Chapter 2
Machining Stability

Mark J. Jackson, H. Zhang, and J. Ni

Abstract Since the early days of regenerative chatter theory, it has been noticed that the phase difference between the current and the previous passes of machining self-excited vibration is correlated closely to the machining stability. However, an analytical proof of this fact has not been investigated, especially based on a nonlinear machining chatter models. In this chapter, an approach for determining the machining stability is presented in terms of the phase difference. The machining stability is demonstrated by a stability criterion in term of the phase difference sensitivity. By investigating the stability of the approximate solution of a nonlinear delay differential equation as the machining chatter model under small perturbations about an equilibrium state, the stability criterion is established. Through this approach, a theoretical proof of the relationship between the machining stability and the phase difference is given in terms of internal energy of the machining process. The analysis is in agreement with the numerical simulations and experimental data. Once the parameters of the machining system are identified, the stability criterion can be employed to predict the onset of machining chatter. The stability criterion identified for specific machining operations is of critical importance especially when using nanostructured coated cutting tools in turning and milling operations.

Keywords Machining • Stability • Chatter • Tools • Coatings • Nanomaterials

2.1 Introduction

It is a well-known fact that the phase difference between the vibration $x(t)$ in the current pass and the vibration $x(t-T)$ in the previous pass determines the machining stability [1–3]. When the phase difference is within 0–180°, the machining system is stable, and no regenerative chatter is observed, while when the phase difference is within 180–360°, the machining system is unstable, and regenerative

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chatter occurs, and the rotating direction of the chatter marks is seen as a right-hand spiral pattern on the workpiece. However, the analytical proof of this phenomenon has not been investigated, especially for the case when the machining chatter is modeled with nonlinearity. The study of phase difference and its sensitivity will provide a meaningful approach for machining chatter control: once the parameters of the machining system are identified, the stability criterion based on the phase difference and its sensitivity analysis can be employed to predict the onset of machining chatter. Chatter control is even more essential than before when high precision manufacturing becomes the common task, and the flexible, frequently changing working conditions increase the possibility of unstable machining [4].

The machining chatter with finite amplitude involves certain nonlinearity [5, 6]. The dynamic machining system can be described as the interaction between the elastic structure of the machine tool and the cutting process. In normal machining processes, the elastic structure has only very slight nonlinearity, mainly the nonlinearity lies in the cutting force. In this chapter, an analysis is based on a well-accepted nonlinear machining chatter model, which is a nonlinear delay differential equation [7, 8]. It is difficult to analytically find the exact solution of this equation. Although the solution can be provided by employing numerical integration algorithms and the step method [9], this procedure is only able to show the response of the nonlinear machining system to a particular set of initial values and initial function values with a certain set of parameter values. Therefore, the analytical relationship between the phase difference and the machining stability cannot be revealed this way. In order to characterize the relationship, an equivalent linearization technique is employed. Equivalent linearization techniques, which are used in ordinary differential equations, are shown to be valid in delay differential equations (called as “difference-differential equations” in early literature) as well [10, 11]. The procedure of the equivalent linearization is to replace the system nonlinearity with a linear gain, which is not constant, but rather a function of the amplitude and frequency of the system oscillation, and which renders the similar system responses to the same sinusoidal input. Based on the equivalent linearization of the nonlinear delay differential equation, sensitivity analysis of the phase difference can give a possible way to investigate the relative stability of the self-excited vibration that will provide a theoretical proof of the relationship between the phase difference and the machining stability. As an application, the phase difference sensitivity analysis can be used to interpret the mechanism of the spindle speed variation (SSV) method, which is receiving increasing attention for machining chatter suppression [12–25].

This chapter is organized in four sections. The physical interpretation how the phase difference is related to the machining stability is presented in Sect. 2.2. In Sect. 2.3, the nonlinear chatter model with single regenerative effect is introduced, then it is equivalently linearized, the sensitivity analysis of the phase difference is presented, the machining stability is studied based on the information of the phase difference sensitivity, and a stability criterion with the phase difference sensitivity is derived. Section 2.4 is verification of the stability criterion according to the internal energy calculations. The theoretical results are validated by numerical simulations. A set of the experimental data is also employed to support the stability analysis. The detailed derivation of the stability criterion is given in the appendix at the end of this chapter.

2.2 Phase Difference and Machining Stability: A Physical Interpretation

During the cutting process, the cutting tool might deviate from its steady motion due to some random disturbances, thus it vibrates relative to the workpiece at that moment. This vibration results in fluctuations in the dynamic cutting force, which drives the cutting tool continuously to oscillate in the cutting passes. If the amplitude of the relative vibration keeps increasing until its vibration amplitude is limited by the system nonlinearity, the machining system is unstable. Otherwise, the machining system is stable. Figure 2.1 displays the thrust component of the cutting force and the relative vibration between the cutting tool and the workpiece surface in the cutting passes. A phase difference angle between the current pass and the previous one is observed. The phase difference angle is the factor that determines whether or not the relative vibration is greater than before, that is, the phase difference determines the machining stability [25].

