Preface

In this monograph, we study the structure of approximate solutions of linear optimal control problems with nonsmooth integrands. These problems are governed by differential equations whose right-hand side is linear with respect to a state and control variables. We establish a number of new results on properties of approximate solutions which are independent of the length of the interval, for all large intervals. As a matter of fact, these results provide a full description of the structure of approximate solutions of linear optimal control problems.

In our book, we study the turnpike phenomenon arising in the optimal control theory. The term was first coined by P. Samuelson in 1948 when he showed that an efficient expanding economy would spend most of the time in the vicinity of a balanced equilibrium path (also called a von Neumann path). To have the turnpike property means, roughly speaking, that the approximate solutions of the problems are determined mainly by the objective function and are essentially independent of the choice of interval and endpoint conditions, except in regions close to the endpoints. The turnpike property discovered by P. Samuelson is well known in the economic literature, where it was studied for various models of economic growth. Usually for these models a turnpike is a singleton.

Now it is well known that the turnpike property is a general phenomenon, which holds for large classes of variational and optimal control problems. In our research, using the Baire category (generic) approach, it was shown that the turnpike property holds for a generic (typical) variational problem [44] and for a generic optimal control problem [52]. According to the generic approach, we say that a property holds for a generic (typical) element of a complete metric space (or the property holds generically) if the set of all elements of the metric space possessing this property contains a G_δ everywhere dense subset of the metric space which is a countable intersection of open everywhere dense sets. This means that the property holds for most elements of the metric space.

Individual (non-generic) turnpike results and sufficient and necessary conditions for the turnpike phenomenon are of great interest because of their numerous applications in engineering and the economic theory. In particular, we are interested in the cases when a turnpike has a simple structure (a singleton or a periodic trajectory).

In this case, it is possible to find the turnpike (or at least its approximations) numerically. In our research which was summarized in [51], we obtained a number of individual (non-generic) turnpike results for variational problems. In our more recent book [53], we studied the turnpike phenomenon for discrete-time optimal control problems, which describe a general model of economic dynamics and for autonomous variational problems with extended-valued integrands. For these problems, the turnpike property was established with the turnpike being a singleton. In [53], for problems which satisfy concavity (convexity) assumption common in the literature, we also studied the structure of approximate solutions in the regions containing end points and obtained a full description of the structure of approximate solutions.

In this monograph, we are also interested in individual turnpike results but for linear optimal control problems which have important applications in engineering. We study two large classes of problems. The first class studied in Chap. 2 consists of linear control problems with periodic nonsmooth convex integrands. We show that for these problems the turnpike property holds and the turnpike is a periodic trajectory-control pair. The second class studied in Chaps. 3–5 consists of linear control problems with autonomous nonconvex nonsmooth integrands. It is shown that for this class of problems the turnpike phenomenon takes place with the turnpike being a singleton. For these two classes of problems, we study the structure of approximate solutions in the regions containing end points and obtain a full description of the structure of approximate solutions. We show that the structure of approximate solutions is stable under small perturbations of integrands. This stability is an important property from the view of practice if we are interested to find a turnpike or its approximations numerically. In the other chapters of the book, we establish a turnpike property for dynamic zero-sum games with linear constraints (see Chap. 6), obtain the description of the structure of variational problems with extended-valued integrands (see Chap. 8), and study the turnpike phenomenon for dynamic games with extended-valued integrands (see Chap. 9).

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