

Chapter 2

Language, Fuzzy Logic, Metalogic

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Abstract This article envisages two main goals: (1) It stresses the importance of language not just as an instrument but above all as an atmosphere, an ambience, where we grow up, we learn, we love and hate, we have pain and we pray, we think, we make mistakes and we finally die. That's why language, though it is not governed by arbitrary rules, is not either a rigid and inflexible instrument, since it has to express the situations, emotions and experiences of our daily life. (2) It points out that logic has always tried to establish the fundamental rules of language and reasoning. But logic has not been able, till very recent times, to recognize its own possibility conditions, which build up its sense and strength. There is a dogmatic demand of determinacy of sense which should be avoided. Everyday language very often expresses doubt or disagreement, hesitance, ambiguity or vagueness. Indeed, the ideal of exactness is over. Fuzzy logic makes a fundamental contribution to the understanding of this fact. Any sentence can have an exact or a vague meaning depending on its context and purpose. Context is given by ordinary life which imposes conditions to sense and understanding. Fuzzy logic takes into account the specific conditions governing exactness and vagueness in the use of words.

2.1 Language

I will start with something commonly known, namely that language is the most decisive element in shaping human thoughts and consequently human personality and culture. Through language we communicate and express our ideas and opinions, with language we build our inner world, our beliefs, feelings and hopes. Hence, each language is the privileged place for the most personal and intimate life, and at the same time a means of communication and social achievement, in favour of dialogue, coexistence and peace.

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Our world is a real cluster of languages. Some people resent it and complain about this fact, probably because multiplicity of languages makes their lives a bit more difficult, especially when they go around as tourists. Fortunately there are also people and institutions which defend linguistic pluralism and try to preserve language diversity, even if some individuals or groups are much more concerned with the protection of their own languages than with the protection of all languages without discrimination.

So we can affirm both at once: that linguistic pluralism is a fact, a simple and undeniable fact; and, at the same time, that this fact is not experienced as a great opportunity for building strong and rich cultures, but is merely seen as (perhaps) an interesting cultural fruit of past geographical and political divisions. A strange sense for globalization leads many people to believe that a linguistic unity would be better than linguistic diversity. Those people think in terms of practical management of public life, not in terms of real appreciation of goods and values. Those who want to make some contribution to a better understanding among human beings should not be so naïve as to think that they will be better off if they speak finally the same language. Well-being depends on the capacity of expounding and increasing thoughts and experiences. This is what humans do when they exploit all the possibilities offered to them by the atmosphere of their native or acquired languages.

From what I have said I can draw the following conclusion: linguistic capacity is a literally basic, fundamental capacity, without which humans would not be humans anymore, since they would lose their own way of having thoughts and emotions, of expressing ethical judgments, of making projects, of imagining and of communicating with each other. Each language does all these things in its own way, with all the extraordinary possibilities and amazing means that history and tradition have slowly developed. This richness must be preserved as a fundamental heritage of the human species, which shows, in the dialogue of languages, how unity can be better preserved through an active respect and consideration for linguistic plurality.

In that sense, languages should not be diminished to the level of political weapons, but should be considered at their real human value. I am afraid that there is now a very general opinion according to which a language is just a means (an instrument) for human understanding. And certainly it is. But it is not just this. If language were just a means for understanding, this would justify the attempt to achieve the superiority of one language over the others. In that case, even if many languages were to disappear, the mere existence of one language known by all would be enough guarantee to preserve understanding in human relationships. The idea that language is just an instrument for communication is the idea that lies beneath the lack of respect for language diversity. If we only need to communicate with each other, then we do not need many languages but a single one known by everybody.

But the truth is that language is more than a ladder, more than an instrument, more than a means designed to pass messages from one person to the other. Language is not like a phone, which in case of being out of order, can be substituted by other means like a letter, an e-mail or a carrier pigeon. Language is not just a means but a medium, a milieu, not just an instrument but an atmosphere, an ambiance, where we grow up, we learn, we love and hate, we have pain and we pray, we think, we make mistakes

and we finally die. Language embraces and produces our different forms of life. That is why languages cannot be simply substituted for each other. Each language shows a vision of the world—perhaps even a *Weltanschauung*,—each one produces different ways of interpreting and understanding not just what speakers say but also what they do, how they react, what they feel. Each language is a world, a real world. Of course there are connections between those worlds -that is why we translate texts and books-, but we should not forget that translation is an art, and this means that there are not automatic bridges which connect languages. (Translators know better than anyone else how difficult it is to translate poetry properly, for instance.)

