

Chapter 2

The Total Information Carried by the Light

This chapter theoretically establishes the quantum limit on the total information carried by the collected light, using a simple approach analogous to the Heisenberg microscope. This sets a limit on the sensitivity attainable with a perfect measurement within a given optical setup. This ultimate limit provides a view to what may be possible in future; real experimental systems cannot perfectly capture the complex spatial profile of the scattered light, so the limit derived here is not achieved in any experiment, even when the sensitivity is limited by quantum noise. The theory presented in this chapter was published in the following paper [25].

2.1 Introduction

The position sensitivity in optical tweezers is usually limited by technical noise sources, such as laser noise, electronic noise in the detector, or drifts of mirrors in the experiment. Substantial efforts have been made to minimize such technical sources of error [2, 14, 17]. With sufficient improvements in the technical noise, the sensitivity per photon must eventually be limited by noise due to the quantization of light [13]. The resulting quantum noise limit is an important consequence of quantum mechanics, and is becoming increasingly relevant in experiments. So, what is the precision at the quantum limit? And how closely are experiments approaching this? The quantum limit has previously only been derived in the paraxial optics regime with Rayleigh scattering [3, 23], making the calculated limit inapplicable to most experiments.

Here we follow the simple principles of the Heisenberg Microscope to derive the maximum information content of a scattered optical field, and the corresponding quantum sensitivity limit for particle tracking in optical tweezers. A more detailed analysis of the quantum limit following standard quantum formalism is given in Chap. 3. The simple approach followed here can be straightforwardly applied to particles with arbitrary shape and size, yields analytic solutions in the Rayleigh scattering regime, and allows analysis of a wide range of optical setups.

The approach is also relevant to other measurements such as microscopy with fluorescent particles [7]. Our results show that leading experiments are already within two orders of magnitude of the quantum noise limit.

2.2 How Much Information Can a Photon Carry?

Particles in optical tweezers are tracked by measuring their perturbing influence on the electric field as they scatter photons. The scattered light is collected with a high numerical aperture lens, providing an image of the particle. Motion of the particle displaces the image, with the displacement typically measured via a resulting power imbalance on a quadrant detector [9]. Each scattered photon carries some information about the position of the particle from which it scatters. This information is fundamentally constrained by the Heisenberg uncertainty relation, which places a limit on the localization of a photon [8] of $\Delta q_i \geq \frac{\hbar}{2\Delta p_i}$, where Δp_i is the photons momentum uncertainty for each axis $i \in \{x, y, z\}$, z is the direction of optical propagation, x the axis of linear polarization, and $\hbar$ is the reduced Planck constant. Consequently, the location of the scattering event which produced the photon can be established with a minimum uncertainty of Δq_i . This also defines the particle position uncertainty when the scattered field originates from the center of the particle, as is the case for Rayleigh scattering. In general, however, the origin of the scattered field may be offset, lying between the center of the particle and that of the optical trap. For example, if the trapped particle is larger than the trap width and displaced such that its center lies outside the trap, scattering can only originate from the part of the particle contained within the trap field. This leads to a smaller displacement in the scattering origin than that of the particle itself. Such effects increase the absolute uncertainty in the particle position, so that in this case, Δq_i establishes a lower bound on particle position uncertainty.

A useful constraint on momentum uncertainty Δp_i is that it must be within the total photon momentum $p_{ph} = \frac{2\pi n_m \hbar}{\lambda}$, where n_m is the refractive index of the medium¹ and λ is the vacuum wavelength, because

$$\Delta p_i^2 = \langle p_i^2 \rangle - \langle p_i \rangle^2 \leq \sum_{i=x,y,z} \langle p_i^2 \rangle - \langle p_i \rangle^2 = p_{ph}^2 - \sum_{i=x,y,z} \langle p_i \rangle^2. \quad (2.1)$$

We introduce parameters f_i to characterize Δp_i along each axis, such that

$$\Delta p_i = f_i p_{ph}, \quad (2.2)$$

¹The momentum flux of light in a medium has been highly controversial, with Abraham and Minkowski separately deriving it to be $\frac{p_0}{n_m c}$ and $\frac{n_m p_0}{c}$ respectively, and with both forms confirmed by experiments [16]. The different forms have been shown to describe different physical quantities, as described in detail in Ref. [16]. It is the Minkowski form $\frac{n_m p_0}{c}$ which is appropriate to this problem, as this conserves kinetic momentum in light-matter interactions.