As is known, the relative vibration is supported by the energy supplied in the cutting process. In a cycle of the relative vibration, when the cutting tool oscillates into the workpiece, the dynamic cutting force is doing positive work, while when the cutting tool oscillates out, the dynamic cutting force is doing negative work (see Figs. 2.1 and 2.2). The net work in a cycle of vibration is the sum of the two works. If the net work is positive, the dynamic cutting force injects energy to the vibration

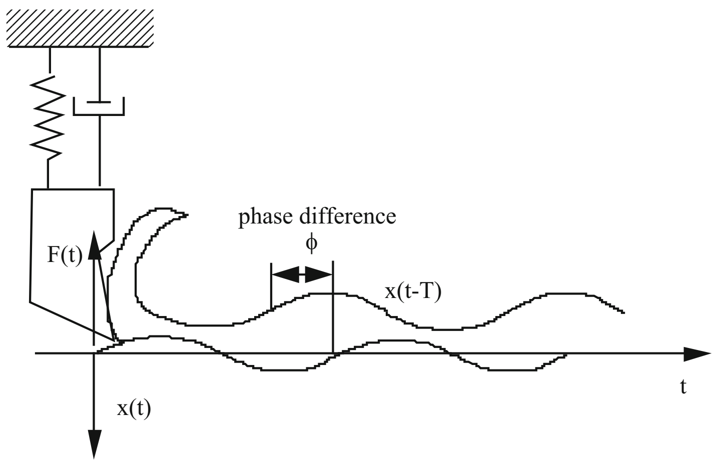

Fig. 2.1 Phase difference in the cutting process

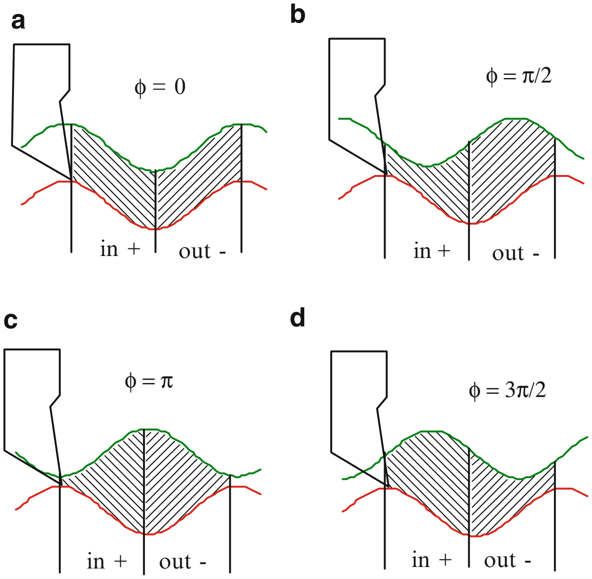

Fig. 2.2 Phase difference and net work in a vibration cycle

system, and the relative vibration becomes larger, thus the machining system is unstable. If the net work is negative, the dynamic cutting force dissipates energy from the vibration system, and the relative vibration decreases, thus the machining system is stable. The value of the net work in a cycle of vibration is determined by the phase difference in the cutting passes.

Figure 2.2 demonstrates several typical cases of the cutting process when the phase difference angle equals to 0°, 90°, 180°, 270°, respectively. The removed area of the workpiece is proportional to the work done by the dynamic cutting force when the cutting tool vibrates in or out the workpiece. For the case shown in Fig. 2.2a, the positive work equals the negative work, so the net work is zero in the vibration cycle, thus the machining system is stable. For the case in Fig. 2.2b, the phase difference is 90°, and the positive work is smaller than the negative work, the net work is negative, that is, the dynamic cutting force is dissipating energy from the system, thus the machining system is stable. When the phase difference is 180° (see Fig. 2.2c), due to symmetry of the areas when the tool vibrates in and out, no work is supplied to the machining system. The case of Fig. 2.2d is the opposite of Fig. 2.2b, the positive net work done by the dynamic cutting force in a cycle of vibration is injected to the vibration system, and results in an unstable cutting process. In general, it is believed that when the phase difference varies within 0–180°, the machining system is stable, while when the phase difference varies within 180–360°, the machining system is unstable.

This is only a physical interpretation. Can we give a theoretical proof on this relationship between the machining stability and the phase difference? Is this relationship still true when the nonlinearity is included in the machining system? In the following sections, the analytical relationship between the phase difference and the machining stability is revealed.

2.3 Sensitivity Analysis of the Phase Difference of Machining Chatter

In general, there are two main nonlinearity functions in the cutting process, i.e., (1) "the tool leaving the cut zone" partially in a cycle of vibration when the amplitude of chatter is large enough and (2) the nonlinear relationship between the cutting force and the chip thickness. Single regenerative effect is considered in the following machining chatter model. The single regenerative effect, of course, occurs in the situation of small vibration amplitude, that is, the cutting tool is always in the workpiece during the cutting process, if

$$A < \frac{s_{\mathrm{o}}}{2\sin\left(\frac{\omega T}{2}\right)} \tag{2.1}$$

where A is the chatter amplitude, s_{o} is the uncut chip thickness, ω is the chatter frequency, and time delay $T = 60/N$, N is the spindle speed in rpm. But the single regenerative effect could be valid for a special situation of "the tool leaving the cut" as well.

In fact the tool leaving the cut does not necessarily lead to multiple-regenerative effects. It can be proved that although "the tool leaving the cut" occurs, only single regenerative effect is involved, if

$$\left|\frac{s_{\mathrm{o}}}{2\sin\left(\frac{\omega T}{2}\right)}\right| < A < \left|\frac{s_{\mathrm{o}}}{\sin(\omega T)}\right| \tag{2.2}$$

The machining system can be described by a nonlinear delay differential equation (NLDDE) with single regenerative effect as follows:

$$\ddot{x}(t) + 2\varsigma\omega_n\dot{x}(t) + \omega_n^2 x(t) = -\frac{\omega_n^2}{k}\Delta F(t)\cos\alpha \tag{2.3}$$

$$\Delta F(t) = PW\left\{s^Y(t) - s_{\mathrm{o}}^Y + Y s_{\mathrm{o}}^{Y-1} C\frac{\dot{x}(t)}{N}\right\} \tag{2.4}$$

$$\begin{aligned} s(t) &= x(t) - x(t-T) + s_{\mathrm{o}}, \ \text{if} \ s(t) > 0; \\ &\text{else} \ s(t) = 0 \end{aligned} \tag{2.5}$$

Under the machining condition of slender shaft turning, the stiffness, k, of the machine tool structure is mainly contributed by the workpiece stiffness at the cutting point. In (2.4), the first two terms represent the dynamic variation of the cutting force, and the third term is the penetration force. Two constants P and Y are the cutting force constants. Besides, when "the tool leaving the cut" occurs, the third term equals to zero because there is no metal cutting at this moment. In this equation, since the nose radius of the cutting tool is small in relation to the depth of cut, the depth of cut is assumed as a linear factor in the dynamic cutting force. In this whole study, we focus on studying the nonlinearity in chip thickness.