Those who pay attention to these very basic facts will be able to understand two complementary things: (a) that languages are not totally arbitrary devices, since they serve not only to unit communities but also to interconnect different linguistic groups with each other; (b) that languages cannot be rigid but that they are rather very flexible structures able to give way to all sort of situations, emotions and realities. Point (a) explains why mankind has always been interested in logic; point (b) justifies the creation and existence of something called «fuzzy logic».

2.2 Logic

Nineteenth century logic was still a very much philosophico-hegelian science, a philosophical discipline which moved between metaphysics and psychology. A new logic, based on strict calculus (sometimes called «mathematical logic»), appears firmly in the twentieth century. Nevertheless, if we look at the classical logic—the one going from Aristotelian logic to nineteenth century logic—and compare it with the new logic of the twentieth century, we easily realize that both, despite their technical differences, have been cultivating extensional logic, i.e. both have being concerned with the exactness of logical predicates they used. Some authors—as Frege and Russell—tried to find exactness through the creation of ideal languages. According to them, science feels uncomfortable with vagueness and needs absolute precision in the use of its terms. Since ordinary language is not useful for the purpose of exactness, it is necessary to build up ideal languages to ensure it.

Other authors, however, looked after exactness through the analysis of natural language. They considered that the surface of natural (everyday) language is vague, but that after closer analysis of our sentences we will eventually discover that exactness is embedded in their deep structure. This is, in fact, the idea proposed by Wittgenstein in his *Tractatus*. His main objective was to isolate all logic vagueness, ambiguity and inexactness. In fact, the idea of exactness has always been part of the bedrock of theoretical knowledge considered as higher knowledge. That is why sciences -including logic and mathematics- have until recently focused their attention on the investigation of fixed entities, permanent, unchanging and almost unchangeable realities.

Predicate logic has also been speaking of «classical (or fregean) predicates» to refer to predicates which only admit two truth values: true and false (0 and 1), with no degrees or shades of any kind. This means that, for any object whatsoever, it is always

decidable whether the object falls or does not fall under the concept expressed by the predicate, as it is shown in examples like «being a Berlin resident» or «being twenty-one years old». This is clearly a binary logic, one of «yes» or «no»; any third possibility is totally excluded from it.

During the twentieth century, mathematics, thanks in part to statistics and probability theory, has expanded its traditional concern and has finally applied to humanities and social sciences. Concern for accuracy has been losing ground in front of the idea of approximation. Mathematical results are no longer measured by their alleged certainty but by their degree of probability. It was also during the twentieth century that logics have been developed accepting more than two truth-values: that was the reason of the appearance of trivalent, tetravalent or, generally, multivalued logics. However, not even those logics can adequately cope with such a basic and general phenomenon as vagueness, which fills ordinary language, our expressions and our reasoning.

2.3 Fuzzy Logic

«Vague» or «fuzzy» predicates do not admit a crisp classification of objects, as it is seen in examples like «being bald», «being rich», «being healthy». Frege hated these predicates because, as already said, he was seeking rigorous exactness without which, according to him, science would not work. In fact, a vague predicate opens an interval of infinite gradation (infinitely divisible between 1 and 0). Ordinary language reflects these gradations, these nuances, and logic has to cope with them if it wants to give real answers to the challenges of everyday life. Nevertheless, when logic does so, the expression «fuzzy logic» may lead to confusion, because it is not logic which is fuzzy but the use of language analyzed by logic. What fuzzy logic is precisely about to demonstrate is that fuzzy predicates can be treated with mathematical rigor if you quantify them correctly. «High» is a vague predicate, but people are 1.70, 1.80, 1.90 m. high, which are very precise heights. Fuzzy logic has to specify the degree of highness of those people since it evaluates each case within the range between 0 and 1.

