Chapter 2
Cosmology of Massive Gravity in the Decoupling Limit

In this chapter we will aim the ambitious task of addressing the burdensome problems of cosmology using the framework of massive gravity. More concrete, we will try to answer the questions of whether or not massive gravity can produce a theoretically reliable self-accelerated geometry and whether or not it can also resolve the cosmological constant problem. On these grounds, we will first dare to tackle these problems in a certain approximation of the full theory. This approximation manifest itself in a way such that the helicity ± 2, ± 1, and helicity-0 modes of the massive graviton decouple from each other in the linearized theory, constituting the so-called decoupling limit of massive gravity. The nonlinear self-interactions, and interactions between these modes, are encoded in a few leading higher-dimensional terms in the Lagrangian, as we introduced in detail in Sect. 1.2. Within this approximation we will study two branches of solutions, in which the graviton can

- either form a condensate whose energy density sources self-acceleration,
- or form a condensate whose energy density compensates the cosmological constant.

In the following we will successfully show that it is indeed possible to construct self-accelerated solutions in the decoupling limit of massive gravity. The acceleration is due to a condensate of the helicity-0 field of the massive graviton, which in the decoupling limit is reparametrization invariant as we showed in Sect. 1.2. At this point, it is worth to emphasize again that the helicity-0 field of the massive graviton is not an arbitrary scalar field like in the Quintessence models, since it descends from a full-fledged tensor field. This is the reason why it has no potential interactions, but enters the Lagrangian via very specific derivative interactions fixed by symmetries de Rham and Gabadadze (2010). These derivative interactions induce an effective negative pressure causing the accelerated expansion. We will show that the fluctuations on top of this self-accelerating background are stable.

A rather unexpected result from this branch of self accelerating solutions is enunciated clearly in the fact that from the observational point of view, the obtained self-accelerating background is indistinguishable from that of the Standard Model of cosmology, the ΛCDM model, at leading order. This result was indeed very surprising since we expected that the helicity-0 could have introduced some differences

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to the fluctuations and consequently to the evolution of the fluctuations. Essentially, we would have expected that the helicity-0 field could give rise to an additional force at cosmological distance scales modifying the growth and distribution of structure [as it was for instance the case for the helicity-0 mode of the DGP model (Lue et al. 2004; Scoccimarro 2009; Chan and Scoccimarro 2009; Afshordi et al. 2009)], while at shorter scales still being strongly screened via the Vainshtein mechanism (Vainshtein 1972). This would then guarantee the recovery of General Relativity with small departures (Vainshtein 1972; Deffayet et al. 2002), that on the other hand may result in measurable small changes (Dvali et al. 2003; Lue and Starkman 2003) in high-precision Laser Ranging experiments. This is exactly what happens in the self accelerating branch of the DGP model. Anyhow, this does not happen on the self-accelerated background in the massive gravity theory in the decoupling limit. Contrary to expectations, the fluctuation of the helicity-0 field of the massive graviton on top of this self accelerating background decouples in the linearized approximation from an arbitrary source. Consequently, the astrophysical sources will not excite this fluctuation, giving rise exactly to the same ΛCDM results. We should emphasize here that this similarity of the self-accelerated solution and its fluctuations to the ΛCDM results hold in the decoupling limit and it could be that it will not hold beyond this limit.

In addition to the self accelerating branch, we will also show that massive gravity can indeed tackle the cosmological constant problem successfully, avoiding S. Weinberg's no-go theorem (Steven 1989). We will proceed as follows: We will accept a large vacuum energy and show that it gravitates very weakly (Dvali et al. 2002, 2003). The large vacuum energy will not manifest itself as strongly as naively anticipated in General Relativity, i.e. it will be *degravitated*, while all the astrophysical sources will still exhibit the General Relativity behavior (Arkani-Hamed et al. 2002). Strictly speaking, one can think of degravitation as a promotion of Newton's constant to a high pass filter operator thereby modifying the effect of long wavelength sources such as a cosmological constant while recovering General Relativity on shorter wavelengths (Dvali et al. 2002, 2003; Arkani-Hamed et al. 2002). Theories of massive and resonance gravitons are particularly adequate for exhibiting the high pass filter behavior to degravitate the cosmological constant (Dvali et al. 2002, 2003) since they are infra-red modifications of General Raltiviy, meaning that they modify the effects of long wavelength sources. Moreover, it was shown in Dvali et al. (2007) that any causal theory that can degravitate the cosmological constant is a theory of massive gravity or resonance gravitons.

It is important to emphasize that in theories of massive gravity degravitation is a causal process. The real measure of whether or not a source is degravitated is given by its time evolution. During inflation for instance, the vacuum energy driving the acceleration of the Universe will not be degravitated for a long time. It is only after long enough periods of time that the IR modification of gravity kicks in and can effectively slow down an accelerated expansion (Dvali et al. 2002, 2003; Arkani-Hamed et al. 2002). Hence, a crucial ingredient for the degravitation mechanism to work is the existence of a (nearly) static solution in the presence of a cosmological constant towards which the geometry can relax at late time (or after some long period of time). Indeed, Dvali et al. (2007) studied linearized massive gravity demonstrating that in this approximation degravitation takes place after a long enough period of time.

We will successfully show that massive gravity accommodates static solutions while evading any ghost issues at least in the decoupling limit. We will illustrate that in this framework an arbitrary vacuum energy can be neutralized by the effective stress-tensor of the helicity-0 component of the massive graviton. Furthermore, we will put constraints on the two parameters of the theory by demanding the small fluctuations around the degravitating solution to be stable.

An intriguing result we find is that the energy scale at which the interactions of the helicity-0 modes become highly nonlinear is affected by the scale of the degravitated cosmological constant. To be more precise, the interaction scale is higher for larger values of the cosmological constant. Unfortunately, this phenomenon creates a problem by postponing Vainshtein's recovery of General Relativity to shorter and shorter distance scales. As a result, the tests of gravity impose a stringent upper bound on the vacuum energy that can be degravitated in this framework without conflicting measurements of gravity. Disappointingly, this upper bound turns out to be of the same order as the critical energy density of the present-day Universe, $(10^{-3}\ \mathrm{eV})^4$—the value that does not need to be degravitated.

