

Chapter 3

Gravitational Waves: Detection and Sources

3.1 Gravitational Wave Detectors

In Sect. 1.4, we have shown that a GW causes the stretching and shrinking of the proper distance between test particles. Therefore, GWs can be observed by measuring these changes in the proper distance. The focus of this work will be on the use of interferometers to measure the influences of GWs. These interferometers are configured to show destructive interference when no distortions are present, as shown Fig. 3.1a. When a GW passes such a detector, the configuration will no longer support the destructive interference and a bright spot can be seen, as shown in Fig. 3.1b. By measuring the intensity of this spot as well as its temporal variation, one will be able to observe the footprints of a GW.

3.1.1 Beam Pattern Functions

Interferometers are sensitive to the relative difference between two distances, the so-called *strain*. Suppose we have an interferometer with its arms pointing along the unit vectors u^i and v^i . One can show [2] that the strain, $h(t)$, is given by

$$\begin{aligned} h(t) &= \frac{1}{2}(h_{ij}u^iu^j - h_{ij}v^iv^j) \\ &= D^{ij}h_{ij}(t), \end{aligned} \quad (3.1)$$

where D^{ij} is referred to as the detector tensor and is given by

$$D^{ij} = \frac{1}{2}(u^iu^j - v^iv^j). \quad (3.2)$$

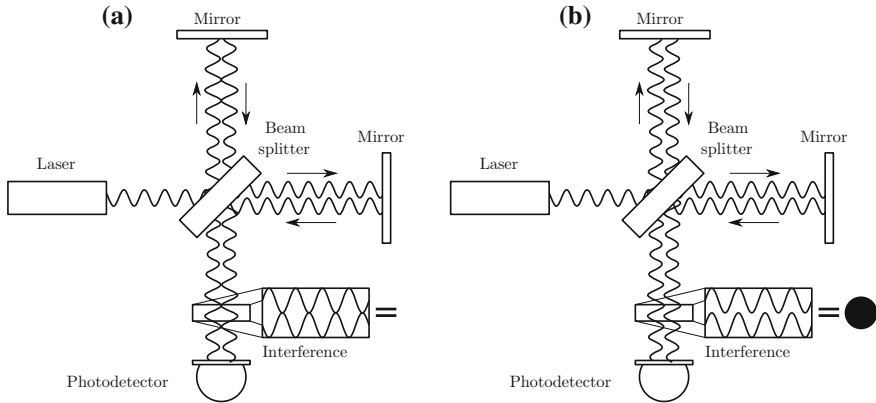


Fig. 3.1 Schematic overview of an interferometric detector [1]. Without a GW present, the interferometer is kept at a destructive interference configuration. A GW will change the relative distance between the arms and cause the interferometer output to deviate from the destructive interference. The intensity of the light and its temporal evolution encode the footprints of GWs. **a** No GW present, **b** GW present

Furthermore, as Eq. (3.1) is linear in h_+ and $h_\times$, we can also write

$$h(t) = F_+ h_+(t) + F_\times h_\times(t), \quad (3.3)$$

where $F_{+, \times}$ are called the *beam pattern functions*.

Suppose we have a detector with arms that are perpendicular to each other, one pointing in the x -direction and the other in the y -direction in a Cartesian coordinate system. This detector frame, denoted by (x, y, z) , is generally different from the GW coordinate system, denoted by (x', y', z') , where the source is conveniently described. To account for such a difference, we first note that when the plus and cross polarisations are not equal in strength, we can rotate the coordinate system by an angle ψ around the z' axis so that the x' and y' axes coincide with the major and minor axis of the associated ellipse. In going from the GW frame to the detector frame, we can rotate the GW frame by an angle θ around the x' axis and an angle ϕ around the z' axis, where the angles (θ, ϕ) denote the direction of propagation of the GW in the detector frame. Applying these three rotations, the beam pattern functions for a detector with perpendicular arms are given by

$$F_+^{(90^\circ)} = \frac{1}{2} \left(1 + \cos^2 \theta \right) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi, \quad (3.4)$$

$$F_\times^{(90^\circ)} = \frac{1}{2} \left(1 + \cos^2 \theta \right) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi. \quad (3.5)$$

The beam pattern functions are shown in Fig. 3.2. Suppose that the arms of the detector are at a 60° angle instead of a 90° angle. In this case, the beam pattern