Equation 2.5 expresses the instantaneous chip thickness. When the chatter amplitude exceeds a critical value, the tool leaves the workpiece material during part of the chatter cycle. The vibration $x(t-T)-s_{\mathrm{o}}$ in the previous pass has a phase difference with the present one. That makes the instantaneous chip thickness change with time (see Fig. 2.2). The uncut chip thickness $s_{\mathrm{o}}=f\sin\kappa$, where f is the feed rate. This model has been verified by experiments and numerical simulation being a valid approach to the real machining process [24, 25].

The relative changes in amplitude and frequency of $x(t)$ over one period of the oscillation are assumed small. This is reasonable for both chatter onset and fully developed chatter, which are usually a slowly changing process with respect to the requirements for the describing function techniques. According to the modified Krylov–Bogoliubov approach and the equivalently linearization, the real and imaginary parts of the characteristic equation of the NLDDE in steady-state case are given as follows:

$$R=-\omega^2+\omega_n^2+a_2-a_2\cos\phi+3.77\frac{A}{\pi}a_3(1-cos\phi)+\frac{3}{2}a_4A^2(1-cos\phi)^2=0 \tag{2.6}$$

$$I=a_1\omega+a_2\sin\phi+3.77\frac{A}{\pi}a_3\sin\phi+\frac{3}{2}a_4A^2\sin\phi(1-cos\phi)=0 \tag{2.7}$$

where

$$a_1=2\varsigma\omega_n+\frac{\omega_n^2}{k}PWYs_{\mathrm{o}}^{Y-1}\frac{C}{N}\cos\alpha$$

$$a_2=\frac{\omega_n^2}{k}PWYs_{\mathrm{o}}^{Y-1}\cos\alpha$$

$$a_3=\frac{1}{2}\frac{\omega_n^2}{k}PWY(Y-1)s_{\mathrm{o}}^{Y-2}\cos\alpha$$

$$a_4=\frac{1}{6}\frac{\omega_n^2}{k}PWY(Y-1)(Y-2)s_{\mathrm{o}}^{Y-3}\cos\alpha$$

$$\phi=\omega T$$

In the above equations, there are three variables: chatter amplitude A, chatter frequency ω, and the phase difference ϕ. Usually, the functions $A(t)$ and $\omega(t)$ can hardly be obtained. Noticing that the phase difference $\phi=\omega T$, if the analysis is directed towards determination of the functions $A^2(\phi)$ and $\omega(\phi)$, the steady-state response of the nonlinear machining system may be appropriately characterized from (2.6) to (2.7) with respect to the phase difference. Since these equations are nonlinear about the chatter frequency ω, the frequency ω_j and the amplitude A_j can be iteratively updated with respect to the instantaneous phase difference ϕ_j and ω_{j-1} at the j-th time interval.

For the steady-state self-excited vibration, the most important features are the amplitude A and the frequency ω of the equilibrium limit cycle state. The stability of the limit cycle can be determined by investigating the system behavior to small disturbances on the amplitude and the frequency of the equilibrium limit cycle state. In order to build the relationship between the phase difference sensitivity and the stability, the proposed sensitivity analysis relates to the differential changes in phase difference ϕ and frequency ω caused by differential changes in the amplitude A, that is,

$$\begin{aligned} S^{\phi} &= \frac{\partial \phi}{\partial A} \\ S^{\omega} &= \frac{\partial \omega}{\partial A} \end{aligned} \tag{2.8}$$

The equations are defined as the phase difference sensitivity and the frequency sensitivity, respectively.

Differentiating (2.6)–(2.7) with respect to the chatter amplitude A, and deriving the close formed expressions of the above sensitivities S^{ϕ} and S^{ω} lead to

$$\frac{\partial R}{\partial \omega} S^{\omega} + \frac{\partial R}{\partial \phi} S^{\phi} + \frac{\partial R}{\partial A} = 0 \tag{2.9}$$

$$\frac{\partial I}{\partial \omega} S^{\omega} + \frac{\partial I}{\partial \phi} S^{\phi} + \frac{\partial I}{\partial A} = 0 \tag{2.10}$$

or

$$\begin{pmatrix} S^{\omega} \\ S^{\phi} \end{pmatrix} = - \begin{pmatrix} \frac{\partial R}{\partial \omega} & \frac{\partial R}{\partial \phi} \\ \frac{\partial I}{\partial \omega} & \frac{\partial I}{\partial \phi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial R}{\partial A} \\ \frac{\partial I}{\partial A} \end{pmatrix} \tag{2.11}$$

where

$$\frac{\partial R}{\partial \omega} = -2\omega$$

$$\frac{\partial R}{\partial \phi} = a_2 \sin \phi + 3.77 \frac{A}{\pi} a_3 \sin \phi + 3 a_4 A^2 \sin \phi (1 - cos\phi)$$

$$\frac{\partial R}{\partial A} = 3 a_4 A (1 - \cos \phi)^2$$

$$\frac{\partial \mathrm{I}}{\partial \omega} = a_1$$

$$\frac{\partial I}{\partial \phi} = a_2 \cos \phi + 3.77\frac{A}{\pi}a_3 \cos \phi + \frac{3}{2}a_4A^2 \left[\cos \phi(1 - cos\phi) + \sin^2 \phi\right]$$

$$\frac{\partial I}{\partial A} = 3a_4A \sin \phi(1 - cos\phi)$$

The closed form expressions of the phase difference sensitivity and the frequency sensitivity can be obtained from (2.11). The value of the phase difference sensitivity is related to the stability of the solution. This relationship is revealed in the following section.