leading to the condition $f_x^2 + f_y^2 + f_z^2 \leq 1$. This relation becomes an equality for a photon which is scattered with no preferred direction ($\langle p_i \rangle = 0$). Using this, a position measurement is limited to an uncertainty of

$$\Delta q_i \geq \frac{\lambda}{4\pi n_m f_i}. \quad (2.3)$$

Perhaps unsurprisingly, this condition has a similar form to the Rayleigh criterion [4], which establishes an approximate limit to the resolution of a given microscope system without regard for the exact features of the scattering process. By contrast, Eq. 2.3 provides a rigorous bound which accounts for the scattered field profile. This is crucial for establishing accurate quantum limits to the precision of particle tracking experiments, particularly in the Mie scattering regime where the scattering exhibits both complex spatial structure and resonance phenomena.

In particle tracking experiment, the position of a particle is measured with $\langle N_s \rangle = \kappa \langle N_0 \rangle$ scattered photons, where N_0 incident photons have a scattering probability of κ . Without quantum correlations in the light, each photon detection event provides an independent measurement of the particle position with uncertainty Δq_i . While the standard deviation of these position measurements will be limited to Δq_i , the particle position can be reliably established with a much smaller standard error δq_i given by

$$\delta q_i \geq \frac{\lambda}{4\pi n_m f_i \sqrt{\kappa \langle N_0 \rangle}}. \quad (2.4)$$

This places a lower limit on the particle position sensitivity achievable with uncorrelated photons. To derive the value of this limit, we only need to calculate the scattering probability κ and the momentum parameter f . As these parameters depend on the trapping field, the particle being tracked, and the refractive index of the surrounding medium, they must be evaluated for specific cases, although neither may ever reach unity. The condition that κ and f remain below one places a general lower bound of $\delta q \geq \frac{\lambda}{4\pi n_m \sqrt{N_0}}$ on the precision, which is imposed even if the scattering properties of the particle were cleverly engineered [12] and the measurement performed perfectly; for all realistic situations, the minimum resolvable displacement is larger than this. To determine the limit for real experiments, we evaluate the quantum limit for the common case of homogeneous spheres in a Gaussian trap.

2.3 Optimal Tracking of a Rayleigh Particle

Both the scattering rate κ and momentum distribution parameter f can be calculated analytically in the Rayleigh scattering regime, in which the particle diameter $d \ll \lambda$. If an incident TEM₀₀ mode is focused on the particle to a waist of w , the scattering rate is

$$\kappa = \frac{128\pi^4 a^6}{3\lambda^4 w^2} \left(\frac{n_p^2 - n_m^2}{n_p^2 + 2n_m^2} \right)^2, \quad (2.5)$$

where n_p is the refractive index of the particle [11]. For Rayleigh scatterers polarized along the x axis, the scattered photons enter the mode ψ , with amplitude given by

$$|\psi|^2 = \frac{3}{8\pi} \frac{1 - (x/r)^2}{r^2}, \quad (2.6)$$

where r is the radial distance [11]. Assuming the photons to be moving radially outward, this mode determines the momentum profile of the scattered light. In most experiments, any scattered photon which reaches the detector has passed through the aperture of an objective. This constrains the momentum range of the detected photons, thereby increasing the position uncertainty. The measurement then only includes the light present in the area A of this aperture. For this measurement, the parameter f is calculated to be

$$f_x^2 = \int |\psi|^2 (x/r)^2 dA - \left(\int |\psi|^2 (x/r) dA \right)^2. \quad (2.7)$$

The second term in Eq. 2.7 is zero for all cases considered here, as photons scattered from centered particles have no preferred transverse direction. The ultimate sensitivity limit is given when we evaluate this over a spherical shell; here this gives $f_x^2 = \frac{1}{5}$, $f_y^2 = \frac{2}{5}$ and $f_z^2 = \frac{2}{5}$. Since the sensitivity predicted from this can only be achieved with perfect imaging of all scattered photons, it also determines the measurement back-action upon the particle. For measurements subject to a limited aperture size, Eq. 2.7 can be integrated analytically, with the corresponding quantum limit plotted in Fig. 2.1. In this, the aperture size is expressed in terms of the NA for a particle suspended in water. This shows near-perfect agreement with the equivalent numerical calculations performed in Ref. [23].

