was coined by these authors and describes the interdisciplinary analysis of empirical economic data using physical models. More than in axiomatic-mathematically-oriented macroeconomics, in this new area of physics an attempt is made to keep research closely connected to the data, which elucidate both distinctive structural characteristics and also invariants through international comparisons.[17] The French theoretical physicist Jean-Philippe Bouchaud[18] pointed out that predictions based on log-normal distributions "with finite variances" by Fischer Black and Myron Scholes were in part responsible for the crisis in 1987, and he affirmed the need for more pragmatic and realistic models.

[16]Mantegna and Stanley [14].

[17]Sinha et al. [15].

[18]Bouchaud [16].

The normal distributions miss out on the paradigm of the black swan,[19] i.e. the occurrence of rare but significant events.

The prediction of truly time-dependent processes is additionally complicated by the variation of their parameters. Bouchaud is of the opinion that the agents of the system (e.g. the stock market) impair the predictability of economic systems through their irrational actions. Most investigations of economic systems have emphasized the financial markets; only in more recent times has there been modeling which takes into account both economic *and* ecological aspects. This can provide a more realistic, if still unfocused picture of the future, since it considers new interactions, but also brings additional uncertainties into play.

Indefinite, in the sense of *uncertain*.

Examples

- Chaotic systems;
- Weather and climate predictions;
- Distributions of wealth with “fat tails”;
- Long-term prognoses, owing to technical or political changes.

2.3 Indeterminacy in Quantum Physics

Physics, as an empirical science, is thought to be the essence of definiteness. It is based upon experiments and observations which can be repeated and reproduced at any time and in every place with the aid of suitable experimental conditions. Ideally, an experiment yields measured values which form the basis for hypotheses. Hypotheses which have been verified by a number of experiments can be merged into natural laws. The basic facts underlying physics originate in Nature itself. They are independent of the models which we use to describe Nature. Are there nevertheless examples of indefiniteness within physics? The answer is “yes”!

One can detect uncertainty in the results of measurements which ought to yield identical values when they are carried out by different groups of experimenters. Even when the experimental conditions are controlled with great care, every measurement apparatus will yield a somewhat different value for each individual measurement. Uncontrollable external influences such as small temperature variations, fluctuations in voltage or motions of the air can lead to errors in a measured value. Measurements are always associated with uncertainties. Measurement errors of this type (*statistical errors*) can be treated using statistical methods as described

[19] Nassim Nicholas Taleb, see footnote 3 in Chap. 1.

in Sects. 2.1 and 2.2. The specific "uncertainty" in quantum physics is particularly fascinating, since it is of a quite different nature. I will deal with this difference in the following.

The basis of our discussion are the indeterminacy relations due to Werner Karl Heisenberg, one of which states that the position and the momentum (momentum = mass times velocity) of a particle cannot be determined simultaneously with unlimited precision. The indeterminacy Δx in the measurement of the particle's position and the indeterminacy Δp in the measurement of its momentum together obey the inequality

$$\Delta x \times \Delta p > h/(4\pi)$$

where h is Planck's constant (also known as Planck's *quantum of action* by analogy to classical mechanics). Max Planck discovered this fundamental physical constant in 1900 through his observation that radiation of a particular wavelength can be emitted or absorbed only in discrete amounts (quanta) of energy. In the indeterminacy relation, Planck's constant limits the precision of a simultaneous measurement of the position and the velocity of a particle, independently of the potential accuracy of the measurement instrumentation. The more precise the determination of the momentary position of the particle, the greater is the indeterminacy *in principle* in its velocity determination. This relation corresponds to an indefiniteness which is inherent in Nature itself and does not depend on the quality of our apparatus or on the observer; therefore one should preferably speak of the "*indeterminacy relation*", since it is an objective fact and we have reserved the concept of "uncertainty" to describe a lack of subjective knowledge.

In quantum mechanics, the status of a particle is described by a function which treats it as a superposition of spatially-localized states. This "wavefunction" Ψ reflects a "virtual" state of the particle, which becomes real with the probability $|\Psi(r)|^2$ when the position of the particle is determined by a measurement at the location r. This probability is objective; it is not based on a subjective lack of knowledge on the part of the observer, as might play a role e.g. in determining the momenta of the individual atoms within a hot gas. The observer of a gas cannot know the velocities and positions of all of its constituent atoms, since it is simply not possible in practice to measure all of them.

The phrase "'virtual' state of the wavefunction" expresses the fact that the wavefunction itself cannot be directly observed. The absolute square $|\Psi(r)|^2$ of the wavefunction is however an observable quantity, and it gives the probability with which the particle is to be found at the position r. The wave nature of a quantum object gives rise to the indeterminacy relations. Broader, spread-out wavefunctions are associated with a greater uncertainty upon measurement.

In a velocity or momentum measurement, it is more expedient to deal with the virtual state within the space of all the possible momenta, i.e. the wavefunction in momentum space, $|\Psi(p)|^2$. The real-space and momentum-space wavefunctions are related to each other through an exact mathematical transformation, which was developed by Jean Baptiste Joseph Fourier. It is useful also for example in the analysis of the frequency spectrum of an acoustic signal. The broader the wavefunction in coordinate space, the sharper it will be in momentum space, and vice versa, as a result of this transformation property.

The indefiniteness in quantum mechanics can be understood by considering the following example: If one passes an electron through a double slit (e.g. a metal screen with two narrow, parallel openings), then one cannot say with certainty through which of the two slits it has passed, nor exactly where it will impact on a detector screen. The quantum-mechanical result for the impacts on the detector screen (cf. Fig. 2.4) corresponds to an intensity pattern as would be expected from *waves* which had passed through the two slits. The image reminds us of the superposition of two circular waves emitted from the centers of the two slits. We can produce and superpose circular waves for example by dropping two stones simultaneously into a calm lake; the distance between the points where they hit the water corresponds to the spacing of the two slits. Wave maxima from one wave which meet up with minima from the other cancel each other out, while two maxima or two minima reinforce each other when they meet. Mathematically, the *phase* of the waves defines the positions where at a given time a maximum or a minimum will be observed. The interference pattern which results from such an experiment is not simply the sum of the individual wave patterns from a single stone or a single slit.

In order to proceed with our discussion, we first need to deal with the probability statements $w(X)$ made by quantum mechanics.

We denote the detection of an electron on the screen by 'A', the passage of an electron through the upper slit by 'B', and passage through the lower slit by 'not B'. In classical mechanics, one would assume that the probabilities of passage through the upper and the lower slits simply add, as is in fact observed in the experiment with bullets (see Fig. 2.4), i.e.

$$w(\mathrm{A}) = w(\mathrm{A} \cap \mathrm{B}) + w(\mathrm{A} \cap \neg\mathrm{B})$$

while in quantum mechanics, we find:

$$w(\mathrm{A}) = w(\mathrm{A} \cap \mathrm{B}) + w(\mathrm{A} \cap \neg\mathrm{B}) + \text{interference term.}$$

Peter Mittelstaedt[20] asserts that the new quantum-mechanical situation is a relative violation of the dictum "*tertium non datur*" of classical logic, since the deviation from two-valued logic is conveyed through the relevant probability. If the events where electrons passed through the upper or the lower slit corresponded to mutually exclusive sets, the formula given above would nevertheless contradict one of Kolmogorow's axioms of probability theory.

It is notable that the result of the quantum-mechanical double-slit experiment approaches the classical result when the vacuum within the apparatus is replaced by a gas that scatters the particles. The double-slit experiment was carried out by Klaus

[20]Mittelstaedt [17].