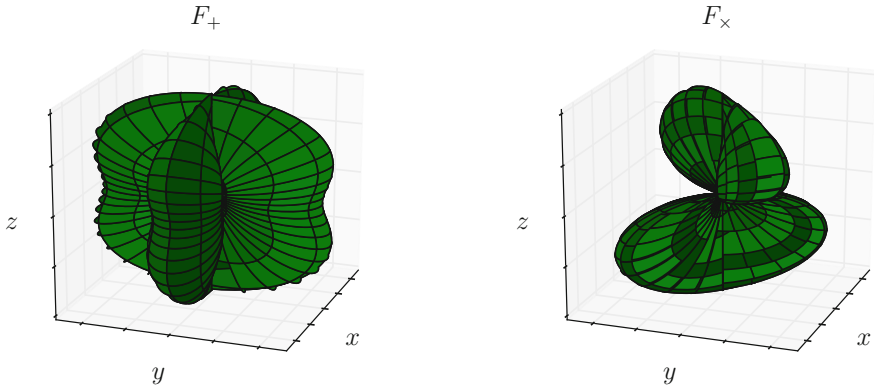


Fig. 3.2 Beam pattern functions $F_+(\theta, \phi, \psi)$ (left) and $F_\times(\theta, \phi, \psi)$ (right) for an interferometric detector with arms at a 90° angle

functions are given by

$$F_+^{(60^\circ)} = \frac{\sqrt{3}}{2} \left[\frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi \right], \quad (3.6)$$

$$F_\times^{(60^\circ)} = \frac{\sqrt{3}}{2} \left[\frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi \right]. \quad (3.7)$$

Compared to a detector with arms at a 90° angle, a detector with a 60° opening angle is less sensitive to GWs. However, with a 60° opening angle, one is able to fit three detectors forming an equilateral triangle in the same location. Especially when the detectors are to be underground, this will significantly save on costs for e.g. digging tunnels.

3.1.2 Ground-Based Detectors

At the moment, there are several ground-based interferometric gravitational waves detectors, either operational or in an upgrade phase. Only GEO (600m arms) in Germany [3] is currently taking data. However, the biggest of these detectors, LIGO (two interferometers, 4km arm length) in the US [4] and Virgo (3 km arm length) in Italy [5] are currently being upgraded from the initial/enhanced configurations (collectively referred to as 1st generation) to the so-called advanced configuration (2nd generation). An aerial view of the LIGO detector is shown in Fig. 3.3. This round of major upgrades is expected to complete in 2014 and will increase their sensitivity significantly. A Japanese detector named KAGRA [6] is currently under



Fig. 3.3 Aerial view of the LIGO detector in Hanford. Photo courtesy of the LIGO Laboratory

construction, and is expected to be operational in 2018. Finally, India aims to have an advanced detector named LIGO-India running by 2020 [7].

Plans are also made for detectors beyond the 2nd generation. The most advanced proposal is the so-called Einstein Telescope (ET), a European initiative for a 3rd generation detector. A conceptual design study has been published proposing three co-located, underground detectors, each with 10km arms with a 60° opening angle [1]. Such a design admits even better sensitivity compared to 2nd generation detectors, up to a factor of ten (see Fig. 3.4).

3.2 Detection of Gravitational Waves

So far we have only considered the effect of a GW on a ring of test particles. In an actual detector, we will not only see the effect of GWs on the test masses (mirrors in case of an interferometer), but also noise contributions such as human traffic. If we denote the noise contribution as $n(t)$ and the gravitational wave induced strain as $h(t)$, then the total output of the detector is given by

$$s(t) = n(t) + h(t). \quad (3.8)$$

Therefore, not only will we have to understand the effects of GWs, it is also important to understand the noise in order to find the GW footprint in the detector output. We will consider each of these contributions in turn.