Here I would like to mention again Wittgenstein's position, since he had a pre-eminent saying in analytic philosophy when fuzzy logic was officially born. Indeed, he showed an own view on vagueness both in the *Tractatus* (1921) and in the *Philosophical Investigations* (1953). These works are often seen as equally maintaining that vagueness is an essential feature of language. According to this interpretation, Wittgenstein's work became an inspiration for some attempts to construct or justify a logic of vagueness. But this was not exactly Wittgenstein's position, since he did not want to construct *a logic* of anything, neither of exactness nor of vagueness. We have seen that the *Tractatus* tried to supersede vagueness by a proper analysis of sentences. Later on, Wittgenstein's turn to everyday language did not mean the acceptance of ordinary language without more ado. Wittgenstein doesn't want to *promote* vagueness; he merely resists the dogmatic demand of determinacy of sense,

that is, he tries to resist the insistence that the *possibility* of doubt or disagreement about the application of an expression must be eliminated. This is a crucial point in Wittgenstein's approach to vagueness and to philosophy. In fact, he tries to distort the ideal of exactness. On this line we can establish the following points:

- (a) There is no single ideal of exactness. The contrast between exact and inexact is relative to a context and a purpose (e.g., whether we are measuring our distance to the sun or the length of a table). An inexact definition is not one which fails to meet the elusive ideal of determinacy, but one which fails to meet the requirements of understanding in a given context.
- (b) No explanation could avert all possibility of indeterminacy, since no system of rules can budget for the countless bizarre possibilities in advance.¹
- (c) Even if vagueness is considered a defect, a proposition with a vague sense still has a sense, just as a vague boundary is still a boundary. If there is only one gap in an enclosure, it is determined that there is only one way out (a fly-bottle is a trap, although there is a way out). For a concept to be useful, all that is required is that it is defined for some cases, so that some things would definitely fall under it, and others definitely would not.
- (d) One might respond in the spirit of the *Tractatus* that although the rules may allow a certain degree of elasticity, that degree must itself be determinate: there may be borderline cases, but it must be exactly determined what counts as such a case. However, this idea leads to a vicious regress. If we try to make the limits of an area more precise by drawing a line, that line has a breadth. If we try to avoid this by using the colour-edge of the line, the only way of determining what counts as overlapping this exact boundary is to draw another line, etc.²

As we can see, Wittgenstein is not working on a fuzzy logic but acts as an objective ally since he accepts limits to exactness and evaluates their importance. Exactness and inexactness (or vagueness) give sense to each other. Wittgenstein's example is important not just for his enormous influence in the history of thought but also because it shows that the future of fuzzy logic does not depend only on its technical improvements but also on the alliances and on the new ranges and applications it can win. These applications are very important when they belong to the domains of linguistics, psychology, literary criticism, ethics or political science.

2.4 Metalogic

Let's now ask the following question: when we talk about «fuzzy logic», are we entering into a new field, into some kind of second level order, from which we will be able to clarify logical problems and prepare new routes? To put it in a more already classical terminology: can we accept the existence of something

¹Cf. Ludwig Wittgenstein: *Philosophical Investigations*, §§80, 84–87.

²Cf. id. *Philosophical Investigations*, §88; Zettel, §§441–442.

called «metalogic»? Carnap's *Logical Syntax of Language* (1937)³ ascribes the origin of the term «metalogic» to the Warsaw logicians. At present, this term is used to refer to second-order reflections about logic (e.g., to proofs of soundness and completeness). In agreement with the position of Wittgenstein on this matter, I would oppose to the necessity of accepting metalogic. We refer to our logical work using terms which can be considered «formal concepts», like «proposition», «name», «predicate», «function», etc. They are used to explain what we do and the terms we use. But this doesn't mean that they have an extra-ordinary status within our logical system. They are simply terms or concepts which help us to explain things, but they do not belong to the signs system. They are of a different order (since strictly speaking we don't use «propositions» but «p», «q», etc.), but they are not of a second-order as if they were of a higher order.