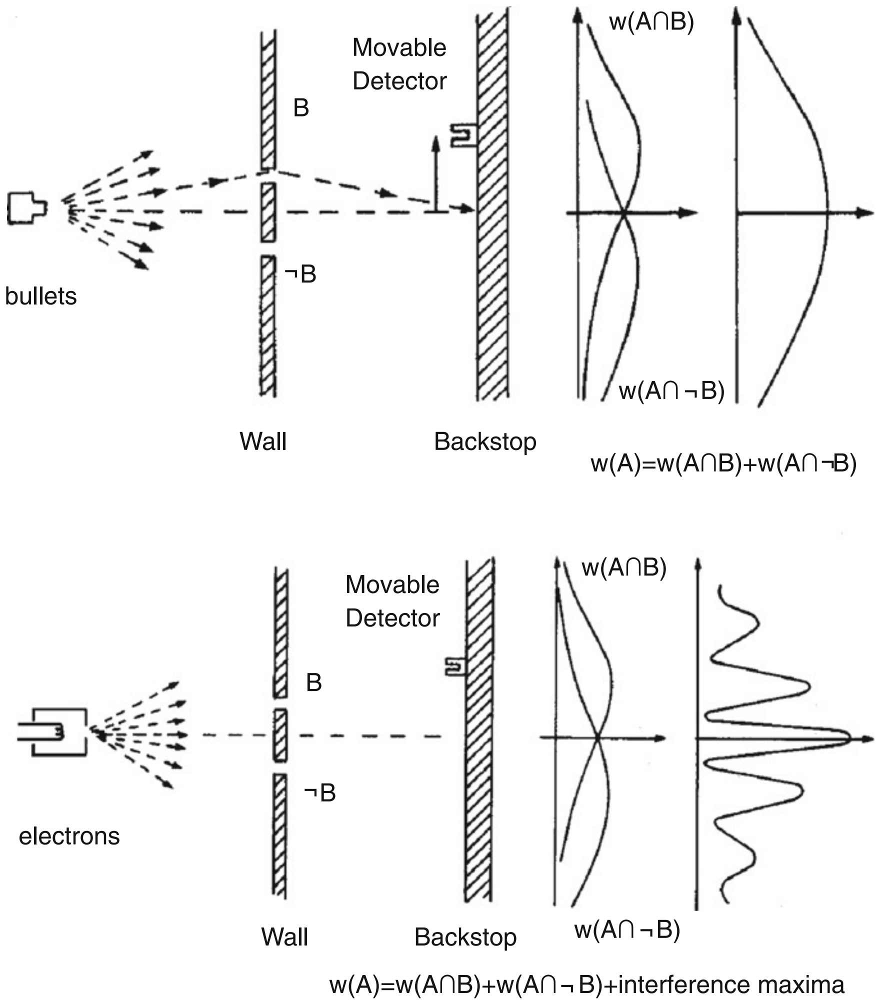

Fig. 2.4 The double-slit experiment: Bullets or electrons from a source (a gun or an electron gun) pass through two slit openings in a wall and impact onto a screen ("backstop"). The resulting pattern on the screen for bullets is simply the sum of the patterns obtained when only one of the two slits is open. In the case of electrons, interference maxima and minima are seen, which are due to the wave nature of matter (i.e. of quantum objects, here electrons)

Homberger et al.[21] using Fullerenes, large spherical molecules composed of carbon atoms. The amount of gas within the apparatus determines the visibility of the interference pattern, which becomes fainter exponentially with increasing gas pressure. This experiment indicates that the measurement process must be explained in terms of the interactions between the system being observed, the

[21]Hornberger et al. [18].

measurement apparatus, and the environment within the apparatus. In this triad, decoherence,[22] i.e. the loss of phase coherence in the superposition of the corresponding wavefunctions, plays an important role. This decoherence can be tested experimentally. It is more difficult to explain why in these product states, only those states survive which correspond to macroscopic "pointer" or "impact" settings. Put simply, a dense gas as experimental environment does not permit fluctuations in the pointer settings, but instead fixes the pointer in equilibrium with the gas molecules which impact upon it.

> The interactions of the measurement apparatus with its environment, i.e. allow only states constructed from eigenvectors of an operator which commutes with the apparatus-environment Hamiltonian. A binary mathematical operation is termed non-commutative when its result depends on the order in which its elements act. The addition of two numbers is commutative, since $a + b = b + a$. Subtraction is however non-commutative, $a - b \neq b - a$. In quantum mechanics, we term two operators non-commutative when their product is $AB \neq BA$. One can suppose that A and B are matrices. In quantum mechanics, position x and momentum p are represented by such non-commutative operators. From this property, we can derive the indeterminacy relations mathematically.

Within the multiplicity of interpretations of quantum mechanics, David Bohm's hypothesis of the "undivided universe"[23] takes on a special position. Bohm considers an electron always together with its quantum field. He introduces a new kind of information, an active information, which can be attributed to the form and not to the intensity of the quantum field. This information is different from Shannon's definition of information (see Sect. 3.1). As an illustration, imagine a ship whose energy and motion are determined by its engines and its fuel reserves. It receives information about its route from the outside world via radio, which communicates directly with the autopilot. As long as radio waves can be received, their intensity is unimportant; only their content determines the course of the ship. This interpretation is unorthodox, because it contains hidden variables. Bohm's indefiniteness comes from the lack of knowledge about the initial distribution of position coordinates of the particle, which contains hidden parameters. All the predictions of quantum mechanics can be reproduced by Bohm's theory. The trajectories of the coordinates within a somewhat weird quantum-mechanical potential determine the results of measurements.

There have been attempts to combine quantum mechanics and gravitation into a unified theory known as quantum gravity.[24] The combination of Planck's constant

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[22]Joos and Zeh [19], Paz and Zurek [20].

[23]Bohm and Hiley [21].

[24]Parallel to these attempts, in the modern literature gravitation is often interpreted not as a fundamental theory, but rather as an emergent or effective theory, which holds only at length scales large compared with the Planck length λ. In that case, it would not be necessary to quantize gravity. The metric tensor would then be analogous to the density distribution of particles in thermodynamics. The new fundamental degrees of freedom are unknown. Indeed, the Einstein equations have a great similarity at a horizon to the thermodynamic equation $T\mathrm{d}S - \mathrm{d}E = P\mathrm{d}V$. See e.g. Padmanabhan [22].

h from quantum mechanics, the speed of light c from special relativity, and the universal constant G of gravitation results in a length scale $\lambda = \sqrt{\hbar G/2\pi c^3} = 10^{-33}$ cm, below which quantum mechanics and gravitation can no longer be treated separately. In order to get an idea of the size of this *Planck length* λ, it is useful to define a logarithmic scale which represents shorter and shorter lengths by a factor of 10 at equal intervals. If we represent the size of a coffee cup by the first mark on this scale, we will have to place 13 marks along the scale to arrive at the size of an atomic nucleus. We would then have to continue three times as far along the scale of size reductions by a factor of 10 to arrive at the Planck length λ. Quantum gravity has evolved around a new indeterminacy relation,

$$\Delta x \times \Delta y > \lambda^2.$$

Here, the smallest elementary length, the Planck length, is on the right-hand side of the inequality. This indeterminacy relation would mean that it is not possible to measure a length more precisely than the Planck length. Every length measurement with increasing precision along the x direction would produce an increased indeterminacy in length measurements along the perpendicular y direction. In quantum mechanics, there is in fact an analogous behavior. The center of the circular orbit of an electron in a homogeneous magnetic field pointing along the z direction cannot be determined exactly.

The associated quantum-mechanical x and y coordinates are represented by operators whose product in this case is $xy \neq yx$; therefore, the center point of the electron's orbit cannot be determined with unlimited accuracy. The indeterminacy relation between x and y is at first sight surprising, since without the magnetic field, x and y are not complementary quantities in the sense of quantum mechanics, in contrast to the position x and the corresponding momentum p_x.