In spite of this low upper bound on the vacuum energy, let us emphasize that there still are two important virtues of the degravitating solution with the low value of the degravitated Cosmological Constant we find here:

- It is a concrete example of how degravitation could work in four-dimensional theories of massive gravity without giving rise to ghost-like instabilities.
- The degravitated solution with small values of cosmological constant can be combined with the self-accelerated solution, to give a satisfactory solution that is in agreement with the existing cosmological and astrophysical data.

Last but not least, the solutions found in the decoupling limit do not necessarily imply the existence of the solutions with identical properties in the full theory. Nevertheless, the decoupling limit solutions should capture the local dynamics at scales well within the present-day Hubble four-volume, as argued in Nicolis et al. (2009). On the other hand, at larger scales the full solutions may be very different from our ones. These differences would kick in at scales comparable to the graviton Compton wavelength. Therefore, our solutions should manifest themselves at least as transients lasting long cosmological times.

As we introduced in detail in Sect. 1.2 the dRGT theory of massive gravity reduces in the decoupling limit to the following interactions for the helicity-2 and helicity-0 components of the massive graviton (de Rham and Gabadadze 2010)

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi], \tag{2.1}$$

where the first term represents the usual kinetic term for the helicity-2 field with $(\mathcal{E}h)_{\mu\nu}$ denoting the linearized Einstein operator acting on $h_{\mu\nu}$ defined in Eq. 1.27,

$a_1 = -1/2$, and $a_{2,3}$ are two arbitrary constants, related to the two parameters from the set $\{c_i, d_i\}$ which characterize a given ghostless theory of massive gravity.

The three symmetric tensors $X^{(n)}_{\mu\nu}[\Pi]$ are composed of the second derivative of the helicity-0 field $\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi$. In order to maintain reparametrization invariance of the full Lagrangian the tensors $X^{(n)}_{\mu\nu}[\Pi]$ should be identically conserved. These properties uniquely determine the expressions for $X^{(n)}_{\mu\nu}$ at each order of non-linearity. The obtained expressions agree with the results of the direct calculations of de Rham and Gabadadze (2010). A convenient parametrization for the tensors $X^{(n)}_{\mu\nu}$ which we adopt in this chapter is as follows:

$$
\begin{aligned}
X^{(1)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^{\beta}{}_{\rho\sigma}\Pi_{\alpha\beta}, \\
X^{(2)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_{\gamma}\Pi_{\alpha\beta}\Pi_{\rho\sigma}, \\
X^{(3)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma\delta}\Pi_{\alpha\beta}\Pi_{\rho\sigma}\Pi_{\gamma\delta}\,.
\end{aligned}
\tag{2.2}
$$

The Lagrangian in the decoupling limit (2.1) represents the *exact* Lagrangian in the sense that it has a finite number of interactions.[1] All the terms higher than quartic order vanish in this limit, making (2.1) a unique theory to which any nonlinear, ghostless extension of massive gravity should reduce in the decoupling limit (de Rham and Gabadadze 2010). Moreover, note that the stress-tensor of external sources only couple to the physical metric $h_{\mu\nu}$. In the basis used in (2.1) there is no direct coupling of π to the stress-tensors. Therefore the Lagrangian (2.1) is invariant with respect to the shifts and the galilean transformations in the internal space of the π field, $\partial_\mu \pi \to \partial_\mu \pi + b_\mu$, where b_μ is a constant four-vector. The latter invariance guarantees that there is no mass nor potential terms generated for π by the loop corrections.

The tree-level coupling of π to the sources arises only after diagonalization: The quadratic mixing $h^{\mu\nu}X^{(1)}_{\mu\nu}$, and the cubic interaction $h^{\mu\nu}X^{(2)}_{\mu\nu}$, can be diagonalized by a nonlinear transformation of $h_{\mu\nu}$, that generates the following coupling of π (de Rham and Gabadadze 2010)

$$
\frac{1}{M_{Pl}}\left(-2a_1\eta_{\mu\nu}\pi + \frac{2a_2\partial_\mu\pi\,\partial_\nu\pi}{\Lambda_3^3}\right)T^{\mu\nu}\,. \tag{2.3}
$$

Moreover, the above transformation also generates all the Galileon terms for the helicity-0 field, introduced in a different context in Nicolis et al. (2009). In this approximation of massive gravity the coupling of the Galileon field to matter after diagonalization is not only given by $\pi\, T$ as considered in the original Galileon theory (Nicolis et al. 2009), but also includes more generic derivative mixing of the form $\partial_\mu \pi\, \partial_\nu \pi\, T^{\mu\nu}$.

Since the Galileon terms are known to exhibit the Vainshtein recovery of General Relativity at least for static sources (Nicolis et al. 2009), so does the above theory with

[1] Recall that we excluded the helicity-1 part since it only appears quadratically.

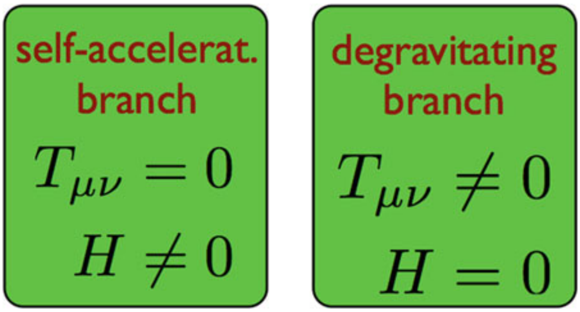

Fig. 2.1 There are two branches of solution: the self-accelerating solution in which the graviton forms a condensate sourcing the acceleration and the degravitating solution where the graviton forms a condensate whose energy density compensates the cosmological constant

$a_3 = 0$. The quartic interaction $h^{\mu\nu}X^{(3)}_{\mu\nu}$, however, cannot be absorbed by any local redefinition of $h_{\mu\nu}$. It is still expected though to admit the Vainshtein mechanism.

However, as we will show in the next section, on the self-accelerated background the fluctuation of the helicity-0 field decouples from an arbitrary source, making the predictions of the theory consistent with General Relativity already in the linearized approximation. This decoupling is a direct consequence of the self-accelerated background and the specific form of the coupling (2.3).

In the following we will explicitly study the two branches of solutions in the decoupling limit: the self-accelerating solution and the degravitating solution (Fig. 2.1).