Between the machining system and the periodic solution of the NLDDE there is an interesting duality, that is, no chatter in the machining system corresponds to the stable trivial solution of the NLDDE, and the onset of chatter is the case of the unstable trivial solution of the NLDDE, and the fully developed chatter with a finite amplitude is the steady-state nontrivial solution of the NLDDE. The stability of the trivial solution of the NLDDE is more interesting to us. The phase difference ϕ is a parameter that is related to the stability of the trivial solution of the NLDDE. There is a bifurcation value ϕ^* of the phase difference ϕ ($0° \sim 360°$). If the phase difference ϕ is sufficiently small ($\phi < \phi^*$), the stable trivial solution is obtained. If ϕ increases up to the bifurcation values $\phi = \phi^*$, the system is just on the threshold of instability. For $\phi > \phi^*$, the self-excited vibration appears. Due to the nonlinear relationship of the machining cutting force, it is easier using the sensitivities to determine the relative stability of the machining system.

For steady-state case, stability of the limit cycle is determined in terms of the behavior of the system under small perturbations of the limit cycle state, i.e., under small perturbations of the amplitude and/or frequency of the limit cycle. If the limit cycle returns to its original equilibrium state after the perturbations, the system is considered *stable*, while if either its amplitude or frequency determinedly deviates from the original equilibrium state until another equilibrium state is reached, it is *unstable*. Then, from the variational relationship of replacing A by $A + \Delta A$ and ω by $\omega + \Delta\omega + j\Delta\sigma$, where $\Delta\sigma = -\dot{A}/A$, a stability index ($S_{\mathrm{I}}$) in terms of the phase difference sensitivity S^{ϕ} can be derived as follows (see Appendix):

$$S_{\mathrm{I}} = \left(\frac{\partial I}{\partial \omega}\cdot\frac{\partial R}{\partial A} - \frac{\partial R}{\partial \omega}\cdot\frac{\partial I}{\partial A}\right) + \left(\frac{\partial R}{\partial \phi}\cdot\frac{\partial I}{\partial \omega} - \frac{\partial R}{\partial \omega}\cdot\frac{\partial I}{\partial \phi}\right)S^{\phi} > 0 \tag{2.12}$$

When the left-hand side of the above equation is greater than zero, the machining system is stable. Additionally, the value of the left-hand side is also an index of the degree of the relative stability. The greater the value, the more stable the machining process.

In order to investigate the stability of the trivial solution of the NLDDE, we set the amplitude, $A \approx 0$, and use the above criterion to determine the stability. A set of parameters used in the chatter model and in the stability criterion are experimentally identified: the damping ratio $\zeta = 0.0384$, the natural frequency of the vibration system $\omega_{\mathrm{n}} = 101$ Hz, the stiffness of the system $k = 45{,}000$ N/m, the cutting force

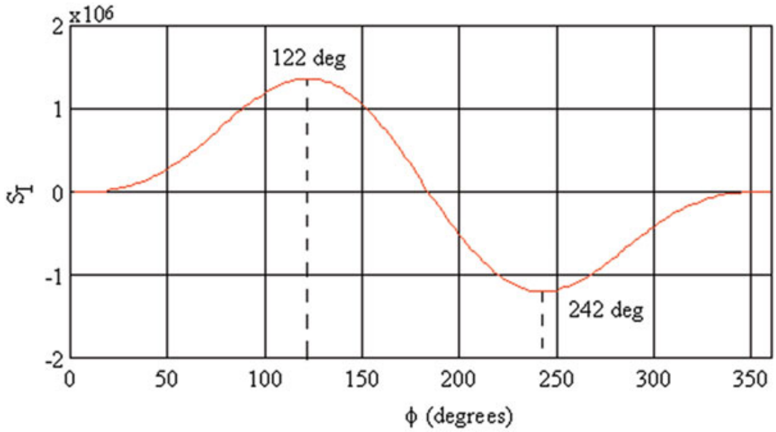

Fig. 2.3 Phase difference and the stability of the machining system

coefficient $P = 1{,}558.4$ N, the cutting force exponential $Y = 0.77$, and the penetration factor $C = 0.004$ [24, 25]. For a certain phase difference ϕ, the amplitude A and the frequency ω are determined by the equations of real values and imaginary values of the steady-state characteristic equation.

The phase difference sensitivity S^{ϕ} can be found from (2.11). Figure 2.3 displays the simulation result of the stability criterion with the phase difference ϕ varying with $0° \sim 360°$. The stability criterion indicates that when the phase difference ϕ is within $0° \sim 180°$, the trivial solution of the NLDDE is stable, i.e., the machining system is stable; and when the phase difference ϕ is within $180° \sim 360°$, the trivial solution is unstable, i.e., chatter occurs. The phase difference bifurcation value $\phi^* = 180°$. This result of the stability criterion exactly tallies with the physical interpretation on the phase difference and the machining stability, and it demonstrates that when the phase difference ϕ is varying within $180° \sim 360°$, the machining process is unstable, i.e., the regenerative chatter occurs. This phenomenon remains true for linear cutting processes. For the nonlinear cutting processes, the phase difference bifurcation value, ϕ^*, is not exactly equal to 180°, but it is still very close to 180°. The effect of nonlinearity in the cutting processes on the phase difference bifurcation value will be discussed in the following section.