It follows from the indeterminacy relations of quantum mechanics that smaller and smaller details of the elementary particles can be investigated by using particle accelerators of higher and higher energies. These accelerators work like microscopes with a high resolution for matter waves.

This would no longer be the case if we were to reach the scale of the smallest elementary length. Leonard Susskind,[25] a resolute supporter of string theory, has extrapolated the hypotheses of that theory to collisions at the highest energies. While in the usual quantum mechanics, higher energies E allow us to study smaller and smaller length scales, $L = hc/(2\pi E)$, this behavior would change in such a string theory at extremely high energies.

In very high-energy collisions, a black hole would be created, which could emit only low-energy Hawking radiation whose energy would be of the order of the inverse Schwarzschild radius R. The Schwarzschild radius $R = 2GE/c^2$, which corresponds to the energy of the Hawking radiation, defines a horizon (the *event horizon*) behind which the black hole is hidden. The importance of gravitational effects enters through the gravitational constant. In

[25] Susskind and Lindesay [23].

astronomy, black holes are stars that have collapsed under the effect of the gravitational force; their gravitational potential is so strong that even light can no longer escape from the black hole. Therefore, such objects appear black in the sky. In principle, there could also be miniature versions of these black holes ('mini black holes'), which are created in collisions of particles at high energies.

There have even been legal proceedings which were aimed at preventing experiments with the new accelerators at the Brookhaven National Laboratory in the USA and at CERN in Geneva, because the plaintiffs feared that small black holes would be produced that could swallow up the Earth.

Experience to date with the similarly energetic cosmic radiation, along with the extremely short lifetimes of the hypothetical mini black holes, contradict the assumptions of those lawsuits. The experiments which have been carried out recently yield lower limits for the possible masses of these objects. No such small black holes have been observed. The Schwarzschild radius of the black holes defines the event horizon, behind which information appears to vanish. Correspondingly, black holes have no characteristic properties aside from their overall electric charge, their angular momentum and their mass. However, quantum mechanics requires that the state of the wavefunction at a given time determines the state of the wavefunction at every later time. One might say that the information contained in the wavefunction is conserved. Now, if one were to throw a book containing information into a black hole, the information would be annihilated, i.e. the above fundamental assumption of quantum mechanics would be violated. This contradiction[26] has recently been resolved through an improved understanding of the role of the Hawking radiation which is emitted by the black holes.

A horizon is also defined for us by the expansion of the universe. At the *cosmic horizon*, at a distance of 45 billion (4.5×10^{10}) light years from us, matter is moving away so fast that no light can reach us from further away. Behind this horizon the universe is inaccessible to us. The astounding homogeneity of the cosmic microwave background radiation can be explained by applying an inflationary model, i.e. an extremely rapid expansion at the beginning of the universe. Without this rapid expansion following the Big Bang, parts of the observable universe would not have been able to communicate and would have different temperatures, in contradiction to the observed homogeneity.

In the preceding summary of the indefiniteness in the quantum world, we have seen how the indeterminacy relations play an extremely important role in physics. They set limits to our possible knowledge, beyond which we can no longer peek at Nature's secrets. It was long thought that the intervention of the observer was the source of the indeterminacy or fuzziness of quantum measurements. Our modern understanding of quantum physics is however that these uncertainty relations must be attributed to Nature herself; they are *indeterminacy* relations. At the same time, these relations have become central, structuring elements of the associated theories.

[26]See Susskind and Lindesay, *ibid*.

Indefinite, in the sense of *indeterminate.*

Examples

- Simultaneous measurement of the position and the momentum of a quantum object;
- Impact of the quantum object at a particular location on a detector;
- Measurement of the position coordinates in different directions in quantum gravitation.

2.4 Vagueness in Language

In our exploration of the typology of indefiniteness, we meet up with phenomena whose origin is not to be found in Nature, but instead in language, in memory and in human thought. These are common in everyday life and have therefore been under discussion since ancient times. The same problems are topics of research today in modern linguistics, psychology, and philosophy. In the following account, I will describe both the ancient beginnings as well as the modern scholarly treatment of these questions. By their comparison, it can be seen how much effort has been necessary to achieve even a small amount of progress.

Linguistic indefiniteness[27] refers in the main to "discrete" sets, whose elements can be enumerated using whole numbers. A standard example is the question: "When is a certain man X bald?" In principle, this question can be answered by looking at X's head. But in making the assessment, one might have doubts as to just how few hairs justify applying the attribute "bald". As a result, the statement "X is bald" remains vague. The following question originated in ancient times: What is a sandpile? Is an accumulation of grains a pile? How many grains do we need in order to make a pile? This example gave rise to the Sorites paradox (from the ancient Greek *σορὸσ* = "pile"), which results from applying induction. The first step in the logical chain of reasoning states: An accumulation of *one* grain is *not* a sandpile. The second step asserts that if an accumulation of n grains is not a sandpile, then an accumulation of $n + 1$ grains is also not a sandpile. By induction, one then arrives at the paradoxical result that an accumulation of a thousand grains is still not a sandpile.

How have physicists dealt with the vagueness in the definition of a sandpile? A sandpile is an open system which is not in stable equilibrium and which continually exchanges energy and matter with its surroundings through drifting sand. Sand avalanches occur when the slope of the pile exceeds the angle of repose; then the pile abandons its metastable

[27] Kemmerling [24].

equilibrium and sand flows down its flanks. Per Bak, Chao Tang and Kurt Wiesenfeldder[28] have developed a simple model of how a sandpile can form. The basal area of the pile is divided into squares; in each square, there is room for up to K sand grains. Grains which are carried to the pile are randomly distributed over the squares. If a square contains more than K grains, the excess grains are moved onto adjoining squares. In this way, clusters are formed on the basal area, whose sizes are determined by the avalanches that occur within each square. The number $D(s)$ of the clusters as a function of their size s follows a power law, $D(s) \approx s^{-\tau}$, with $\tau = 0.98$. This distribution is independent of the quantity K, which could perhaps be compared to the size of the sandpile in the Sorites discussion. The pile's structure can be explained without having to fine-tune the parameter K; this is an advantage of Bak's model. As an aside, we note that in real experiments,[29] the critical behavior of the mathematical model given above cannot always be demonstrated. If e.g. rice grains are used, and they are too round, one observes deviations.

To give rise to the Sorites paradox, a set (e.g. of sandpiles) must be specified which cannot be broken up into smaller units. The induction step in the Sorites paradox leads us to believe that there are only gradual changes without qualitative differences. Taking as a counterexample the vague concept of childhood, we note that the transition from child to adult passes through puberty, which is accompanied by significant changes.

All of the discussions which have been carried on in philosophy seem to offer little practical guidance for finding a solution. I will now describe how engineers or computer scientists attack this problem: They interpret the continuous transition of one state into another as "fuzzyness".

In 1965, the systems analyst Lofti Asker Zadeh[30] introduced to this end the concept of "fuzzy sets". Elements belong to fuzzy sets only to a certain extent, not exclusively. A vague statement such as "this room is warm" does not define the temperature of the room, but it nevertheless often suffices to characterize the state of the room for its occupants. In order to describe the vague meaning of the word "warm" more precisely, it is useful to imagine an ensemble of warm rooms and then to compare the particular room under consideration with these rooms, to evaluate how appropriate or inappropriate the statement was. The foundation for this comparison is a range of values or *basis* for the problem at hand. In this example, the basis is a range of temperatures T, which for the purposes of this discussion and without loss of generality I will define as "warm" between 0 and 30 °C. This eliminates absurdities such as the Sorites paradox right from the beginning, namely that one could, by means of small reductions of the threshold for "warm" in steps of 1 °C followed by an induction step, arrive at the ridiculous idea that −1 °C must still be considered as warm. In the next step, we define a *membership function* for all of the elements t in T, which attributes a value between 0 and 1 to each element t. I will also call $m(t)$ with $0 < m < 1$ an "opinion function", since this term makes it clearer that we are dealing with a subjective assignment rather than an actual fixed meaning. This qualifies the problem of multi-valued logic right from the beginning. Opinions can be

[28]Bak et al. [25].