2.1 The Self-Accelerated Solution in the Decoupling Limit

The universality of the decoupling limit Lagrangian (2.1) for the class of ghostless massive gravities, suggests the possibility of a fairly model-independent phenomenology of the massive theories that should be captured by the limiting Lagrangian (2.1). In the present section, we will be interested in the cosmological solutions in these theories. We will directly work in the decoupling limit, which implies scales much smaller than the Compton wavelength of the graviton. In the case of the self-accelerated de Sitter solution for instance, this corresponds to probing physics within the Hubble scale, which as one would expect, is set by the value of the graviton mass.

2.1.1 Self-Accelerating Background

Below we look for homogeneous and isotropic solutions of the equations of motion that follow from the Lagrangian (2.1). The helicity-0 equation of motion reads as follows:

$$\partial_\alpha\partial_\beta h^{\mu\nu}\left(a_1\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^\beta{}_{\rho\sigma}+2\frac{a_2}{\Lambda_3^3}\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^{\beta\gamma}{}_\sigma\Pi_{\rho\gamma}\right.$$
$$\left.+3\frac{a_3}{\Lambda_3^6}\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^{\beta\gamma\delta}\Pi_{\rho\gamma}\Pi_{\sigma\delta}\right)=0\,, \tag{2.4}$$

while variation of the Lagrangian w.r.t. the helicity-2 field gives

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}+\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi]=0. \tag{2.5}$$

We are primarily interested in the self-accelerated solutions of the system (2.4)–(2.5). This solution is obtained by choosing the configuration for π such that the second factor in (2.4) vanishes. This has for consequence to kill the first order mixing between $h_{\mu\nu}$ and π and hence the coupling of π to matter at leading order (which arises after diagonalization of the mixing term). As a consequence the perturbations around the self-accelerated solution we obtain here do not couple to matter. This will be presented in more details in what follows.

For an observer at the origin of the coordinate system, the de Sitter metric can locally (i.e., for times t, and physical distances $|\mathbf{x}|$, much smaller than the Hubble scale H^{-1}) be written as a small perturbation over Minkowski space-time Nicolis et al. (2009)

$$ds^2=[1-\frac{1}{2}H^2x^\alpha x_\alpha]\eta_{\mu\nu}dx^\mu dx^\nu. \tag{2.6}$$

The linearized Einstein tensor for the metric (2.6) is given by

$$G^{\mathrm{lin}}_{\mu\nu}=\frac{1}{M_{\mathrm{Pl}}}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}=-3H^2\eta_{\mu\nu}. \tag{2.7}$$

For the helicity-0 field we look for the solution of the following isotropic form

$$\pi=\frac{1}{2}q\Lambda_3^3x^\alpha x_\alpha+b\Lambda_3^2t+c\Lambda_3\,, \tag{2.8}$$

where q, b and c are three dimensionless constants.

The equations of motion for the helicity-0 and helicity-2 fields (2.4)–(2.5), therefore, can be recast in the following form

$$H^2\left(-\frac{1}{2}+2a_2q+3a_3q^2\right)=0, \tag{2.9}$$

$$M_{\mathrm{Pl}}H^2=2q\Lambda_3^3\left[-\frac{1}{2}+a_2q+a_3q^2\right]. \tag{2.10}$$

Solving the quadratic equation (2.10) for q (for $H \neq 0$), we obtain the Hubble constant of the self-accelerated solution from (2.10). Its magnitude, $\mathrm{H}^2 \sim \Lambda_3^3/\mathrm{M_{Pl}} = \mathrm{m}^2$, is set by the graviton mass, as expected (positivity of H^2 is one of the conditions that we will be demanding below). It is not hard to convince oneself that there exists a whole set of self-accelerated solutions, parametrized by a_2 and a_3. This range, however, will be restricted further by the requirement of stability of the solution, which is the focus of the next section.

2.1.2 Small Perturbations and Stability

Here we investigate the constraints that the requirement of stability imposes on a possible background. Let us adopt a particular solution of the system (2.9)–(2.10) and consider perturbations on the corresponding de Sitter background

$$\mathrm{h}_{\mu\nu} = \mathrm{h}^{\mathrm{b}}_{\mu\nu} + \chi_{\mu\nu}, \quad \pi = \pi^{\mathrm{b}} + \phi, \tag{2.11}$$

where the superscript b denotes the corresponding background values, and ϕ here stands for the perturbation of the helicity-0 mode. The Lagrangian for the perturbations (up to a total derivative) reads as follows

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + 6(\mathrm{a}_2+3\mathrm{a}_3\mathrm{q})\frac{\mathrm{H}^2\mathrm{M_{Pl}}}{\Lambda_3^3}\phi\Box\phi - 3\mathrm{a}_3\frac{\mathrm{H}^2\mathrm{M_{Pl}}}{\Lambda_3^6}(\partial_\mu\phi)^2\Box\phi\\ &+\frac{\mathrm{a}_2+3\mathrm{a}_3\mathrm{q}}{\Lambda_3^3}\chi^{\mu\nu}\mathrm{X}^{(2)}_{\mu\nu}[\Phi] + \frac{\mathrm{a}_3}{\Lambda_3^6}\chi^{\mu\nu}\mathrm{X}^{(3)}_{\mu\nu}[\Phi] + \frac{\chi^{\mu\nu}\mathrm{T}_{\mu\nu}}{M_{\mathrm{Pl}}},\end{aligned} \tag{2.12}$$

where Φ denotes the four-by-four matrix with the elements $\Phi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$. The first term in the first line of the above expression is the Einstein term for $\chi_{\mu\nu}$, the second term is a kinetic term for the scalar, and the third one is the cubic Galileon. The second line contains cubic and quartic interactions between $\chi_{\mu\nu}$ and ϕ, which are identical in form to the corresponding terms in the decoupling limit on Minkowski space-time (2.1). None of these interactions therefore lead to ghost-like instabilities (de Rham and Gabadadze 2010), as long as the ϕ kinetic term is positive definite. Most interestingly, however, there is no quadratic mixing term between χ and ϕ in (2.12), i.e. there is no mixed term like $\chi^{\mu\nu}\mathrm{X}^{(1)}_{\mu\nu}$. Since it is only $\chi_{\mu\nu}$ that couples to external sources $\mathrm{T}_{\mu\nu}$ in the quadratic approximation, then there will not be a quadratic coupling of ϕ to the sources generated in the absence of the quadratic $\chi - \phi$ mixing. Therefore, for arbitrary external sources, there exist consistent solutions for which the fluctuation of the helicity-0 is not excited, $\phi = 0$. On these solutions one exactly recovers the results of the linearized General Relativity. The above phenomenon provides a mechanism of decoupling the helicity-0 mode from arbitrary external sources! This mechanism is a universal property of the self-accelerating solution in ghostless massive gravity.