A reflection is a reflection, no matter which sequential order they take. There is no need to accept a second-order logic which allegedly would be more comprehensive and better articulated than the previous one. We can expect no gain derived from such an acceptance. Logic determines what is necessary, but there is no metalogic which makes logic necessary to control previous levels. In fact, all concepts which philosophy uses in describing ordinary language are themselves ordinary. The same happens with logical concepts, which are, all of them, at the same level.

When I talk about language (words, sentences, etc.) I must speak the language of every day. Is this language somehow too coarse and material for what we want to say? Then *how is another one to be constructed?*—And how strange that we should be able to do anything at all with the one we have! In giving explanations I already have to use language full-blown (not some sort of preparatory, provisional one); this by itself shows that I can adduce only exterior facts about language.⁴

This does not exclude that fuzzy logic—or logic *tout court*—may offer many interesting aspects of discussion and reflection. In many cases logic uses terms and concepts which are also important in other areas of knowledge. This is the case, for instance of «ambiguity» («good life depends on a liver»), that affects semantics, pragmatics and particularly rhetoric, or «vagueness» («John is bald»), that affects also semantics, but it is less related to pragmatics and more linked to syntax and logic; for that reason it is an absolutely central concept for fuzzy logic.

There is still another term which deserves a major attention, but which in the shortness of this text it will only be treated—as it has been done with other terms—in a rather informal way. I am referring to «uncertainty» or «uncertain». Language is a field without walls or doors. Indeed, the borders between the uses of language are always tenuous and vague, but we can also say that they are uncertain. In fact, we often live in the uncertainty (which is not simply a doubt), for instance when we do not know if we make good use or misuse of some terms. This indicates that there are at least two major uses of the word «uncertain»: our uses of terms may not only be vague but also uncertain, and we may of course feel uncertain about things. If I say «the epidemic outbreak caused by contaminated food is uncertain», I'm

³Translation from *Logische Syntax der Sprache*, 1934.

⁴Ludwig Wittgenstein: *Philosophical Investigations*, §120.

saying it fails to show clear profiles. This would be «objective uncertainty». But I can also say that I am uncertain about some situation, for example when I say «my involvement in the business is uncertain», which means that it is not safe, that I did not yet take a final decision on it. This might be called «subjective uncertainty».

And still we can make a third use of the term «uncertain» which I will call «amphibological», which consists in having both at once (that a fact is uncertain and that I feel uncertain about a fact), that is, when the objective and subjective uncertainty match. This happens, for example, when some experts involved in a political debate say that «the impact of the broadcasted debate is very uncertain». They mean the impact itself is staggering, unequal, that it depends on small, unpredictable, almost imperceptible details, and yet they also want to say that they are unsure about what they do, about what they should do to positively influence the electorate. (This amphibological character is also a special case of ambiguity: indeed, an expression is amphibological when it has different meanings at once, that is when it cannot simply have them and can play with them. Typically, the ambiguity is unique and demands an alternative interpretation, as seen in the example given above. The term «uncertain», however, admits an amphibological use, i.e. it allows a simultaneous interpretation of its ambiguity, but doesn't impose it.)

The subjective component of «uncertain» has surely provoked that it had little interest in fuzzy logic. But the very fact that uncertainty has sometimes a psychological component of subjective nature confirms the interest shown in its analysis by scholars of the humanities and social sciences. Indeed, it seems to me uncontroversial that in the future it will be crucial to give to psychological uncertainty the importance it has been given so far to vagueness in physics and biology.

I'm not saying that logic returns to psychology. I'm only proposing that we try to model and analyze our psychological capacities without having to accept the behaviorist model, which relies on the analysis of behavior without sufficiently examining the linguistic terms of its analysis. Terms like «certain» and «uncertain» (which don't imply respectively «knowledge» or «ignorance») are crucial to that purpose.