[29]Frette et al. [26].

[30]Zadeh [27].

inconsistent and need not always be logically justifiable. The value 1 of the opinion function, i.e. $m(t) = 1$, corresponds to the opinion that the statement is completely true, while $m(t) = 0$ corresponds to the opinion that the statement is totally false. The opinion function for the statement that A is *not* correct is defined as follows:

$$m(\neg \mathrm{A}) = 1 - m(\mathrm{A}).$$

The goal of this mathematical formulation is to quantify the indefiniteness of the judgements of experts. Note that this indefiniteness is quite different from the uncertainty which arises from the variation among different temperature measurements. The opinion function varies between 0 and 1, making it similar to a probability. It is however not normalized, and the indefiniteness that it denotes is not the same as the entropy or information derived from probabilities, as we shall see. We continue with our example by discussing various opinion functions (cf. Fig. 2.5). We first consider the "wishy-washy" expert (*ww*), who has no strong opinion and would therefore set the value of the opinion function to $m = ½$ over the whole range. In contrast, we have the "warm-blooded" expert (*w*), who finds the upper part of the range of temperatures to be quite warm and has a linearly increasing opinion curve. Our third expert is a "coddled" Middle European (*cw*) who finds only the range between 20 and 30 °C to be genuinely warm, and whose opinion function exhibits a sharp maximum at 25 °C. These various functions are shown in Fig. 2.5. We can then use information theory[31] to define a measure for the indefiniteness $U(\mathrm{A})$, which fulfils the condition that $U(\neg\mathrm{A}) = U(\mathrm{A})$,

$$U(\mathrm{A}) = -\int \mathrm{d}t\{m(t)\log[2, m(t)] + ((1 - m(t))\log[2, (1 - m(t))])\}.$$

Here, we use the logarithm to the basis 2; then we find for the indefiniteness of the three experts $U_{ww}(\mathrm{A}) = 30$ °C, $U_w(\mathrm{A}) = 21.6$ °C, and $U_{cw}(\mathrm{A}) = 7.2$ °C. This quantitative determination agrees with our impression that the *ww* expert had the least definite opinion, while the *cw* expert is rather sharply opinionated.

As the next step, I will define the opinion function for the simultaneous occurrence ("and") of the statements A and B by making use of the minimum function:

$$m(\mathrm{A} \cap \mathrm{B}) = \min[m(\mathrm{A}), m(\mathrm{B})].$$

The "or" conjunction can be similarly defined using the maximum function. These specifications allow us to quantify to what extent additional information would help to determine the state of the room as "warm".

Additional information could be a statement such as, "If children are wearing short pants" (B), and I assert that it is completely true ($m = 1$) if the temperature is above 18 °C.

[31] DeLuca and Termini [28]. See also: Bandemer and Näther [29].

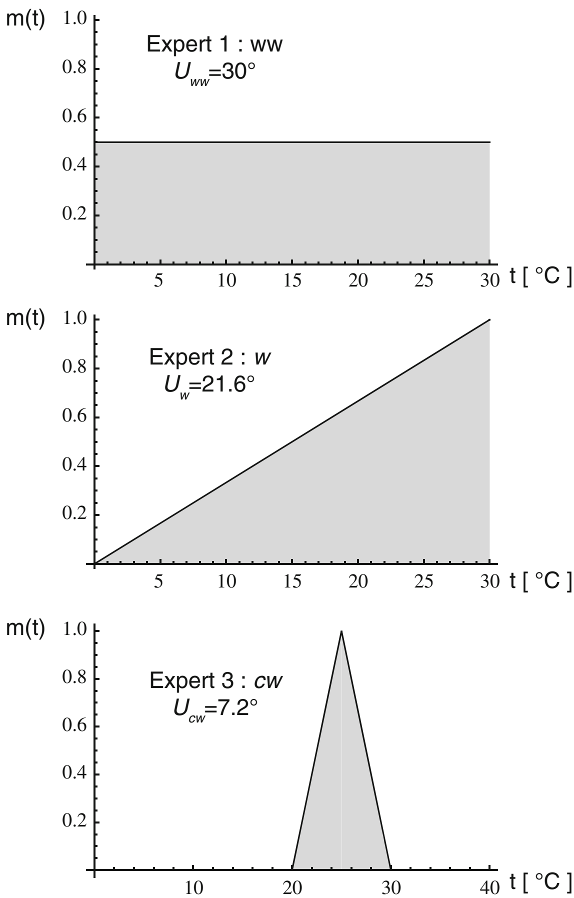


Fig. 2.5 Membership or opinion functions denote to what extent an expert considers the temperatures given on the horizontal axis to be 'warm'. From above, the opinion functions of the "wishy-washy" expert (*ww*), the "warm-blooded" expert (*w*) and the "coddled" expert (*cw*) are shown as functions of the temperature in degrees celsius. The values U_{ww} (A) = 30 °C, U_w (A) = 21.6 °C, and U_{cw} (A) = 7.2 °C give the indefiniteness of the opinions of the various experts in units of temperature, i.e. in degrees celsius; see the quantification of the indefiniteness (in the main text)

In Fig. 2.6, one must then take the minimum of the linear function and the rectangular function; the latter is equal to zero below 18°. Above 18°, this minimum coincides with the linear function.

This additional information reduces the indefiniteness of the opinions of the first two experts (*ww* and *w*), which were particularly vague. The indefiniteness of the opinion of the 'coddled' expert (*cw*) is not changed by the additional information.

Compare:

$$U_{ww}(\mathrm{A}) = 30\ ^\circ\mathrm{C}, U_w(\mathrm{A}) = 21.6\ ^\circ\mathrm{C} \text{ and } U_{cw}(\mathrm{A}) = 7.2\ ^\circ\mathrm{C}$$

with

$$U_{ww}(\mathrm{A} \cap \mathrm{B}) = 12.0\ ^\circ\mathrm{C}, U_w(\mathrm{A} \cap \mathrm{B}) = 7.8\ ^\circ\mathrm{C} \text{ and } U_{cw}(\mathrm{A} \cap \mathrm{B}) = 7.2\ ^\circ\mathrm{C}.$$

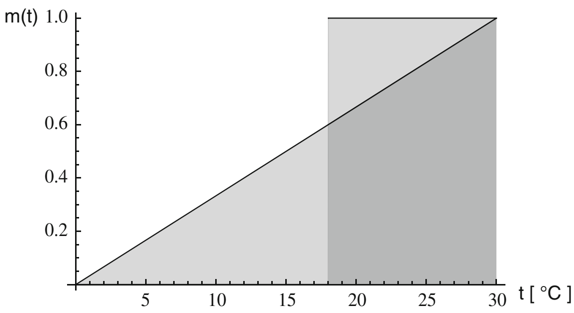


Fig. 2.6 Due to the conjunction (∩) of the two statements "the room is warm" and "the children are wearing short pants", we find new opinion functions $m(t) = 0$ for $T < 18$ °C and $m(t) = T/30$ °C for $T > 18$ °C (*darkly shaded area*). This leads to an indefiniteness of 7.8 °C for the "warm-blooded" expert (w)

The gain from an additional specification is clear for the first two experts; the second approaches the quantitative indefiniteness of the third when the new information is added.

