PREFACE

The book is the outcome of lectures and seminars on various aspects of differentiable manifolds that I have given over the years at the Indian Statistical Institute, Calcutta, and at other universities in India. The purpose of these lectures was to provide the necessary background for, and to train the students in the use of some fundamental tools of differential topology. The book may be used as an orientation course for advanced-level research students or for independent study. The prerequisites are an elementary knowledge of linear algebra, multivariate calculus, general topology, analysis, and some algebraic topology. I have tried to make demands on the reader's knowledge of background materials as modest as possible by brushing them up adequately whenever they are needed.

The book provides a systematic and comprehensive account of the theory of differentiable manifolds. More explicitly, the book has two objectives. The first is to serve as an introduction to the subject. The aim of the second part is to acquaint the reader with some epochal discoveries in the field of manifolds, mainly the earlier works of Stephen Smale for which he was awarded the Fields Medal. The topics covered include (1) Thom transversality, (2) Morse theory, (3) Theory of handle presentation, (4) h-cobordism theory and generalised Poincaré's conjecture. However, while trying to achieve these two objectives, I have not made any sharp division between them. They intermix in a natural way.

Chapter 1 begins with the basics of differentiable manifolds. These are introduced mainly by means of examples. Chapter 2 addresses the problems of various approximations leading to Whitney's embedding theorem. The main ingredient is Sard's theorem. Chapter 3 introduces tangent spaces, vector fields and flows, and the exterior algebra. The main results of this chapter are the Darboux-Weinstein theorem on symplectic structures, and an analogous theorem for contact structures. Chapter 4 discusses Riemannian manifolds, geodesics, and the exponential map. The main result of this chapter is the Hopf-Rinow theorem. Chapter 5 is a brief introduction to the concepts of differentiable vector bundles. We include Atiyah's construction of vector bundles, the homotopy property, and orientations. Chapter 6 is devoted to elementary transversality theory with simple applications. The main result here is Hopf's degree theorem.

The subject matter of Chapter 7 is tubular neighbourhoods and collar neighbourhoods. Their existence and uniqueness are obtained by using the isotopy extension theorem. Also included in this chapter are discussions of straightening corners of manifolds, and constructions of manifolds by the gluing process. Chapter 8 outlines topics and results concerning spaces of differentiable maps, and spaces of jets. We prove here Thom's transversality theorem, and the multi-jet transversality theorem. The main applications are Whitney's immersion and embedding theorems. Chapter 9 is concerned with Morse theory, its applications in computation of homology groups, and triangulation of differentiable manifolds. Chapter 10 offers the theory of handle presentation and its simplification leading to the h-cobordism theorem, and the solution of the generalised Poincaré's conjecture. The writing of this chapter has been influenced mainly by the book of J. Milnor on h-cobordism, and also by the lecture notes of C.T.C. Wall on differentiable manifolds.

I do not presume to give all the basic information in the field, or a survey touching on every topic. Rather, I treat various selected topics in differential topology, which interest me, from a point of view that I hope the reader will find appealing. There are also matters which I have simply mentioned without elaboration. I welcome comments, suggestions, and corrections from readers of the book so that a later edition may benefit from experience with this one.

The book may be divided into three parts. The first part comprising Chapters 1 to 4 is foundational. It will be useful to general students of pure mathematics even if they are not going to take up research in mathematics. It can be used to design a course at the M.Sc. level in Indian universities. The second part consists of Chapters 5 to 7. It caters to researchers in the areas of Topology, Differential or Algebraic Geometry and Global Analysis. It touches on advanced topics which are of general interest to serious people in these areas. These topics help in an in-depth understanding of these particular areas. Finally the third part, which is the remainder of the book, is meant for those desirous of working in the field of Differential Topology itself.

The present book is a revised version of an earlier book with the title "Topics in Differential Topology". The significant difference between this edition and the earlier one is that here we have omitted the last chapter (Chapter 11) on Gromov theory of homotopy principle of certain partial differential relations. Because this topic is too technical for beginners, and also because it deserves a separate volume. I have added a new section in Chapter 5 on integration of differential forms on manifolds and Stokes' theorem, although we have not used this topic in the rest of the book.

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