Hence, there are no instabilities in (2.12), as long as $a_2 + 3a_3 q > 0$. The latter condition, along with the requirement of positivity of H^2, and the equations of motion (2.10), requires that the following system be satisfied:

$$-\frac{1}{2} + 2a_2 q + 3a_3 q^2 = 0,$$
$$M_{Pl} H^2 = 2q\Lambda_3^3 \left[a_2 q + a_3 q^2 - \frac{1}{2} \right] > 0, \quad a_2 + 3a_3 q > 0,$$

for the self-accelerating solution to be physically meaningful. The above system can be solved. The solution is given as follows

$$a_2 < 0, \quad -\frac{2a_2^2}{3} < a_3 < -\frac{a_2^2}{2}, \tag{2.13}$$

while the Hubble constant and q are given by the following expressions

$$H^2 = m^2[2a_2 q^2 + 2a_3 q^3 - q] > 0, \quad q = -\frac{a_2}{3a_3} + \frac{(2a_2^2 + 3a_3)^{1/2}}{3\sqrt{2}a_3}. \tag{2.14}$$

It is clear from (2.13), that the undiagonalizable interaction $h^{\mu\nu} X^{(3)}_{\mu\nu}$ plays a crucial role for the stability of this class of solutions: All theories without this term (i.e. the ones with $a_3 = 0$) would have ghost-like instabilities on the self-accelerated background.

We therefore conclude that there exists a well-defined class of massive theories with the parameters satisfying the conditions (2.13), which propagate no ghosts on asymptotically flat backgrounds, and also admit stable self-accelerated solutions in the decoupling limit.

As seen from the decoupling limit Lagrangian (2.1), the helicity-0 mode π provides an effective stress-tensor that is felt by the helicity-2 field:

$$\begin{aligned} T^{\pi}_{\mu\nu} &= M_{Pl} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] \\ &= -6q M_{Pl} \Lambda_3^3 \left[-\frac{1}{2} + a_2 q + a_3 q^2 \right] \eta_{\mu\nu}\,. \end{aligned} \tag{2.15}$$

It is this stress-tensor that provides the negative pressure density required to drive the acceleration of the Universe. Supplemented by the matter density contribution, it leads to the usual ΛCDM—like cosmological expansion of the background in the sub-horizon approximation used here.

As already mentioned, irrespective of the completion (beyond the Hubble scale) of the self-accelerated solution, it is locally indistinguishable from the ΛCDM model. At horizon scales, however, it is likely that these two scenarios will depart from each other: As we emphasized before, the solutions found in the decoupling limit do not necessarily imply the existence of full solutions with identical properties. A given solution in the decoupling limit can just be a transient state of the full solution. Significant deviations of the latter from the former should kick in at distance/time scales comparable to the graviton Compton wavelength.

2.2 Screening the Cosmological Constant in the Decoupling Limit

One explicit realization of degravitation is expected to occur in massive gravity, where gravity is weaker in the IR, and the graviton mass could play the role of a high-pass filter (Dvali et al. 2002, 2003). In this section we show explicitly how the dRGT theory of massive gravity successfully screens an arbitrarily large cosmological constant in the decoupling limit, while evading any ghost issues and preserving Lorentz invariance.

For convenience we recall the decoupling limit Lagrangian of (2.1) coupled to an external source

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{Pl}} h^{\mu\nu} T_{\mu\nu} \,. \tag{2.16}$$

The equations of motion for the helicity-0 and 2 modes are then

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] = -\frac{1}{M_{Pl}} T_{\mu\nu} \,, \tag{2.17}$$

and

$$\begin{aligned} &\left(a_1 + \frac{a_2}{\Lambda_3^3} \Box\pi + \frac{3a_3}{2\Lambda_3^6}\left([\Pi]^2 - [\Pi^2]\right)\right)\left[\Box h - \partial_\alpha \partial_\beta h^{\alpha\beta}\right] \\ &+ \frac{1}{\Lambda_3^3}\left(a_2 \Pi_{\mu\nu} - 3\frac{a_3}{\Lambda_3^3}\left(\Pi^2_{\mu\nu} - \Box\pi\, \Pi_{\mu\nu}\right)\right)\left[2\partial^\mu \partial_\alpha h^{\alpha\nu} - \Box h^{\mu\nu} - \partial^\mu \partial^\nu h\right] \\ &- \frac{3a_3}{\Lambda_3^6}\left(\Pi_{\mu\alpha}\Pi_{\nu\beta} - \Pi_{\mu\nu}\Pi_{\alpha\beta}\right)\partial^\alpha \partial^\beta h^{\mu\nu} = 0 \,. \end{aligned} \tag{2.18}$$

We now focus on a pure cosmological constant source, $T_{\mu\nu} = -\lambda \eta_{\mu\nu}$, and make use of the following ansatz,

$$h_{\mu\nu} = -\frac{1}{2}H^2 x^2 M_{\rm Pl}\, \eta_{\mu\nu}\,, \tag{2.19}$$
$$\pi = \frac{1}{2}\, q\, x^2 \Lambda_3^3\,. \tag{2.20}$$

The equations of motion then simplify to

$$\left(-\frac{1}{2}M_{\rm Pl}H^2 + \sum_{n=1}^{3} a_n\, q^n \Lambda_3^3\right)\eta_{\mu\nu} = -\frac{\lambda}{6M_{\rm Pl}}\eta_{\mu\nu}\,, \tag{2.21}$$
$$H^2\left(a_1 + 2a_2 q + 3a_3 q^2\right) = 0\,. \tag{2.22}$$

As we will see below, this system of equations admits two branches of solutions, a degravitating one, for which the geometry remains flat (mimicking the late-time part of the relaxation process), and a de Sitter branch which is closely related to the standard GR de Sitter solution. We start with the degravitating branch before exploring the more usual de Sitter solution and show that the stability of these branches depends on the free parameters $a_{2,3}$, as well as the magnitude of the cosmological constant.