I denote the opinion function which corresponds to the simultaneous occurrence of the statements "A" and "not A" as the *ambiguity*:

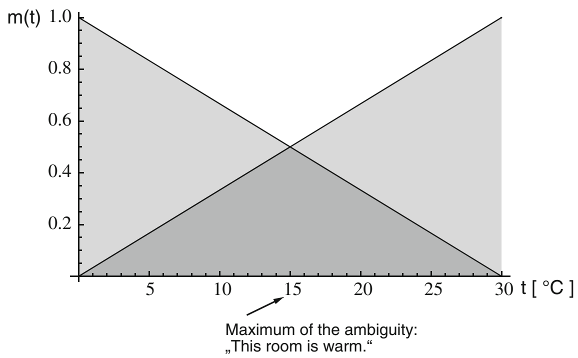
$$\mathit{ambiguity}(\mathrm{A\ and\ \neg A}) = m(\mathrm{A} \cap \neg \mathrm{A}).$$

This definition leads to useful results. In particular, for the opinion of the first expert (ww), we obtain a constant ambiguity of ½, while the ambiguity of the second expert (w) has a maximum at 15 °C (Fig. 2.7).

Thus it is most difficult for him at the temperature 15° to decide if the room is warm or not warm, i.e. to give a definite opinion. At the two endpoints of the range, i.e. at 0 and 30 °C, the situation is clear and the ambiguity vanishes.

The advantage of fuzzy set theory now consists in the fact that one can implement vague instructions on a machine. It is important for control technology to

Fig. 2.7 The ambiguity of the second expert (w) with respect to the statement "This room is warm" (*darkly shaded area*). His opinion is least strong where the two lines cross each other, i.e. the ambiguity is maximal at 15 °C



define goals as broadly as possible; for example, a command to the heating control system “Keep this room warm!” cannot be carried out properly when the definition of “warm” is limited to the range 19.5–19.8 °C. The thermostat would continually turn the heating system on and off.

Fuzzy logic is used for example in the subway system in Sendai, Japan; it has been in operation since 1988 without human drivers.[32] Human drivers or conventional computer-controlled systems accelerate or brake the train by observing marks which tell them how far the train is from the station platform. With this kind of control according to a fixed plan, the train’s velocity is often abruptly increased or decreased. The Hitachi engineers applied flexible controls in Sendai by making use of fuzzy logic, and these keep the train’s motion more uniform. These control rules take into account how often and to what extent the velocity has been changed and how near it is to the maximum allowed velocity. This system has reduced the travel time and energy consumption by 10 % in comparison to a conventional system.

Those philosophers who may be sitting in the train will still not be convinced that an extension of Aristotelian logic to fuzzy logic is acceptable. Thus, T. Williamson has stated that, “Many-valued logic is particularly popular among those who lack logical sophistication”.[33] I think that this objection is appropriate if one interprets the membership or opinion functions as *truth functions*.[34] If we take the statement A to be “Max is short” and attribute to it a “truth value” $t(\mathrm{A}) = 0.5$, then the statement “Max is short and Max is not short” has a truth value of $t(\mathrm{A}$ and $\neg\mathrm{A}) = 0.5$. This is clearly not what one expects; for the latter case, $t = 0$ would be a reasonable value, since both assertions cannot be true. Similar contradictions are found for the logical conjunction “or”. In the weaker form of fuzzy logic using opinion functions, these contradictions do not occur, since opinion functions are not truth functions. I find that half-truths and characterizations such as “very, very true” in contrast to “very true” are not very convincing.

Semantic aspects of indefiniteness play an important role in translating from one language to another and in interpreting legal texts. Vagueness prevents over-regulation and the associated blocking of the system. A reasonable level of control is possible only when a certain tolerance remains to allow for flexible responses. Herbert Lionel Adolphus Hart et al.[35] advocate formulating laws in such a way that they retain an “open texture”, and in their interpretation, an adaptive case law can be applied as required by the situation at hand.

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[32] Kosko and Isaka [30].

[33] Williamson [31].

[34] van Deemter [32].

[35] Hart [33]: “Whichever device, precedent [authorative example] or legislation, is chosen for the communication of standards of behavior […]; they will have what has been termed an *open texture*. […] It is, however, important to appreciate why apart from this dependence on language as it actually is, with its characteristics of open texture, we should not cherish, even as an ideal, the conception of a rule so detailed that the question whether it is applied or not to a particular case was always settled in advance, and never involved, at the point of actual application a fresh choice between open alternatives.”, p. 124.

On the other hand, power can more easily be (mis)used as a result of vague laws and rules. This is the topic of Michel Foucault's[36] "Deployments of Power". These 'deployments' ('*dispositifs*' in the original French) refer to a mixture of scientific records, administrative directives and regulatory decisions, which are not formulated clearly and are therefore suitable as a means of manipulating people. Foucault himself is not very precise in speaking of them: "[…] un ensemble résolument hétérogène, comportant des discours, des institutions, […], des énoncés scientifiques, des propositions philosophiques, bref: du dit, aussi bien que du nondit, […]".[37] He emphasizes the not-said, which characterizes the indefinite, that which is simply presumed or to be presumed.

How is one to proceed when defining precise limits has serious consequences and there is a need for prompt action, for example in the case of a medical diagnosis? The purely epistomological approach to this problem is based upon the realization that a limit exists, but we do not know where it lies. This attitude represents an optimistic approach to the problem.

It remains unclear whether or not, as a consequence of semantic indefiniteness, the classical theory of concepts itself is in need of revision. This classical theory propounds that concepts are to be defined through sufficient and necessary conditions. Fine-tuning a theory (*tinkering*, *bricolage*) and the tentative nature of theories are strong arguments against a priori or analytic judgments as an element of how we formulate scientific concepts. Claude Lévi-Strauss, in his book "The Savage Mind",[38] describes the *tinkerer*, who can make the best of what is at his disposal and knows how to 'fix things up'. Lévi-Strauss assumes a critical stance which does not sharply distinguish between rational thinking and the primitive (i.e. 'savage') mind.

At the beginning of this chapter, I dismissed the philosophical discussion of vagueness as impractical. That judgment must now be qualified. The artificiality of the scholarly and scientific world corresponds to the limits of our possibilities and abilities to formulate theories, to interpret the world and to change it. True progress demands new concepts and methods. Thus, the progress of the natural sciences is closely linked to the logical-conceptual. Max Bense[39] comments on this connection between logical thinking and practical action as follows: "Now, a statement is the representation of a thought. An application is an action based on a thought. The attention of a logician is focused on the statement, while that of a technician turns to the application. […] Regarding the formal character of a statement, its truth is

[36]Foucault [34].

[37]Foucault, *Dits et Ecrits*, *ibid*.

[38]Lévi-Strauss [35]. "The 'bricoleur' is adept at performing a large number of diverse tasks; but, unlike the engineer, he does not subordinate each of them to the availability of raw materials and tools conceived and procured for the purpose of the project. His universe of instruments is closed and the rules of his game are always to make do with 'whatever is at hand', […]. […] He [the physicist] is no more able than the 'bricoleur' to do whatever he wishes when he is presented with a given task. He too has to begin by making a catalogue of a previously determined set consisting of theoretical and practical knowledge, of technical means, which restrict the possible solutions." pp. 17, 19.

[39]Bense [36].

equivalent to freedom from contradictions; regarding its instrumental character, truth means essentially applicability."

He emphasizes the analogous character of logical and practical actions. To be consistent, one should not underestimate the significance of semantic indefiniteness for the "functioning" or the "failure" of our technological world. A closer examination of the double-slit experiment in physics however also points up the limits of purely logical certainty. Logical necessity is an entirely intellectual construction and cannot replace experience.

Indefinite, in the sense of *vague*.

Examples

- Vague statements and their paradoxical results;
- Fuzzy sets and opinion functions;
- The indefiniteness of opinion functions can be calculated.