2.4 Sensitivity in the Mie Scattering Regime

Optical tweezers experiments usually operate with particles which are too large to be accurately approximated as Rayleigh scatterers. For these particles, the scattering profile is complicated by such effects as multiple internal reflections and the interference between optical paths of different length. For spherical particles in focused optical fields, this scattering regime is described mathematically by extended Mie theory [11]. We evaluate the scattering profiles numerically with the Optical Tweezers Toolbox [19]. These profiles are integrated as described in Eq. 2.7 to find the quantum sensitivity limit for particle tracking. The quantum limit is shown in Fig. 2.2 for polystyrene beads suspended in water, measured with 1064 nm light and an NA = 1.3

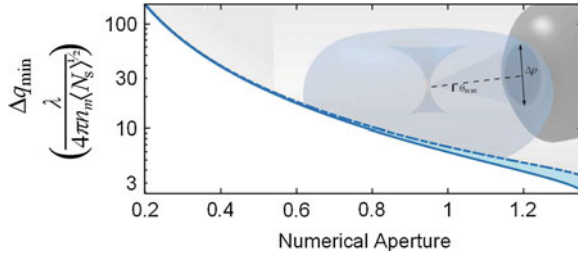


Fig. 2.1 Light scatters from a small particle with a defined spatial profile, shown here as Rayleigh scattering. The field which enters the objective aperture is measured to determine the particle position. The scattered photons which propagate towards the aperture ($\theta > \theta_{\min}$) have a momentum distribution Δp which defines the fundamental lower bound on sensitivity with which the particle position can be determined, as plotted here as a function of the condenser NA for Rayleigh particles suspended in water, illuminated by linearly polarized light. The sensitivity limit is shown for motion both perpendicular (*solid line*) and parallel (*dashed line*) to the polarization axis. Sensitivity improves with NA, as high NA lenses collect scattered photons with a larger momentum range. Because of the anisotropic scattering profile, this improvement is greatest for motion which is perpendicular to the polarization axis. Together with the scattering rate κ , this sensitivity defines the quantum limit on displacement resolution per incident photon

objective and condenser. Examining this, we see a number of noteworthy features. For small particles, where Rayleigh scattering is valid, forward and back scatter offer an equally sensitive measurement. However, as the particle size increases, the scatter becomes preferentially forward, with less information carried by back-scattered photons. Polarization has little effect on the sensitivity for beads larger than the wavelength. Two different resonant effects are evident in Fig. 2.2 for large particles. The direction of scattering is periodically modulated by an effect which is approximately given by thin-film interference. This causes a significant modulation of the sensitivity available through back-scatter measurements; however, maximizing back-scattered sensitivity may not be advantageous for high-refractive index particles, as this regime also has the weakest trapping potential [22]. In addition to this, Mie resonances suppress the particle position information encoded in the phase of the outgoing field when propagation through the particle increases the optical path length by an integer multiple of the wavelength (see Fig. 2.2 insets). This degrades the measurement sensitivity without affecting the trapping potential, as the trapping is determined purely by the optical amplitude.

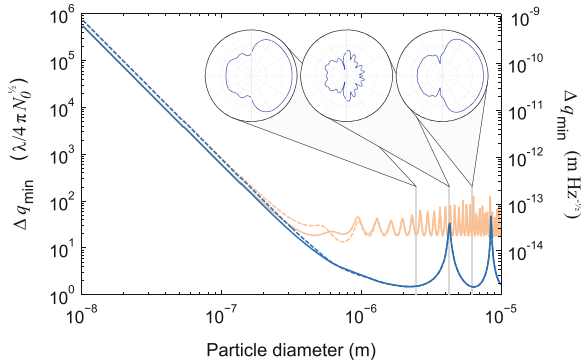


Fig. 2.2 The quantum limit on position sensing polystyrene beads ($n = 1.58$) suspended in water with a 1.3 NA objective as a function of diameter. The faint and dark lines representing measurement of back-scatter and forward-scatter respectively, and *dashed* and *solid* lines representing measurements parallel and perpendicular to the plane of polarization. The axis on the right shows the best sensitivity possible for a measurement based on 1 mW of optical power. *Insets* Logarithmically scaled scattered field intensity profiles for bead diameters of 2.5, 4.3 and 6.2 μm , calculated in the plane of polarization. The 4.3 μm bead forward-scatters far less light than either of the other two, because resonant effects reduce the interaction of the incoming field with the bead, such that most of the light remains in the incident mode. However, the back-scatter is of a similar magnitude for all three