2.2.1 *The Degravitating Branch*

It is easy to check that the geometry can remain flat i.e. $H = 0$ and $g_{\mu\nu} \equiv \eta_{\mu\nu}$, despite the presence of the cosmological constant. Such solutions are possible due to the presence of the extra helicity-0 mode that carries the source instead of the usual metric. With $H = 0$, Eq. (2.22) is trivially satisfied, while the modified Einstein equation (2.21) determines the coefficient (which we denote by q_0 here) for the helicity-0 field in (2.20),

$$a_1 q_0 + a_2 q_0^2 + a_3 q_0^3 = -\frac{\tilde{\lambda}}{6}\,, \tag{2.23}$$

in terms of the dimensionless quantity $\tilde{\lambda} = \lambda/\Lambda_3^3 M_{\rm Pl}$. Notice that as long as the parameter a_3 is present, Eq. (2.23) has always at least one real root. There is therefore a flat solution for arbitrarily large cosmological constant.

Let us now briefly comment on the stability of the flat solution, as this has important consequences for the relaxation mechanism behind degravitation. We consider the field fluctuations above the static solution,

$$\pi = \frac{1}{2}q_0 \Lambda_3^3\, x^2 - \phi/\kappa, \tag{2.24}$$
$$T_{\mu\nu} = -\lambda \eta_{\mu\nu} + \tau_{\mu\nu}, \tag{2.25}$$

where q_0 is related to λ via (2.23) and the coupling κ is determined by

$$\kappa = 2(a_1 + 2a_2 q_0 + 3a_3 q_0^2)\,. \tag{2.26}$$

To the leading order, the action for these fluctuations is then simply given by

$$\mathcal{L}^{(2)} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{2}h^{\mu\nu}X^{(1)}_{\mu\nu}[\Phi] + \frac{1}{M_{Pl}}h^{\mu\nu}\tau_{\mu\nu}\,, \tag{2.27}$$

with $\Phi_{\mu\nu} = \partial_\mu\partial_\nu\phi$. The stability of this theory is better understood when working in the Einstein frame where the helicity-0 and -2 modes decouple. This is achieved by performing the change of variable,

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \phi\,\eta_{\mu\nu}\,, \tag{2.28}$$

which brings the action to the following form

$$\mathcal{L}^{(2)} = -\frac{1}{2}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{3}{2}\,\phi\,\Box\,\phi + \frac{1}{M_{Pl}}\left(\bar{h}^{\mu\nu} + \phi\,\eta^{\mu\nu}\right)\tau_{\mu\nu}. \tag{2.29}$$

Stability of the static solution is therefore manifest for any region of the parameter space for which κ is real and does not vanish. As already mentioned, if $a_3 \neq 0$ there is always a real solution to (2.23), which is therefore stable for $\kappa \neq 0$. This suggests the presence of a flat late-time attractor solution for degravitation. The special case $a_3 = 0$ is discussed separately below.

2.2.2 de Sitter Branch

In the presence of a cosmological constant, the field equations (2.21) and (2.22) also admit a second branch of solutions; these connect with the self-accelerating branch presented in Sect. 2.1, and we refer to them as the de Sitter solutions. The parameters for these solutions should satisfy

$$a_1 + 2a_2 q_{dS} + 3a_3 q_{dS}^2 = 0\,, \tag{2.30}$$

$$H_{dS}^2 = \frac{\lambda}{3M_{Pl}^2} + \frac{2\Lambda_3^3}{M_{Pl}}\left(a_1 q_{dS} + a_2 q_{dS}^2 + a_3 q_{dS}^3\right)\,. \tag{2.31}$$

This solution is closer to the usual GR de Sitter configuration and only exists if $a_2^2 \geq 3a_1a_3$. The stability of this solution can be analyzed as previously by looking at fluctuations around this background configuration,

$$\pi = \frac{1}{2} q_{\rm dS}\, \Lambda_3^3\, x^2 + \phi\,, \tag{2.32}$$

$$h_{\mu\nu} = -\frac{1}{2} H_{\rm dS}^2\, x^2\, \eta_{\mu\nu} + \chi_{\mu\nu}\,, \tag{2.33}$$

$$T_{\mu\nu} = -\lambda \eta_{\mu\nu} + \tau_{\mu\nu}\,. \tag{2.34}$$

To second order in fluctuations, the resulting action is then of the form

$$\mathcal{L}^{(2)} = -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + \frac{6H_{\rm dS}^2 M_{\rm Pl}}{\Lambda_3^3}(a_2 + 3a_3 q_{\rm dS})\,\phi\,\Box\,\phi + \frac{1}{M_{\rm Pl}}\chi^{\mu\nu}\tau_{\mu\nu}\,. \tag{2.35}$$

It is interesting to point out again that the helicity-0 fluctuation ϕ then decouples from matter sources at quadratic order (however the coupling reappears at the cubic order). Stability of this solution is therefore ensured if the parameters satisfy one of the following three constraints, (setting $a_1 = -1/2$ and $\tilde{\lambda} > 0$)

$$a_2 < 0 \quad \text{and} \quad -\frac{2a_2^2}{3} \le a_3 < \frac{1 - 3a_2\tilde{\lambda} - (1 - 2a_2\tilde{\lambda})^{3/2}}{3\tilde{\lambda}^2}\,, \tag{2.36}$$

or

$$a_2 < \frac{1}{2\tilde{\lambda}} \quad \text{and} \quad a_3 > \frac{1 - 3a_2\tilde{\lambda} + (1 - 2a_2\tilde{\lambda})^{3/2}}{3\tilde{\lambda}^2}\,, \tag{2.37}$$

or

$$a_2 \ge \frac{1}{2\tilde{\lambda}} \quad \text{and} \quad a_3 > -\frac{2}{3}a_2^2\,. \tag{2.38}$$

These are consistent with the results (2.13) found for the self-accelerating solution in the absence of a cosmological constant. Notice here that in the presence of a cosmological constant, the accelerating solution can be stable even when $a_3 = 0$. This branch of solutions therefore connects with the usual de Sitter one of GR.

2.2.3 *Diagonalizable Action*

In Sect. 2.1 we have emphasized the importance of the contribution of $X^{(3)}_{\mu\nu}$ for the stability of the self-accelerating solution. However, in the presence of a nonzero cosmological constant, this contribution is not a priori essential for the stability of either the degravitating or the de Sitter branches. Furthermore, since the helicity-0 and -2 modes can be diagonalized at the nonlinear level when $a_3 = 0$, as was explicitly shown in de Rham and Gabadadze (2010), we will study this special case separately below. In particular, we will show that it leads to certain special bounds both in the degravitating and de Sitter branches of solution.