2.5 Indistinct Memories of the Past

Although recollections can be stored for many years in our long-term memory, occasionally it happens that invoking them calls up an image which is pale or unfocused in comparison to the original scene. Where and when does indefiniteness enter into memory? Is one impression not uniquely linked to another? Or was a perception simply 'filed in the wrong drawer'? Is the stored information erroneous or vague?

Reconstruction of the past is an attempt to establish causal relations between past and present events. Our memory is the location where the definite and the indefinite from the past are mixed together. We speak of the effort of remembering in order to express the fact that searching our memories for the past is distinct from the subconscious processes which occur in the brain parallel to our conscious thought processes. Therefore, I will include the brain in these considerations of our ability to remember. Neurophysiological research has found that different parts of the brain are active when it is recalling a particular event. The exchange of memory content between different areas of the brain is not understood, and the question remains open as to whether it is controlled by a program. Often, we can only dimly sense a connection between different recollections. One then speaks of implicit memories, which are formed indistinctly and without our conscious intervention, by association or habit. Modern psychology has developed test procedures to analyze impulsive judgments. Imaging techniques make it possible to localize this memory content.

Sometimes we find that we cannot connect a name with a particular face, nor can we remember the place where we saw it for the first time. Psychological experiments have shown that test subjects can recall only about one-fourth of what they had originally committed to memory from a particular experience. They tend to make up new 'information' to complement what is indistinct in their memories. For example, witnesses at a trial may identify the wrong person if he or she is simply wearing the same clothing as the actual person in the past.

Why should we investigate the biophysics of memory before studying its philosophical implications? Our own personal experience can lead us astray when we are dealing with our brains. Wolf Singer[40] has expressed the opinion that we are trained through observations of our surroundings to understand *linear* phenomena, in which a small change in an action gives rise to a corresponding small change in the reaction. In his understanding, we are unable to grasp the complex nonlinear interconnections in the human brain. In trying to comprehend memory processes in the brain, we have to consider a network of 10,000 million nerve cells. Each individual nerve cell (neuron) communicates with roughly ten thousand others which send it input data, and it passes these data on to an equal number of other neurons. This takes place through many interconnections, similarly to the World Wide Web. The shape of the neurons is asymmetric. So-called dendrites form finely branched input connections. A long axon conducts the impulses as excitation potentials in the range of millivolts along the length of the nerve cell. Insofar as an axon of cell A repeatedly excites another cell B, the cells modify themselves so that the effectiveness of the excitation is amplified. The strength of the neural interconnections is thus fundamental in the storage of recollections. Learning is based on a fine adjustment of the connections (synapses) between the nerve cells, which transfer the excitations from cell to cell. They permit the recognition of patterns that have been stored in the network. Descartes' "*res cogitans*" is not a computer. The brain functions in a completely different fashion from a digital computer in which every bit of information is allocated to a particular storage site.

Neurons fire at intervals of milliseconds, i.e. they emit short signals (action potentials) which lead to transport processes in the ion channels of other neurons which are interconnected to them. Positive ions flow through the interior of the nerve cells and increase the potential there. The basic equation of associative memory considers the rate of firing of each neuron in the network as a function of the potential of that neuron. This equation becomes a closed system when one takes into account that the potential of each neuron itself depends linearly on the rate of firing of other neurons that are providing input to it. Learning means adjusting the strengths of the synapses. Positive and negative synaptical strengths correspond to excitation or damping of neuronal stimulation.

Neurophysiological research attempts to understand how the still-undefined individual content from the various sensory organs is combined into a recognizable pattern or sensory object. Spatial, color or nonvisual—i.e. heard or felt—impressions arrive in different areas of the cerebral cortex and are assembled together in a process which involves the frontal lobes (basal ganglia) and the thalamus (near the brain stem). This synchronizes the various nerve signals. In the case of visual

[40]Singer [37].

information, the synaptic connections take the form of maps, whose topology corresponds to the correlations of the input signals. Gerald Maurice Edelmann[41] has proposed the hypothesis that the reciprocal exchange between such maps provides support for the synchronization of memories. He has developed the concept of *reentry* which he defines as the recursive interchange of signals that occurs in parallel between brain maps, and which continuously interrelates these maps to each other in time and space.

In contrast to the philosophy of the mind, neurophysiology sees consciousness as an indefinite correlate of a dynamic neuronal core structure, which is defined through causal relations. The variability of this core structure permits the formation of "attractors", i.e. patterns towards which the brain gravitates almost independently of the particular initial conditions. Memory content can be considered to comprise such patterns. By creating such attractor patterns, our brains can adapt to the requirements of a permanently changing environment.[42] The freedom of humans thus resides not in the possibility of our "freely" making decisions; it is rather to be found in the individual structuring of our likes and dislikes. Does this creativity also allow the memory to combine indefinite bits of content into scenarios which produce a coherent past? Indeterminacy appears here in individual reconstructible processes whose combined action is beyond our understanding. Wolf Singer asserts that "The dynamic states of billions of neurons in the cerebral cortex display a degree of complexity which is far beyond anything that we can imagine. That does not necessarily mean that we will not be able or are not willing to develop analytical methods which can identify these systemic states and follow their temporal evolution; their description will however be abstract and vague and will bear no similarity to our everyday ideas and concepts, which themselves are based upon the functioning of these neuronal states." Interdisciplinary cooperations can focus on the intersection "consciousness/preconsciousness". Scholars of the humanities and the social sciences investigate and interpret consciousness and its cultural concretions, while neurophysiologists study the preconscious.

At the beginning of the past century, various psychologists including Sigmund Freud, Carl Gustav Jung, Alfred Adler and others were attempting to bring structure into the indefinite aspects of our memory. They started with empirical case histories which they had encountered in their clinical practice as psychotherapists. Freud assumed that a part of our soul, the indefinite Es, lies hidden from the conscious Ego. He called this part the *unconscious* and made it into a fundamental component of psychoanalysis. The Freudian unconscious is accessible to the psychoanalyst only when he can bring it into the conscious mind. The psychiatrist and philosopher Thomas Fuchs[43] commented on the process of internalization from the neurobiological viewpoint: "Every contact with others leaves traces on the neuronal level through synaptic learning; to be sure, not necessarily in the form of localisable

[41]Edelman [38].

[42]Edelman: *Wider than the Sky*, *ibid*., pp. 145 ff.

[43]Fuchs [39].

"memories", "images", or "representations" of the interactions or the contact persons, stored in fixed locations, but rather as dispositions or perceptions, feeling and behavior." In the collective consciousness of groups, individual experiences are generalized and stored with the help of symbols. History deals with such memories and their concretization in historical documents. The options of historians are however limited by the brief horizon of memory. "The true image of the past scurries away rapidly; we can capture it only as a picture which flashes up fleetingly at the moment of its perceptibility and then disappears forever. […] For it is an unrecoverable image of the past that threatens to vanish in every present which fails to recognize itself as reflected in that past."[44]

Closely related to remembering is forgetting. While memory can be investigated in experiments with animals, it is more difficult to study forgetting in that way. Forgetting is not only a weakening of recollections, but is often an active suppression of unpleasant, unwanted memories. The systems analyst Niklas Luhmann considers forgetting to be a kind of indefiniteness which is produced by our memory itself, as a creative act which opens up space for new possibilities of creative choice. Here, we can discern similarities to the collective forgetting by a whole society, which is called into play through disagreeable or painful events. It is however questionable whether such forgetting can lead to a genuine renewal.