2.4 Verification of the Stability Criterion

As mentioned before, the self-excited vibration is supported by the net work done by the dynamic cutting force during a vibration cycle. The net work can be calculated by the following equation:

$$\begin{aligned} E_f &= \int_0^{2\pi/\omega} -\Delta F(t) \cos \alpha \dot{x}(t) dt \\ &= -\int_0^{2\pi/\omega} Pw \left\{ y(t) + Y s_o^{Y-1} \frac{C}{N} \dot{x}(t) \right\} \cos \alpha \dot{x}(t) dt \end{aligned} \tag{2.13}$$

where $\Delta F(t)$ is given by (2.4), and

$$\begin{aligned} y(t) &= s^Y(t) - s_o^Y = [x(t) - x(t-T) + s_o]^Y - s_o^Y \\ &\approx Y s_o^{Y-1}[x(t) - x(t-T)] + \frac{1}{2} Y(Y-1) s_o^{Y-2} |x(t) - x(t-T)| [x(t) - x(t-T)] \\ &+ \frac{1}{6} Y(Y-1)(Y-2) s_o^{Y-3} [x(t) - x(t-T)]^3 \end{aligned} \tag{2.14}$$

The net work in a vibration cycle can be derived as follows:

$$\begin{aligned} E_f &= PWA^2 Y \pi s_o^{Y-1} \left[\frac{C}{60} \phi - \sin \phi \right] \\ &+ PWA^4 \pi \frac{1}{6} Y(Y-1)(Y-2) s_o^{Y-3} \left[-\frac{3}{4} \sin \phi + \frac{3}{2} \sin \phi \cos \phi - \frac{3}{4} \sin \phi \cos^2 \phi - \frac{3}{4} \sin^3 \phi \right] \end{aligned} \tag{2.15}$$

Figure 2.4 illustrates the relationship of the net work versus the phase difference ϕ when the chatter amplitude A is very small. The numerical simulation shows the work done by the dynamic cutting force during the phase difference varying from 0° to 360° (with the chatter amplitude of 0.001 mm, the chatter frequency of 101 Hz, the depth of cut, $w = 3$ mm, the uncut chip thickness $s_o = 0.1$ mm, and the spindle speed $N = 600$ rpm). It can be observed that for the phase difference angle varying from zero to about 180°, the energy is negative (the dynamic cutting force is dissipating energy from the vibration system in this vibration cycle), thus

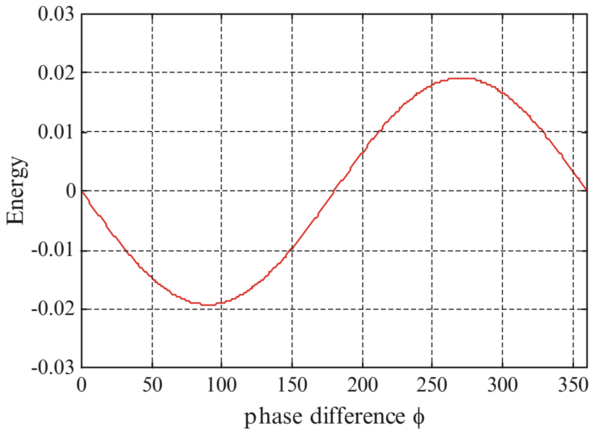

Fig. 2.4 The net work versus the phase difference ϕ (with $A = 0.001$ mm, energy unit, $N \cdot$ mm)

the vibration amplitude decreases, and if in the successive cycles the dynamic cutting force has tendency to dissipate more or the same energy from the vibration system, the machining process is stable; for the phase difference angle varying from about 180° to 360°, the energy is positive (the dynamic cutting force is injecting energy into the vibration system), thus the vibration amplitude increases, and if in the successive cycles the dynamic cutting force has tendency to input more or the same energy into the vibration system, the machining process is unstable. At about 180°, the energy is zero, and the system is just on the threshold of instability.

The following discussions can be made: (1) For the trivial solution case ($A \approx 0$), if the penetration effect is not considered, we have

$$E_f = PwA^2 Y \cos\alpha \{-s_{\mathrm{o}}^{Y-1}\pi \sin\phi\} \tag{2.16}$$

From $E_f = 0$, the bifurcation value of phase difference angle ϕ_{o} is found exact 180°. (2) For the trivial solution case ($A \approx 0$), if the penetration effect is considered, we have

$$E_f = PwA^2 Y \cos\alpha \left\{-s_{\mathrm{o}}^{Y-1}\pi C\frac{\omega}{N} - s_{\mathrm{o}}^{Y-1}\pi \sin\phi\right\} \tag{2.17}$$

The bifurcation value of phase difference ϕ_{o} is not exactly 180°, but around 180° (the simulation result gives 180.02°, at the spindle speed of 490 rpm). When the spindle speed is large enough, the penetration effect can be neglected, the bifurcation value of phase difference angle ϕ_{o} is exactly 180°. For nontrivial solutions with stable amplitude, if the penetration effect is not considered, the bifurcation value of the phase difference angle is calculated as 180.16°.

Figure 2.5 demonstrates the relationship of the net work versus the phase difference ϕ when "the tool leaving the cut" occurs and only single regenerative effect is involved. The following discussions can be made: For the nontrivial solution with stable amplitude, with the penetration effect and the nonlinear approach, the bifurcation value of the phase difference angle is calculated as 180.15°.