2.5 Relevance of This Limit

2.5.1 Resolving Zero-Point Motion

One application of this calculation is to determine the conditions required to observe the zero-point motion of a levitating sphere. When this can be measured, it is possible to generate squeezed mechanical states or cool to the ground state [1], and also provides a means to search for the non-Newtonian gravity predicted at small scales [6]. The average amplitude of zero-point motion is given by [1]

$$\Delta q_{\text{zpm}} = \left(\frac{\hbar}{2m\Omega} \right)^{1/2} \quad (2.8)$$

for a mechanical frequency Ω and mass m . For most interesting applications, the measurement time must be short compared to the average time for one bath phonon to enter or leave the mechanical mode [1], which is given by $\tau = \frac{2\pi}{\Gamma n}$ for a mean phonon occupancy $n = \frac{k_B T}{\hbar \Omega}$ and decay rate Γ . Although feedback is often used to cool motion, it does not influence this coupling rate, as feedback can only reduce the phonon occupancy by introducing mechanical dissipation which increases Γ , keeping the product unchanged. In order to measure the movement Δq_{zpm} over a time τ , the incident power must exceed the minimum threshold which we calculate

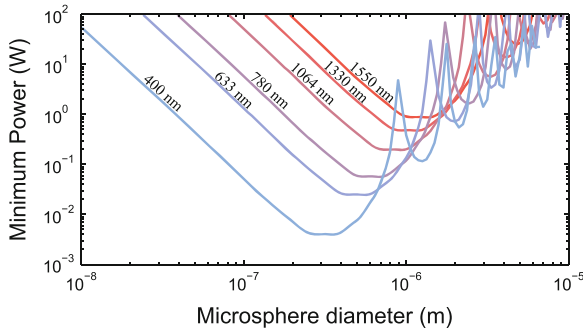


Fig. 2.3 This is the optical power required to observe the zero-point motion of a levitating silica bead in vacuum, for six different laser wavelengths. A refractive index of 1.46, a temperature of $T = 298$ K, and a decay rate of $\Gamma = 2\pi \times 0.1$ Hz have been used, along with $\text{NA} = 0.8$ objective and condenser. This plot can be easily extrapolated to other physical parameters because the required power scales linearly with both the decay Γ and temperature T . Although the decay rate used for this calculation is similar to that in recent experiments [15], future experiments may attain much lower decay rates, with predicted rates of order 10^{-6} Hz [1, 6]

here. Conveniently, an increase in the mechanical frequency Ω reduces both the mechanical amplitude Δq_{zpm} and phonon occupancy n such that the power threshold is independent of the frequency. This means that the trapping potential need not be considered, and the derived power limit is valid even if there are additional trapping fields. The power threshold is calculated as a function of sphere radius and shown in Fig. 2.3. This shows that for a given laser, the optimal bead diameter is somewhat smaller than the wavelength. In the Rayleigh scattering regime, the required power scales as λ^7 , making a short wavelength laser a practical choice. For large beads, the preferred wavelength is determined entirely by the Mie resonances. The dependence of the zero-point motion on particle mass generally makes its observation more difficult in larger beads. To observe zero-point motion in any silica bead with a diameter larger than $6 \mu\text{m}$ would be prohibitively difficult at the assumed decay rate of 0.1 Hz, as it requires a quantum limited measurement of over 10 W of optical power.