On the basis of recent results from brain research, the historian becomes a kind of knowledge archeologist, who is confronted with contradictory recollections. The historian Johannes Fried[45] describes in his book "*Schleier der Erinnerung*" how the two physicists Werner Karl Heisenberg and Niels Bohr retained quite different memories of their meeting in Copenhagen in 1941. From their reports, Fried finds, the meeting took place "at an undetermined time, in an undetermined place, under undetermined circumstances, and dealt with undetermined topics." We are accustomed to the fact that the historical fragments from ancient times often contain little original knowledge, but stand at the origins of well-thought-out stories and legends. History, which deals with memories and recollections, has to accept[46] that these become less and less clear with the passage of time, i.e. they become indistinct. The objects of history are unique; their reconstruction entails indefiniteness.

Indefinite, in the sense of *indistinct*.

Examples

- Calling up memories and recollections;
- Synapses learn dispositions;
- The reconstruction of the historical past.

[44]Benjamin [40].

[45]Fried [41].

[46]Schneidmüller [42].

2.6 Indefinite Ontology

What is the origin of the universe? Is there a reason why the world came into being? I call such questions and their respective indefinite answers *ontological*, since they are related to our image of the whole of the world (all of existence, "being"); they are not limited to just parts of it. Are these questions well defined? As scientists, we are accustomed to treating such questions with caution. In this chapter, I will carry out the "experiment" of comparing the modern physical theory of *cosmogeny*, the formation of the world, with the philosophical thinking of Plato on the foundations of the world. Ancient Greek philosophy has always been of great interest to physicists, since many of our modern concepts, such as that of the atom, first arose at that time and place. I have chosen the ancient philosopher Plato (424-348 B.C.) because the indefinite and the definite played a central role in his thought. The comparison will also illustrate just how differently physics and philosophy go about searching for the origins of the universe.

Let us follow the history of our universe, presumed to be (13.5–14) billion (10^9) years old, to its beginnings, by taking a look backwards; we choose the moment of the Big Bang as the zero point of our time scale. Then we find the birth of our galaxy around a billion years later. Up to 300,000 years following the origin of the universe, it is still opaque; i.e. there are no electromagnetic signals from this early era. At a temperature of 3000 K, a thousand times hotter than the current temperature of interstellar space, electrically neutral atoms can form by the combining of electrons and protons (0 °C corresponds to 273 K, room temperature to 293 K). Light was then free to move outwards to the boundaries of the universe, since its energy was not high enough to ionize the atoms and it was therefore no longer absorbed by them. This is termed the *decoupling* of radiation and matter. At the time the atoms were formed, the light in the universe had a Planck spectrum similar to that of the sun today (corresponding to 5800 K), just somewhat cooler. The earlier we examine the universe, the hotter it was; i.e. the greater the energy scale which plays a role in the processes taking place. The atomic nuclei were formed one second after the Big Bang, at a temperature of 10^{10} K. The neutrinos play an important role in the synthesis of nuclei, and the existence of three light neutrinos in the Standard Model agrees well with the distribution of the elements throughout space. In the early universe, the macroscopic astrophysics of the universe and the microscopic physics of elementary particles act together, since the increasingly high temperatures permit higher and higher energetic excitations. Some microseconds (10^{-6} s) after the Big Bang, the quarks and gluons combined to form nucleons. Going back to 10^{-10} s, we can apply knowledge obtained and tested in laboratory experiments in order to describe the hot fireball of the early universe, which had a temperature at that time corresponding to 100 GeV (10^{15} K). This corresponds to our state of knowledge including the recent discovery of the Higgs boson. One expects that at still higher temperatures, all the known interactions would have been equally strong. Quantum field theories contain virtual processes which renormalize the couplings. Through these corrections, the strength of the couplings varies with resolution or energy, leading to an increase of the weak coupling constants and a decrease of the strong coupling at high energies or temperatures.

The hypothesis of an initial point at which time and space were created, which is in common use by cosmologists today, is controversial in terms of the question of just how singular that beginning really was—was there already a 'time' at the moment of the Big Bang? The higher the energy density of the universe, the stronger the curvature of space and time. At the moment of the Big Bang itself, one can therefore no longer speak of space and time. The quantum effects on gravitation which would then occur are currently not understood.

The cosmic background radiation which was produced at the time when atoms formed can be observed today at a temperature of slightly below 3 K. This low temperature arises from the thousand-fold expansion of space since the time when this radiation appeared. The temperature of the background radiation is constant down to deviations of one part in 100,000, and it thus gives precise information about the density fluctuations within the universe, which gave rise to the galaxies. The measurements indicate a flat universe, with no curvature. How can a flat universe correspond to a solution of the Einstein equations, although that solution is particularly unstable and requires a very carefully chosen initial value for the energy density of the universe? Why is the background radiation so homogeneous? Various spatially separated parts of the universe could hardly have been in close (causal) contact at the moment of decoupling after the Big Bang. Why should they now have the same temperature? The explanation requires a mechanism which is still not fully understood by present-day theory. The Einstein equations permit an exponentially increasing expansion of the universe, in the case that there is a scalar field which evolves with an extremely small kinetic energy, so that its predominantly potential energy can drive the expansion; the latter is then extremely fast. Within 10^{-35} s, the universe would expand by many orders of magnitude. The scalar inflationary field blows up the universe so rapidly that parts of the universe which are widely separated by the time of the formation of atoms could still have been in causal contact with each other. This would explain the extreme homogeneity of the 3 K-microwave radiation.

Cosmologists have estimated from the mass distributions of the spiral galaxies that the energy density of the visible universe is only 4.8 % of the critical mass density required for a flat universe. There must therefore be non-observable matter, known as *dark matter*. It is called dark because it neither reflects nor emits nor absorbs light. From the combined analysis of galactic clusters, supernovae and the microwave background radiation, an upper limit for the percentage of dark matter which contributes to the critical density can be found. The result is that this cold, dark matter contributes 26.2 % to the critical density. The rest (69 %) is attributed to an either weakly time-dependent or time-independent *dark energy*. Albert Einstein introduced a cosmological constant into his original equations of general relativity (to make the universe stable, since its expansion had not yet been observed at the time); this constant can accurately play the role of the dark energy. The value of the cosmological constant can be determined from observations of the accelerating expansion of the universe. Its magnitude and sign are not yet theoretically understood. We have already pointed out this problem in discussing the results of string theory (Sect. 2.1). The increase of our knowledge has led to an increase of the boundaries of that knowledge. The dark, unobserved parts of the universe now constitute 95.2 % of the whole. These results were obtained by satellite measurements from the cosmic microwave spectrum in combination with lensing reconstruction and other external data.[47] Dark matter bends the light rays of galaxies and makes multiple images of the same background source appear. This effect, known as "gravitational lensing", can be used to map out the distribution of dark matter.

Great indefiniteness calls up a need for more knowledge and less uncertainty, and it thus spurs on efforts to reduce the indefiniteness. Precisely this effort to escape from the fog of indefiniteness was the driving force for Plato in his late work "*Philebus*", in which he declares the definite and the indefinite to be fundamental components of the universe. One could find here an analogy to modern physics, which presumes 4.8 % definite and 95.2 % indefinite matter/energy in the universe at present. But let us look deeper into Plato's philosophy.

The following figures play roles in "Philebus": *Socrates*, who asks questions and instructs; the knowledgeable and flexible *Protarchus*; and, in the introduction, *Philebus*. Socrates begins the trialog by citing the thesis of the Hedonist Philebus, to the effect that well-being, pleasure and enjoyment are good for all living beings. He counters this thesis by maintaining that wisdom, correct opinions and true reasoning are more important. The conversation seems to revolve around the proper mixture of wisdom and pleasure. Then suddenly the main theme emerges—there are four original components of being, four elements: the indeterminate (*apeiron*), the determinate (*peras*), a mixture of the two (*meixis*), and the cause of the union of the indeterminate and the determinate (*aition*). The indeterminate can vary gradually and is thus unlimited. "Everything of which we see that it becomes more or less, and strong or weak, and all similar properties, all that must be collected under the category of the unlimited (the indeterminate) in unity." Today, we would speak of *continuous* properties.