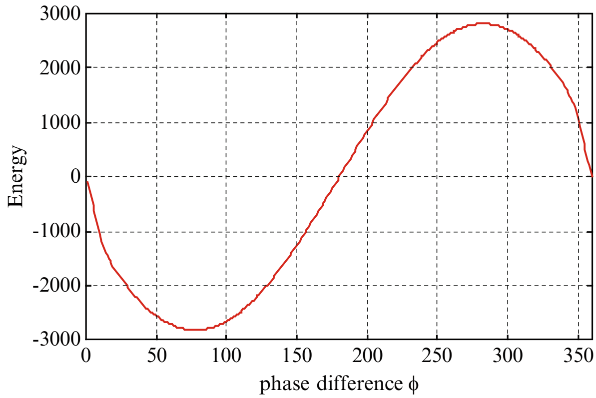

Fig. 2.5 The net work versus the phase difference ϕ (with $A = 0.5487$ mm, energy unit, $N \cdot$mm)

Although the nonlinearity in the cutting process plays an increasingly notable role along with the increment of the chatter amplitude, and yield some distortions of the net work curve, the basic characteristics remain almost unchanged: For the phase difference ϕ varying roughly within $0° \sim 180°$, the net work is negative, and the dynamic cutting force is dissipating energy from the vibration system, thus the machining process is stable. For the phase difference ϕ varying roughly within $180° \sim 360°$, the net work is positive, and the dynamic cutting force is supplying energy to the vibration system, thus the machining process is unstable. The bifurcation value of the phase difference ϕ* is about 180°. This result coincides with the conclusion of the stability criterion.

It is obvious that the phase difference angle ϕ is related to the stability of the machining system. In the following sections, we will derive a stability criterion in terms of the phase difference angle and its sensitivity, which can be used to indicate the degree of stability of the machining system.

The most stable and unstable phase difference angles can also be studied. Let

$$\frac{\partial E}{\partial \phi} = \int_0^{2\pi/\omega} -\frac{\partial}{\partial \phi}\Delta F(t)\cos\alpha\dot{x}(t)\,dt \tag{2.18}$$

that is,

$$\begin{aligned}\frac{\partial E}{\partial \phi} &= PWYs_o^{Y-1}A^2\cos\phi\pi + \frac{1}{2}PWA^4Y(Y-1)(Y-2)s_o^{Y-3}\\ &\left(cos\,\phi\frac{\pi}{4} + sin\phi + \cos\phi\pi + \cos^3\phi\frac{3}{4}\pi + \sin^2\phi\frac{3}{4}\pi\right)\\ &+ PWYs_o^{Y-1}A^2\frac{C}{60}\pi = 0\end{aligned} \tag{2.19}$$

The phase difference angles that satisfy the above equations are determined by the parameters s_o, Y, A, and C. Several case conditions are studied:

(*CASE 1*) For the case of $Y = 1$ (the cutting force is assumed linear), (2.19) can be simplified as

$$PWA^2\pi\left(\cos\phi + \frac{C}{60}\right) = 0 \tag{2.20}$$

The most stable phase difference angle, on which the net work done by the dynamic cutting force during a vibration cycle is a minimum, is given by

$$\phi_{\text{most stable}} = 90° + \cos^{-1}\left(\frac{C}{60}\right) \tag{2.21}$$

And the most unstable phase difference angle, on which the net work done by the dynamic cutting force during a vibration cycle is a maximum, is given by

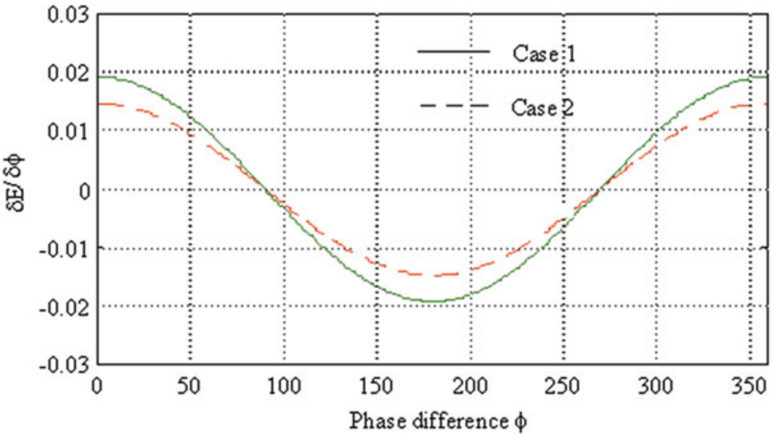

Fig. 2.6 The most stable and the most unstable phase difference angles (linear and small chatter amplitude cases)

$$\phi_{\text{most unstable}} = 270^\circ + \cos^{-1}\left(\frac{C}{60}\right) \tag{2.22}$$

Since the parameter C is very small, the most stable and most unstable phase difference angles are about 90° and 270°, respectively (see Fig. 2.6, Case 1).

(*CASE 2*) For the case of small chatter amplitude, $A \approx 0$, and $Y \neq 1$, (2.19) can be simplified as

$$PWA^2\pi Y s_{\mathrm{o}}^{Y-1}\left(cos\phi + \frac{C}{60}\right) = 0 \tag{2.23}$$

The most stable and unstable phase difference angles are the same as those in case 1, as shown in Fig. 2.6 Case 2.

(*CASE 3*) For the case of large chatter amplitude and nonlinear cutting force, the most stable and the most unstable difference angles are determined by (2.19). It can be observed in Fig. 2.7 that the most stable and the most unstable phase difference angles are still about 90° and 270°, respectively.