Stability: To start with, when $a_3 = 0$, the degravitating solution only exists if

$$2a_2\tilde{\lambda} < 3a_1^2\,. \tag{2.39}$$

This bound ensures the absence of ghost-like instabilities around the degravitating solution. Assuming that the parameters $a_{1,2} = \mathcal{O}(1)$ take some natural values then the situation $a_2 > 0$ implies a severe constraint on the value of the vacuum energy that can be degravitated. This is similar to the bound in the non-linear realization of massive gravity (de Rham 2010), as well as in codimension-two deficit angle solutions, $\lambda \lesssim m^2 M_{\rm Pl}^2$. The situation $a_2 < 0$ on the other hand allows for an arbitrarily large cosmological constant.

On the other hand, the bound $a_2^2 \geq 3a_1a_3$ for the existence of the de Sitter solution is always satisfied if $a_3 = 0$. However, the constraints on the parameters (2.36)–(2.38) which guarantee the absence of ghosts on the de Sitter branch imply that

$$2a_2\tilde{\lambda} > 3a_1^2\,. \tag{2.40}$$

In this specific case then, we infer that when the Sitter solution is stable, the degravitating branch does not exist, and when the degravitating branch exists the de Sitter solution is unstable. Therefore, at each point in the parameter space there is only one, out of these two solutions, that makes sense. In the more general case where $a_3 \neq 0$ the situation is however much more subtle and it might be possible to find parameters for which both branches exist and are stable simultaneously.

Einstein's frame: Let us now work instead in the Einstein frame, where the helicity-2 and -0 modes are diagonalized (which is possible as long as $a_3 = 0$). The transition to Einstein's frame is performed by the change of variable (de Rham and Gabadadze 2010)

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1\pi\,\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\pi\,\partial_\nu\pi, \tag{2.41}$$

such that the action takes the form

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\bar{h}^{\mu\nu}(\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2\,\pi\,\Box\,\pi - \frac{6a_2a_1}{\Lambda_3^3}(\partial\pi)^2[\Pi] \\ & + \frac{2a_2^2}{\Lambda_3^6}(\partial\pi)^2\left([\Pi^2] - [\Pi]^2\right) \\ & + \frac{1}{M_{\rm Pl}}\left(\bar{h}_{\mu\nu} - 2a_1\,\pi\,\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\pi\,\partial_\nu\pi\right)T^{\mu\nu},\end{aligned} \tag{2.42}$$

and the structure of the Galileon becomes manifest. Notice however, that the coefficients of the different Galileon interactions are not arbitrary. Furthermore, the coupling to matter includes terms of the form $\partial_\mu\pi\,\partial_\nu\pi\,T^{\mu\nu}$, absent in the original

Galileon formalism (Nicolis et al. 2009). Both of these distinctions play a crucial role in screening the cosmological constant—the task which was thought impossible in the original Galileon theory. Here, however, as long as the bound (2.39) is satisfied, the solution for π reads

$$\pi = \frac{1}{2} q_0 \Lambda_3^3 x^2 \qquad \text{with} \qquad a_1 q_0 + a_2 q_0^2 = -\frac{\tilde{\lambda}}{6}, \tag{2.43}$$

while the helicity-2 mode $\bar{h}_{\mu\nu}$ now takes the form

$$\bar{h}_{\mu\nu} = \left(\frac{\xi}{2} - \frac{\lambda}{6M_{Pl}}\right) x^2 \eta_{\mu\nu} + \xi\, x_\mu x_\nu, \tag{2.44}$$

with ξ being an arbitrary gauge freedom parameter. Fixing $\xi = -2a_2 q_0^2 \Lambda_3^3$, the physical metric is then manifestly flat:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{M_{Pl}}\left(\bar{h}_{\mu\nu} - 2a_1\, \pi\, \eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3} \partial_\mu \pi\, \partial_\nu \pi\right) \\ &= \eta_{\mu\nu} - \frac{\lambda_3^3}{M_{Pl}}\left(a_1 q_0 + a_2 q_0^2 + \frac{\tilde{\lambda}}{6}\right) x^2 \eta_{\mu\nu} + \frac{x_\mu x_\nu}{M_{Pl}}\left(\xi + 2a_2 q_0^2 \Lambda_3^3\right) \\ &\equiv \eta_{\mu\nu}. \end{aligned}$$

To reiterate, the specific nonlinear coupling to matter that naturally arises in the ghostless theory of massive gravity is essential for the screening mechanism to work. This allows us to understand why neither DGP nor an ordinary Galileon theory are capable of achieving degravitation. As we already pointed out, the Galileon interactions arise naturally after diagonalization. However, let us summarize the common and different points between Galileon theory and the interactions in the decoupling limit after diagonalization.

Common	Differences
IR modification of gravity as due to a light scalar field with non-linear derivative interactions ($\rightarrow$ Vainshtein mechanism)	Non-diagonazable interaction $\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ which is important for the self-accelerating solution
Respects the symmetry $\pi \rightarrow \pi + c + b_\mu x^\mu$	Extra coupling $\partial_\mu \pi\, \partial_\nu \pi\, T^{\mu\nu}$ which is important for the degravitating solution
Second order equations of motion, containing at most two time derivatives	Only two free parameters
Non-renormalization theorem applies	Observational differences

2.2.4 Phenomenology

Let us now focus on the phenomenology of the degravitating solution. This mechanism relies crucially on the extra helicity-0 mode in the massive graviton. However tests of gravity severely constrain the presence of additional scalar degrees of freedom. As is well known in theories of massive gravity, the helicity-0 mode can evade fifth force constraints in the vicinity of matter if the helicity-0 mode interactions are important enough to freeze out the field fluctuations, Vainshtein (1972).