That which is defined by a limit, in contrast, is clearly distinguished from its opposite; it has *discrete* properties: "Thus that which does not take on those properties, but rather everything opposite to them, starting with the equal and equality, and after equality twice as much and every whole number in a ratio to numbers, and every measure in a ratio to measures, if we count all that among the limited (the determinate), we would be on the right path".[48]

Plato in "Philebus" explains the indeterminate, which we have called the indefinite, undefined or unknown (cf. Sect. 1.2). He sets the finite and defined as an opposite to the indefinite, as a class of concepts or objects which can be clearly identified and enumerated with numbers. The determinate belongs to a class (paragraph 25c) which puts an end to all differences and opposites.

In the class of the determinate objects, there is no discussion as to whether a particular element belongs to it or not, but rather a clear and unique assignment can

[47]Planck satellite: 2015 results. XIII. Cosmological parameters. Table 4, *astropreprints* 1502.01589.

[48]Plato, *Philebos*, 25b. All the quoted paragraphs correspond to those in the German edition, Hamburg (2007), pp. 442 ff. For the Greek and English versions, see e.g. *Philebus*, http://www.perseus.tufts.edu/hopper/text?doc=Perseus%3Atext%3A1999.01.0174%3Atext%3DPhileb.%3Asection%3D25b. The paragraph numbering is similar.

be made of a first, second and third element of this class (25c). Here we find a veneration of numbers which recalls the presocratic Pythagoras (570-495 B.C.), where numbers determine among other things the fundamental intervals in music. The octave, the fifth and the fourth are defined by the ratios of numbers (2:1), (3:2) and (4:3) on a vibrating string. If one were to change the length of the string continuously, corresponding more or less to the indefinite, there would be no harmonic overtones. “And in the acute and the grave, the quick and the slow, which are unlimited, the addition of these same elements (the determinate, discrete, finite) creates a limit and establishes the whole art of music in all its perfection, does it not?” (26a).

Protarchus develops the discussion further by recognizing that a mixture of the indeterminate with the determinate represents a new formative element: “Thou evidently wish to say that if these two are mixed, certain results are produced in each instance”.[49] Socrates agrees: “And thence arise the seasons and all the beauties of our world, when the unlimited and that which is limited within itself are mixed together” (26a). Plato wants to understand the mixture as a new unit or element; he sees the process “from Becoming to Being” (26d).

Then Socrates begins to speak of the *cause*: “But we said there was, in addition to three classes, a fourth to be investigated. Let us do that together. See whether you think that everything which comes into being must necessarily come into being through a cause” (26e). Physics recognizes causes, which always precede their effects.

> Students of physics encounter *causes* in Newton’s first law of motion, which maintains that a body which is not acted upon by any external forces remains at rest or in a state of uniform linear motion. Later, when they attempt to derive electromagnetic radiation (from Maxwell’s equations), they encounter the principle of causality. This principle states that effects can propagate (from their causes) with at most the velocity of light. Thus in spacetime, only events within the so-called past light cone can influence events in the present; likewise, the absolute future is limited by the future light cone. Since there are very many equivalent inertial frames of reference, special relativity theory states that an event can never be associated with a particular time coordinate. But cause and effect maintain the same temporal order under Lorentz transformations, which conserve the structure of spacetime.

The “Platonic cause” is to be distinguished from physical causes; it can to be sure also play that role within causality, but it has a still deeper significance. ‘Nothing is without a cause’ could be a brief summary of this principle. Here, the final cause (“*causa finalis*”) is meant, as it represents the purpose of the universe. In Plato’s work “*Timaeus*”, Timaeus explains the formation of the world in all its details. At the beginning, he warns: “Wherefore, Socrates, if in our treatment of a great host of matters regarding the Gods and the generation of the Universe we prove unable to give accounts that are always in all respects self-consistent and

[49]Plato, *Philebus*, *ibid*., 25e.

perfectly exact, be not thou surprised".[50] Read naively, fire, earth, water and air become the original components or elements—in complete agreement with traditional ancient cosmology. But the real elements are the forms, namely triangles (53d). In Timaeus, we recognize the theme of Philebus once again: "Midway between the Being which is indivisible and remains always the same and the Being which is transient and divisible in bodies, He (the Demiurge) blended a third form of Being compounded out of the twain" (35a).

Paul Natorp[51] interprets this cosmogeny in terms of the paired but opposite concepts represented by the indeterminate and the determinate. He understands their combination as an 'ensoulment' or a harmonization. In his book "*Platos Ideenlehre*", he explains the fourth element, the *cause*, or *reason*, as an additional ingredient to complement the other three. "Cause is in general none other than the law or the determinant of the indefinite". Natorp explicates the aspects of the Philebus text in poetic language.

Plato's emphasis of the quantitatively indefinite fascinates me, along with its opposite, the mathematically-ordered definiteness. Here, I see a line of development leading to structuralism, which sees symbols as a mixture of significant data, an indefinite object, and theory. In the case of physical and biological concepts and theories, one can readily comprehend[52] how new knowledge improves our definitions of objects. How, though, does the determining element mix with the indeterminate? Is there a Platonic cause for this?

What is the Platonic cause? In this chapter, I have attempted to show that one has to count on indefiniteness, but that one can also count with it. An essential point was the distinction between knowledge (e.g. astronomy) and "interpretation" (e.g. astrology). If we can explain only 4.8 % of the energy density of the cosmos with the Standard Model of elementary particles, then a large amount of the indefinite remains, and it cries out for continued research. These indefinite elements are necessary in order to arrive at a consistent description of the observed flat universe through Einstein's equations. The mixture of definite and as yet indefinite physics is to be found in gravitational theory. The astronomer-physicist is not searching in vain.

The philosopher may want to challenge the cause, or reason: "Is the final word that we can say of being: 'Being means reason'? Or does not the essence of humanity, does not our belonging to being, does not the nature of being yet remain, and they remain still the disconcertingly memorable? Dare we, if it should be so, yield up this most memorable in favor of simply calculating thought and its enormous successes? Or is it not our responsibility to find paths on which our thinking can answer to the memorable, instead of sneaking past it, bewitched by calculating thought? That is the question. It is the universal question of thinking. Its

[50]Plato: *Timaios*, Hamburg 2007, 29c. English: e.g. *Timaeus*, http://www.perseus.tufts.edu/hopper/text?doc=Perseus%3atext%3a1999.01.0180%3atext%3dTim.

[51]Natorp [43].

[52]Pirner [44].

answer will decide what is to become of the Earth and of the existence of humanity on this Earth".[53]

This quote from Martin Heidegger would seem to engender both emptiness and a deep meaning at the same time. Have the determinate and the indeterminate mixed together in his thinking to yield something genuinely new? Or do the definite thoughts which he produced just point to a certain origin and thereby make their own value questionable? I for one cannot make head nor tail of this philosophy, because its association to Being is circumscribed only very vaguely. A more balanced view of Heidegger's involvement with the natural sciences and natural philosophy is given in an article by Joseph Rouse, who states that recent examples in the history of physics "cannot be appropriately regarded as impositions of a predetermined orientation toward calculative control upon nature as a plastic resource, for what it matters to understand collectively and what is at stake has shifted. Such shifts instead reflect an openness within science to allowing things to show themselves intelligibly in new ways".[54] Modern cosmology is yet another example.

Indefinite, in the sense of *undefined*.

Examples

- Dark matter and dark energy in cosmology are (still) undefined;
- The Undefined, the Unlimited in Plato's philosophy;
- The mixture of the indefinite and the definite.

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[53]Heidegger [45].

[54]Rouse [46].

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