The above discussions imply that no matter whether the chatter amplitude is large or not, and whether the nonlinearity in the cutting process is considered or not, the most stable and the most unstable phase difference angles are around 90° and 270°, respectively. However, as shown in Fig. 2.3, the largest positive value of the stability criterion is found at 122° of the phase difference angle (the most stable phase difference angle), and the smallest negative value of the stability criterion is at 242° of the phase difference angle (the most unstable phase difference angle). These differences are due to the fact that the stability criterion is derived from the steady-state characteristic equations, and the influences of the derivatives of σ and

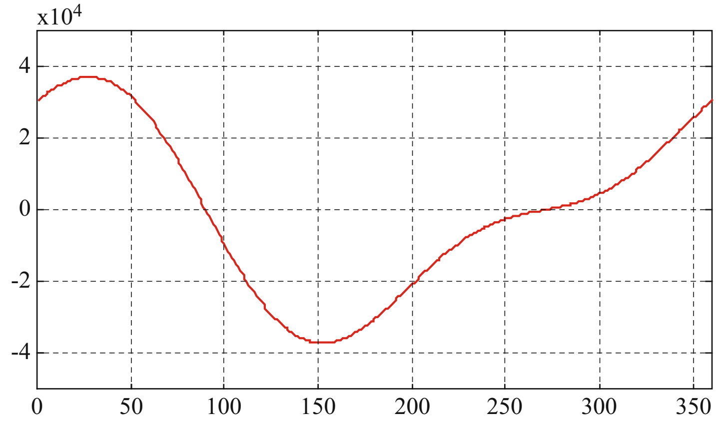

Fig. 2.7 The most stable and the most unstable phase difference angles (nonlinear case with large chatter amplitude $A = 0.5487$ mm)

the derivatives of ω are ignored. Fortunately, this simplification does not interfere with the correct indication of stability. Gelb [4] also claimed that stability criterions derived from the steady-state characteristic equations are almost always correct indices of stability.

The above results of machining stability and phase difference can also be verified by experimental results. Kasahara et al. [11] made their effort to investigate the phase characteristics of self-excited vibration in cutting. It is meaningful to point out that Kasahara's results provided an experimental support to our theoretical analysis. Figure 2.8 displays the spindle speed N, self-excited vibration signal $x(t)$, chatter frequency ω, and phase difference ν. In Fig. 2.8, four unstable machining conditions can be observed (let A represents the fully developed machining chatter amplitude):

(a) $N = 580$ rpm, $\omega = 162$ Hz, $\phi = 270°$, $A = 50$ μm.
(b) $N = 475$ rpm, $\omega = 157$ Hz, $\phi = 320°$, $A = 90$ μm.
(c) $N = 600$ rpm, $\omega = 162$ Hz, $\phi = 290°$, $A = 70$ μm.
(d) $N = 750$ rpm, $\omega = 156$ Hz, $\phi = 264°$, $A = 52$ μm.

Notice that the phase difference $\phi = 2\tilde{\pi}\nu$, where ν is the defined phase difference in Kasahara et al. [11], and ϕ is the defined phase difference in this work in agreement with the time delay concept. Nr is the number of designated revolution during the measurement. First of all, the experimental results display that when the phase difference ϕ is within 180–360°, the machining process is unstable. In all of these four cases, the fully developed chatters occurred.

Secondly, the effectiveness of our stability criterion equation can also be verified: According to Kasahara's work, the feed rate $f = 0.05$ mm/rev, the depth of cut $W = 0.5$ mm, the natural frequency $\omega_n = 156$ Hz, and the stiffness $k = 2$ N/μm. Assuming the cutting force parameters Y and P, and the penetration factor C are the

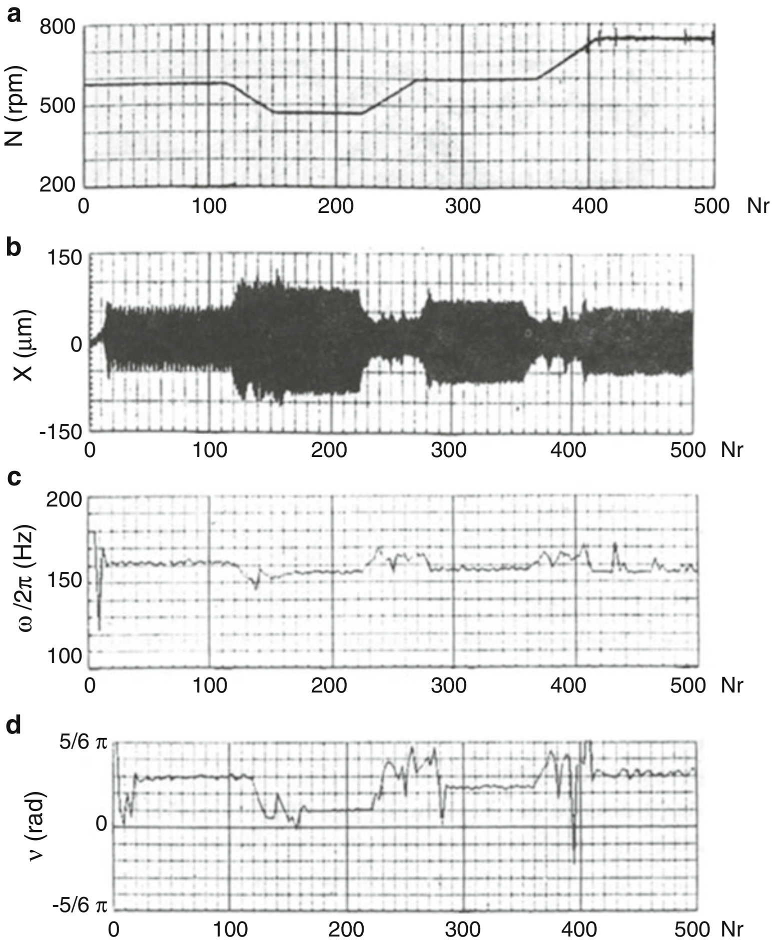

Fig. 2.8 Machining chatter experimental results (see Kasahara et al. [11])

same as that identified in our laboratory researches, i.e., $Y = 0.77$, $P = 1{,}558.4$ N, and $C = 0.004$. Then the stability index S_{I} can be calculated as follows: (a) $S_{\mathrm{I}} = -3{,}185.1$, (b) $S_{\mathrm{I}} = -747.7$, (c) $S_{\mathrm{I}} = -2{,}885.7$, and (d) $S_{\mathrm{I}} = -3{,}683.9$. All of them are negative, that is, according to the stability criterion with the phase difference sensitivity, these four cutting processes are unstable. It matches with the experimental results. Furthermore, with the experimental parameters identified by Kasahara, the similar stability chart as shown in Fig. 2.3 can also be obtained with the bifurcation phase difference angle at about 180°.