This brings me to say that, in this twenty-first century, logic researchers are able to make some connected statements that surely they could not have made some years ago. I will organize those statements around two important points:

1. *Language, considered in abstract terms, is neither exact nor vague.* Any sentence can have an exact or a vague meaning depending on its context and purpose. Vagueness is not an exception in language but a common possibility. I do not mean that there is an essence of vagueness in language but that any term can be used or interpreted in a vague sense, not previously foreseen by yourself or others. This doesn't bring us to paralysis. Vagueness works because we constantly try to narrow it for understanding purposes. Whenever we fail or are wrong, we have self-correcting mechanisms. That's why logic and analysis of language must be increasingly flexible and subtle.
2. *Exactness is the limit of vagueness.* Vagueness, on its turn, has also its limits. That is why we often accept many expressions as adequately accurate even if we

know that they could be interpreted vaguely. This means that in our everyday life we pragmatically accept the limits of the interval between 0 and 1.

It is, therefore, important to reverse the classical assumption according to which vagueness is merely the limit of exactness. This is an important change of perspective which allows me to point out to some consequences which are relevant for contemporary culture and which go far beyond the field of mathematics and logic. I will briefly mention only six of those consequences:

- (a) The theoretical shift towards understanding inexactness means a radical change of perspective in the way humans look at reality. Therefore, we are not facing a new theory, a new hypothesis or a new work project. This is rather a Copernican turnaround that has already begun to have huge consequences for industry and technology, but also for domestic life, education, medicine and the rest of our lives.
- (b) The shift towards understanding vagueness and inexactness redefines the epistemological optimism of many modern scientists and thinkers. They thought they might arrive to exact knowledge of everything, being able to capture everything with precision and accuracy. Sometimes they even assumed that what they could not understand with total precision had no interest or was worthless. Knowledge has now become cautious. In the past exactness had been linked with knowledge of reality. Inexactness and vagueness are now only linked with our way of speaking about reality, without any ontological claim, without any attempt to know if reality is accurate or inaccurate, simply because we just talk about it. This looks like a more humble position but, paradoxically, it is more «realistic».
- (c) The shift towards understanding inexactness also redefines the concept of description. What does it mean now the attempt «to describe reality»? Over a hundred years ago nonfigurative art started this debate and questioned the notion of plain description. Indeed, facts accept multiple descriptions. It is precisely the kind of descriptions people use what shows us whether they are describing the same thing in a different way or whether they are talking about completely different things.
- (d) The binomial true/false has lost its static and abstract character. Strictly speaking, we should only refer to truth or falsity in terms of degree or range. To assign a degree allows us to be both flexible and rigorous, because in assigning a degree of truth we are also assigning a degree of falsity or of ignorance. This is not a defense of relativism—which only makes sense when opposed to absolutism—, but it highlights the relational nature of truth, since nothing is true or false in itself or in an abstract way, but always with respect to some criterion and within some system or framework. A new problem arises when we ask whether we can compare different frames of reference with each other. This is a capital issue in social sciences, when they are about to make, for example, intercultural statements.
- (e) No ideal of exactness can be laid down. Such an ideal have only been supported by the belief that what is not predictable is irrelevant. But an

inexact predicate is not useless; on the contrary we can obtain from it a lot of very relevant information. Consider, for example, weather forecasts which, though often very inaccurate, provide valuable information. After all, we establish, not always in a very accurate way, our ideals of exactness depending on time, subject, people and age. We should always ask: «exactness what for?» The lack of exactness cannot be used for blame. The lack of rigor should certainly be, because you can be very rigorous in your treatment of inexactness.

In the field of ethics this is very important. Let's see what happens in a hospital: a doctor has no longer to ask what can be done to save a life. Now doctors have to ask what should be done, because they have many resources available. Now you must choose and have preferences and priorities. Usually this has not been the case before.

- (f) The shift towards understanding inexactness makes explicit the approach of science to society. Fortunately science has abandoned its selfish dogmatism. Now we are confronted with enormous possibilities which remain open to analysis and treatment in large areas of knowledge. This happens in a far more vivid form in those areas which, due to its elasticity and flexibility, had been virtually marginalized by a too narrow logico-mathematical perspective.

Indeed, nothing is negligible in principle. In a parody of Pascal we might say that, if the heart has reasons that our head does not understand, then our head has to do a greater effort. Or you can say with Goethe: «For those who think they know everything, almost everything is ridiculous. For those who are reasonable, almost nothing is.» Leibniz expresses the same thought schematically, i.e., in a sentence which is clear and vague at the same time: «Je ne méprise presque rien.»

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