Around the degravitating solution, the scale for helicity-0 interactions are no longer governed by the parameter Λ_3, but rather by the scale determined by the cosmological constant $\tilde{\Lambda}_3 \sim (\lambda/M_{Pl})^{1/3}$. To see this, let us pursue the analysis of the fluctuations around the degravitating branch (2.24) and keep the higher order interactions. The resulting Lagrangian is then

$$\mathcal{L}^{(2)} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{2}h^{\mu\nu}\left(X^{(1)}_{\mu\nu}[\Phi] + \frac{\tilde{a}_2}{\tilde{\Lambda}^3}X^{(2)}_{\mu\nu}[\Phi] + \frac{\tilde{a}_3}{\tilde{\Lambda}^6}X^{(3)}_{\mu\nu}[\Phi]\right) + \frac{1}{M_{Pl}}h^{\mu\nu}\tau_{\mu\nu}, \tag{2.45}$$

with

$$\frac{\tilde{a}_2}{\tilde{\Lambda}^3} = -2\frac{a_2 + 3a_3 q_0}{\Lambda_3^3\kappa^2}, \quad \text{and} \quad \frac{\tilde{a}_3}{\tilde{\Lambda}^6} = -\frac{2a_3}{\Lambda_3^6\kappa^3}. \tag{2.46}$$

Assuming $a_{2,3} \sim \mathcal{O}(1)$, a large cosmological constant $\tilde{\lambda} \gg 1$, implies $q_0 \gg 1$, so that $a_3 q_0^2 \gg a_2 q_0 \gg a_1$ and $\kappa \sim a_3 q_0$ such that

$$\frac{\tilde{a}_2}{\tilde{\Lambda}^3} \sim \frac{1}{\Lambda_3^3 a_3 q_0^3} \sim \frac{1}{\Lambda_3^3\tilde{\lambda}} \sim \frac{M_{Pl}}{\lambda} \tag{2.47}$$

(notice that this result is maintained even if $a_3 = 0$), and similarly

$$\frac{\tilde{a}_3}{\tilde{\Lambda}^6} \sim \left(\frac{M_{Pl}}{\lambda}\right)^2. \tag{2.48}$$

To evade fifth force constrains within the solar system, the scale $\tilde{\Lambda}$ should therefore be small enough to allow for the nonlinear interactions to dominate over the quadratic contribution and enable the Vainshtein mechanism. In the DGP model this typically imposes the constraint, $\tilde{\Lambda}^3/M_{Pl} \lesssim (10^{-33}\ \text{eV})^2$, while this value can be pushed by a few orders of magnitude in the presence of Galileon interactions, (Nicolis et al. 2009; Burrage and Seery 2010). Therefore, the allowed value of vacuum energy that can be screened without being in conflict with observations is fairly low, of the order of $(10^{-3}\ \text{eV})^4$ or so.

An alternative would be to impose a hierarchy between the dimensionless coefficients a_i. Since the Galilean interactions satisfy a non-renormalization theorem (Nicolis and Rattazzi 2004), which we will discuss in more detail in Chap. 5 such a tuning would remain technically natural. To explore this avenue in a simple way, let us set $a_3 = 0$. In that case, the effective strong coupling scale is given by

$$\tilde{\Lambda}^3 = \Lambda_3^3 \frac{\frac{3}{4} - 2a_2\tilde{\lambda}}{a_2} . \tag{2.49}$$

The strong coupling scale can then be tuned to small values by adjusting the parameter a_2 within the very small window

$$|a_2\tilde{\lambda} - \frac{3}{8}| \lesssim \frac{(10^{-33}\mathrm{eV})^2 M_{\mathrm{Pl}}}{\Lambda_3^3} . \tag{2.50}$$

Therefore even when allowing a hierarchy between the parameters, once they are fixed only very restricted values of the degravitated cosmological constant would be compatible with solar system tests. The previous argument would have been unaffected if we had set $a_3 \neq 0$.

The above constraint on the vacuum energy that can be degravitated makes the present framework not viable phenomenologically for solving the old cosmological constant problem. There may be a way out of this setback though: As mentioned previously, one may envisage a cosmological scenario in which the neutralization of vacuum energy takes place before the Universe enters the epoch for which the Vainshtein mechanism is absolutely necessary to suppress the helicity-0 fluctuations. Such an epoch should certainly be before the radiation domination. During that epoch, however, the cosmological evolution should reset itself—perhaps via some sort of phase transition—to continue subsequent evolution along the other branch of the solutions that exhibits the standard early behavior followed by the self-acceleration. This scheme would have to address the cosmological instabilities discussed in Grisa and Sorbo (2010), Berkhahn et al. (2010). Moreover, the viability of such a scenario would depend on properties of the degravitating solution in the full theory—which are not known. Therefore, we do not rely on this possibility.

Nevertheless, there are certain important virtues to the degravitating solution with the low value of the degravitated cosmological constant. This is an example of high importance in understanding how S. Weinberg's no-go theorem can be evaded in principle. As already emphasized in Rham et al. (2008, 2009), such mechanisms evade the no-go theorem by employing a field which explicitly breaks Poincaré invariance in its vacuum configuration $\pi \sim x^2$, while keeping the physics insensitive to this breaking. Indeed, physical observables are only sensitive to $\Pi_{\mu\nu} = \partial_\mu\partial_\nu \pi$ which is clearly Poincaré invariant, while the configuration of the π field itself has no direct physical bearing. This is built in the specific Galileon symmetry of the theory, and is a consequence of the fact that π is not an arbitrary scalar field but rather descends as the helicity-0 mode of the massive graviton. More precisely, under a

Poincaré transformation, $x^\mu \to \Lambda^\mu_\nu x^\nu + a^\mu$, the configuration for π transforms as $x^2 \to x^2 + v_\mu x^\mu + c$, with $v_\mu = 2a_\nu \Lambda^\nu_\mu$ and $c = a^2$ which is precisely the Galileon transformation for π under which the action is invariant. In other words the Poincaré symmetry is still realized up to a Galilean transformation (or, there is a diagonal subgroup of Poincaré and internal "galilean" groups that remains unbroken by the VEV of the π field).

Thus, we have presented here the crucial steps towards a non-linear realization of degravitation within the context of massive gravity, and this, without introducing any ghosts (at least in the decoupling limit). The arguments presented here only rely on the decoupling limit and it is reasonable to doubt their validity beyond that regime.