Although the power predicted in Fig. 2.3 should make the zero-point motion observable, it will also impart back-action onto the bead motion. The back-action will be greater than the zero-point momentum, since only the forward-scattered photons are included in the measurement. This consideration is particularly severe in the Rayleigh scattering regime, where the back-scatter carries as much information as the forward-scatter. If zero-point motion of a Rayleigh particle were resolvable with the $\text{NA} = 0.8$ condenser considered here, back-action will exceed the zero-point momentum by a factor of 2.9. By contrast, deep within the Mie scattering regime most of the information is in the forward-scattered light, and the zero-point motion can be resolved while imparting back-action which only exceeds the zero-point momentum by a factor of 1.06. However, if the measurement is in any way non-optimal, this will enlarge the relative back-action. In real experiments, optical loss and the use of

split detectors [23] ensure a non-optimal position measurement. This is a significant limitation, as non-optimal measurement prevents ground state cooling and minimum uncertainty squeezing [18].

2.5.2 Experimental Sensitivity

With our calculations, we can characterize the gap between experimental results and the quantum limit. A recent experiment achieved a sensitivity of $1.7 \times 10^{-14} \text{ m Hz}^{-1/2}$ when tracking $1 \text{ }\mu\text{m}$ diameter polystyrene beads in water with 700 mW of 1064 nm light, and collecting only the forward-scatter [2]. Assuming the objective and condenser have $\text{NA} = 1.3$, we find that $\kappa^{1/2} f_x = 0.380$, and the 3.7×10^{18} incident photons per second should limit the sensitivity to $1.1 \times 10^{-16} \text{ m Hz}^{-1/2}$, which is within a factor of 145 of the demonstrated result. Not all of the photons were measured, however, as the field was attenuated by 70 % before the detector, such that the remaining photons were measured with sensitivity within a factor of 80 of the quantum limit. In another recent experiment, silica beads in vacuum were tracked at $3.9 \times 10^{-14} \text{ m Hz}^{-1/2}$ using 120 mW of 1064 nm light, and $\text{NA} = 0.68$ objective and condenser [15]. For this case, $\kappa^{1/2} f_x = 0.203$ and the quantum limit is $5.2 \times 10^{-16} \text{ m Hz}^{-1/2}$, only 75 times lower than the experimental sensitivity. In this experiment, the measurement for each axis was performed on separate detectors, so each detector had fewer photons available for measurement. Although both of these experiments used non-optimal split detectors, they operate well within two orders of magnitude of the quantum limit. If further improvements to sensitivity are required, it would be useful to characterize the contributions to this gap arising from optical loss, non-optimal detection, and various noise sources. This characterization would indicate the aspects of the measurement which could be most effectively improved. Some classical strategies to reduce this gap are discussed in Part II of this thesis. The quantum limit derived can only be overcome by using quantum correlated light to achieve sub-shot noise performance in a classically near-optimal measurement.

2.6 Lessons to Learn

The quantum limit calculated here has important implications for future experiments requiring better sensitivity than currently available. The thermal motion of particles in water has been observed to deviate from Brownian motion on very short time-scales, due to hydrodynamic resonances [5] and ballistic motion [10]. However, there are further predicted effects which remain unobserved, such as oscillations arising from the elastic compressibility of water [10]. Direct observation of this phenomenon requires sensitivity of around $2 \times 10^{-17} \text{ m Hz}^{-1/2}$ [10], which even with a quantum limited measurement requires 19 W of 1064 nm light. This presents a problem, as the surrounding water is heated by approximately 8 K/Watt of 1064 nm

trapping light [20], such that the boiling point is reached when the optical power is around 10 W. The only way to surpass the quantum limit is to use correlated photons which allow more information to be extracted per photon [8, 13]. Such quantum resources can be integrated into particle tracking experiments [24], and with existing technology, offer up to 10 dB of enhancement [21]. With this quantum enhancement, measurement of the elastic compressibility of water would require a more achievable 2 W of optical power. Based on our calculations, and the advanced state of recent experiments, we conclude that quantum resources will play an important role in the next generation of high precision tracking experiments.

From the calculation presented here, it is clear that the minimum resolvable displacement improves as the objective NA increases, and as more scattered photons are measured. High sensitivity experiments should therefore utilize high NA objectives and capture as many scattered photons as possible. Neither of these conclusions should come as a surprise. The particles tracked in optical tweezers are assumed to be located at the centroid of the optical image, which is well known to improve with both of these parameters. This principle is applied in Chap. 5, where a method is presented to remove unscattered trapping light from the detection, and thus increase the scattered photon number which can be measured without saturating the detector.

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