2.5 Conclusions

A theoretical proof on the well-known phenomenon of the phase difference and the machining stability is provided. The bifurcation value of the phase difference is about 180° in both linear and nonlinear cases. The phase difference sensitivity analysis can be used to investigate the stability of the machining system. A stability criterion in terms of the phase difference sensitivity is derived, and verified by numerical simulation and experimental data. Energy analysis on the well-known relationship of the phase difference is performed as a proof of the stability criterion. Once the parameters of the machining system are identified, the stability criterion could be used to predict the onset of machining chatter.

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Derivation of the Stability Criterion with the Phase Difference Sensitivity

The steady-state characteristic equation can be expressed as

$$R(A, \omega, \phi) + jI(A, \omega, \phi) = 0 \tag{2.24}$$

where A and ω are the steady-state amplitude and frequency of the machining chatter, and ϕ is the phase difference angle. Assume that the small perturbations in the chatter amplitude A, the change rate of the chatter amplitude $\Delta\sigma$, and frequency ω are caused by the small deviation of the phase difference angle $\Delta\phi$. The chatter amplitude A and frequency ω with small perturbations are given as

$$A- > A + \Delta A, \text{ and } \omega- > \omega + \Delta\omega + j\Delta\sigma \tag{2.25}$$

It is noticed that the small perturbation in the chatter frequency is associated with the change rate of the chatter amplitude, that is, $\Delta\sigma = -\dot{A}/A$, if the approximate solution of the NLDDE machining chatter model is given by the first-order approach. Hence, we have

$$\begin{aligned} &R(A + \Delta A, \omega + \Delta\omega + \Delta\sigma, \phi + \Delta\phi) \\ &\quad + I(A + \Delta A, \omega + \Delta\omega + j\Delta\sigma, \phi + \Delta\phi) \\ &\quad = 0 \end{aligned} \tag{2.26}$$

The Taylor series first-order expansion of (2.26) about the equilibrium state yields

$$\begin{aligned}&\frac{\partial R}{\partial A}\Delta A+\frac{\partial R}{\partial \omega}(\Delta\omega+j\Delta\sigma)+\frac{\partial R}{\partial \phi}\Delta\phi\\&\quad+j\frac{\partial I}{\partial A}\Delta A+j\frac{\partial I}{\partial \omega}(\Delta\omega+j\Delta\sigma)+j\frac{\partial I}{\partial \phi}\Delta\phi=0\end{aligned}\tag{2.27}$$

Both the real and the imaginary parts will vanish separately, if the above equation is satisfied:

$$\begin{aligned}&\frac{\partial R}{\partial A}\Delta A+\frac{\partial R}{\partial \omega}\Delta\omega+\frac{\partial R}{\partial \phi}\Delta\phi-\frac{\partial I}{\partial \omega}\Delta\sigma=0\\&\frac{\partial R}{\partial \omega}\Delta\sigma+\frac{\partial I}{\partial A}\Delta A+\frac{\partial I}{\partial \omega}\Delta\omega+\frac{\partial I}{\partial \phi}\Delta\sigma=0\end{aligned}\tag{2.28}$$

A single relationship among $\Delta\sigma$, ΔA, and $\Delta\phi$ can be obtained by eliminating $\Delta\omega$ from the equation set (2.28):

$$\begin{aligned}\left[\left(\frac{\partial I}{\partial \omega}\right)^2+\left(\frac{\partial R}{\partial \omega}\right)^2\right]\Delta\sigma&=\left(\frac{\partial R}{\partial A}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial A}\frac{\partial R}{\partial \omega}\right)\Delta A\\&\quad+\left(\frac{\partial R}{\partial \phi}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial \phi}\frac{\partial R}{\partial \omega}\right)\Delta\phi\end{aligned}\tag{2.29}$$

Notice that the phase difference sensitivity S^{ϕ} is defined as

$$S^{\phi}=\frac{\partial\phi}{\partial A}\approx\frac{\Delta\phi}{\Delta A}\tag{2.30}$$

Hence, (2.29) can be rewritten in the form

$$\left[\left(\frac{\partial I}{\partial \omega}\right)^2+\left(\frac{\partial R}{\partial \omega}\right)^2\right]\Delta\sigma=\left[\left(\frac{\partial R}{\partial A}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial A}\frac{\partial R}{\partial \omega}\right)+\left(\frac{\partial R}{\partial \phi}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial \phi}\frac{\partial R}{\partial \omega}\right)S^{\phi}\right]\Delta A\tag{2.31}$$

If the equilibrium state is stable, a positive increment ΔA must result in a negative derivative of the chatter amplitude, $\dot{A}$, thus a positive relative change rate of the chatter amplitude $\Delta\sigma$; and similarly, a negative increment ΔA must lead to a negative relative change rate of the chatter amplitude $\Delta\sigma$. Stated another way, for a stable equilibrium state, the following condition must be satisfied:

$$\left(\frac{\partial R}{\partial A}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial A}\frac{\partial R}{\partial \omega}\right)+\left(\frac{\partial R}{\partial \phi}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial \phi}\frac{\partial R}{\partial \omega}\right)S^{\phi}>0\tag{2.32}$$

This is the stability criterion with the phase difference sensitivity.

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