2.3 Summary and Critics

In this chapter we have addressed the potential impact of massive gravity on cosmology. We studied the self-accelerating and degravitating solutions in the decoupling limit of massive gravity and put constraints on the two free parameters of the theory from instability conditions in the cosmological evolution demanding the absence of ghost and Laplacian instabilities. We have shown that massive gravity can be used to construct self accelerating solutions in the decoupling limit. The helicity-0 degree of freedom of the massive graviton forms a condensate whose energy density sources self-acceleration and small fluctuations around these self-accelerating solutions are stable. Furthermore, the fluctuations of the helicity-0 field do not couple to the fluctuations of the helicity-2 field and hence to the matter fields such that the cosmological evolution is exactly as in the standard ΛCDM model. We have also demonstrated that massive gravity can screen an arbitrarily large cosmological constant in the decoupling limit without giving rise to any ghost. Unfortunately, the allowed value of the vacuum energy that can be screened without being in conflict with observations is fairly low. This conflict with the Vainshtein mechanism renders the degravitation solution as found here phenomenologically not viable for solving the old cosmological constant problem. Nevertheless, it is the first time that an explicit model can present a way out from Weinberg's no-go theroem. A possible way out of the conflict between the General Relativity recovery scale and the large Cosmological Constant could be to envisage a cosmological scenario in which degravitation of the vacuum energy takes place before the Universe enters the radiation dominated epoch—say during the inflationary period, or even earlier. By the end of that epoch then the cosmology should reset itself to continue evolution along the other branch of the solutions that exhibits the standard early behavior followed by the self-acceleration. The existence of such a transition would depend on properties of the degravitating solution in the full theory.

As already mentioned before, these solutions found in the decoupling limit do not guaranty the existence of full solutions with identical properties in the full theory. The solutions in the decoupling limit could be considered just as a transient state of the full solution. Since our work, there has been made a quite a lot of progress in studying the self-accelerating solutions, nevertheless the found solutions in the literature seem to be plagued by instabilities. On the degravitating solutions side, there has been little progress in the full theory and these solutions have been left aside so far in the literature. Even if the decoupling limit of massive gravity fails to degravitate an arbitrary large cosmological constant in order to make the Vainshtein mechanism work, it could be that in the full theory there exists a cosmological scenario in which the degravitation of the vacuum energy is not in conflict with the Vainshtein mechanism. This is worth the effort to investigate in the future.

We would like now take a critical viewpoint on the analysis performed in this chapter to discuss the limitations. The first limitation is the fact that the solutions found here are only valid in the approximation we made, on scales smaller than the Hubble scale. The second more worrisome limitation is the negligence of the helicity-1 field. As we emphasized before, the helicity-1 field enters only quadratically, or in higher order terms in the Lagrangian, and therefore we set it to be zero.

Since the appearance of our work there has been a flurry of investigations related to the self- accelerating solutions of the full theory in the literature, which go beyond the study represented here in this thesis. It has been shown that if the fiducial metric is chosen to be flat than for the physical metric there is no flat FLRW solutions (D'Amico et al. 2011) in the full theory beyond the decoupling limit. One way out of this no-go solution is to make the Stueckelberg field carry anisotropies but still keep the geometry FLRW. Nevertheless, solutions found in this way turn out to have strongly coupled degrees of freedom, meaning that some of the degrees of freedom lose their kinetic terms. Alternatively, one can accept non-FLRW solutions but puts constraints on the magnitude of the mass of the graviton coming from the consistency with known constraints on homogeneity and isotropy. This would rely on the successful implementation of the Vainshtein mechanism in the cosmological evolution which so far has not been investigated in detail. Even if flat FLRW has been proven not to exist, there are successful constructions of open FLRW solutions (Gumrukcuoglu et al. 2011). However, at the level of non-linear perturbations instabilities pop up again and render these solutions phenomenologically not viable. These negative outcomes forced the considerations of more general fiducial metric. Indeed, if one assumes a de Sitter reference metric, then one can find FLRW solutions. But the de Sitter reference metric brings other problems along. The Higuchi bound imposes the mass of the graviton to be $m^2 > H^2$ which turns these solutions inconsistent with the observational constraints. Similarly, the generalization of the fiducial metric to a FLRW metric forces a generalized Higuchi bound and one encounters similar problems (Fasiello and Tolley 2012). As we already explained in the introduction the dRGT theory is based on a framework in which the massive graviton propagates on top of a fixed background reference metric. The generalization of dRGT gave rise to theories of ghost-free bimetric gravity in which the reference metric becomes dynamical as well. This bimetric generalization of dRGT gives rise to stable self-accelerating solutions without imposing problems related to the Higuchi bound and so put into operation new exciting research directions (Fasiello and Tolley 2013). There has also been other extensions of massive gravity, like time-depending mass

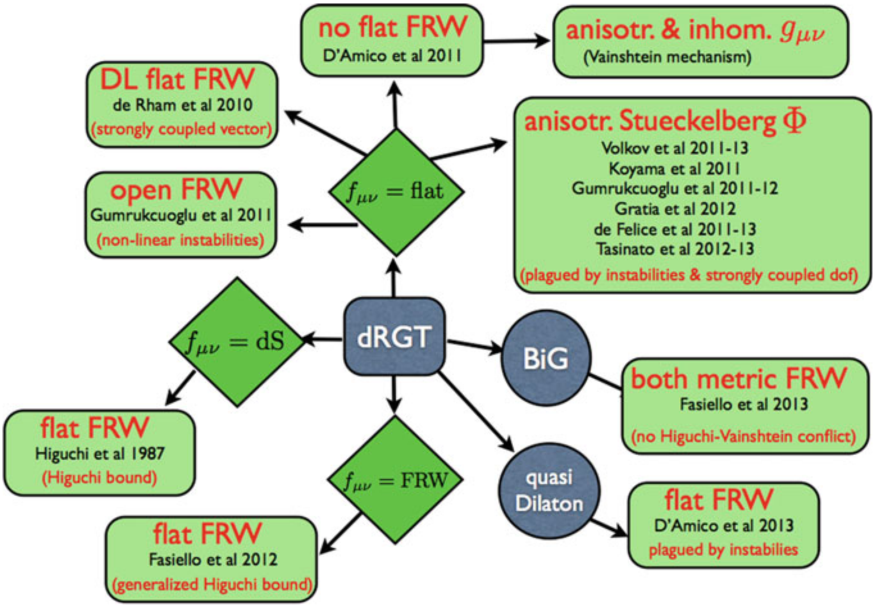

Fig. 2.2 The cosmological "tree" of massive gravity

of the graviton (Huang 2012) or adding additional degrees of freedom [Quasi-dilaton D'Amico et al. (2013)] from which some of the generalization might yield stable self-accelerating solutions (Fig. 2.2).

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