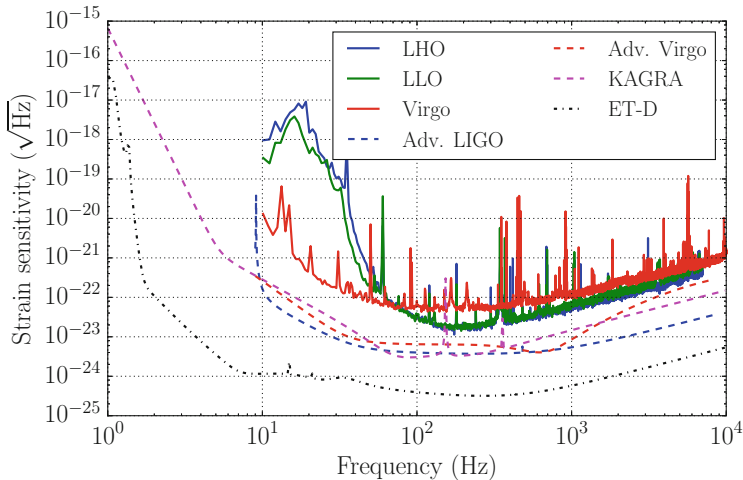


Fig. 3.4 Detector sensitivity for various detectors in terms of the strain sensitivity, $\sqrt{S_n(f)}$. The *solid curves* represent the strain sensitivity of LIGO in Hanford (LHO, *blue*), LIGO in Livingston (LLO, *green*) and Virgo in Pisa (*red*) on their last joint science run, before the upgrade to their advanced configurations. The *dashed curves* show the design sensitivity for the second generation detectors. Advanced LIGO (*blue*), Advanced Virgo (*red*), KAGRA (*magenta*). The *dashed-dotted curve* shows the design sensitivity labelled ET-D for the third generation detector Einstein Telescope (*black*)

3.2.1 Characterising the Noise

Suppose we can describe the noise $n(t)$ by a Gaussian stochastic process [8]. A Gaussian stochastic process is uniquely characterised by its average value and its auto-correlation. Without loss of generality, we can set the average value to be

$$\langle n(t) \rangle = 0, \quad (3.9)$$

where the brackets denote the ensemble average. In practice, one only has access to a single realisation of the detector. Therefore, we assume ergodicity and replace the ensemble average with the time average. The auto-correlation, defined as

$$R(\tau) \equiv \langle n(t + \tau)n(t) \rangle, \quad (3.10)$$

typically decays to zero when $|\tau| \rightarrow \infty$, i.e. the knowledge of the noise at time t reveals little information about the noise at a time $t + \tau$. We can now define the *one-sided power spectral density* as

$$\frac{1}{2} S_n(f) \equiv \int_{-\infty}^{\infty} d\tau R(\tau) e^{i2\pi f\tau}. \quad (3.11)$$

The reality of $R(\tau)$ implies that $S_n(-f) = S_n^*(f)$, while the invariance under time translation implies that $S_n(f)$ is real. Therefore, one has $S_n(-f) = S_n(f)$. With Eq. (3.10) and the inversion of Eq. (3.11), we can write

$$\begin{aligned} R(0) &= \langle n^2(t) \rangle \\ &= \int_0^\infty df S_n(f). \end{aligned} \quad (3.12)$$

This shows that the integrated power spectral density is equal to the variance of the noise. The relation in Eq. (3.12) also allows us to introduce an alternative form of the power spectral density, i.e.

$$\langle \tilde{n}^*(f) \tilde{n}(f') \rangle = \delta(f - f') \frac{1}{2} S_n(f), \quad (3.13)$$

where a tilde denotes the Fourier transform and a star the complex conjugate. The (expected) power spectral densities for various detectors are shown in Fig. 3.4.

3.2.2 Matched Filtering

In the last subsection, we looked at characterising the noise. Naively, one might expect that the signal $h(t)$ can only be detected if $|n(t)| < |h(t)|$. However, for ground-based GW detectors, we are in the regime where $|n(t)| \gg |h(t)|$. It turns out that, provided one has information on $h(t)$, we can still find the GW within the data even if the noise amplitude is bigger than the GW amplitude.

Suppose we know the shape of $h(t)$. One can compute the average of the product of $s(t)$ with $h(t)$ over a period of T

$$\frac{1}{T} \int_0^T dt s(t) h(t) = \frac{1}{T} \int_0^T dt h^2(t) + \frac{1}{T} \int_0^T dt n(t) h(t). \quad (3.14)$$

The integral in the first term on the RHS is positive definite and thus grows as T . In terms of the order of magnitude, we have

$$\frac{1}{T} \int_0^T dt h^2(t) \sim h_0^2, \quad (3.15)$$

where h_0 is the characteristic amplitude of the oscillating function $h(t)$. On the other hand, in the integral in the second term on the RHS, $n(t)$ and $h(t)$ are uncorrelated and the integral thus grows like $T^{1/2}$, similar to a random walk. Therefore, we have

$$\frac{1}{T} \int_0^T dt n(t) h(t) \sim \sqrt{\frac{\tau_0}{T}} n_0 h_0, \quad (3.16)$$

where n_0 is the characteristic amplitude of the noise $n(t)$ and τ_0 is a characteristic timescale, e.g. the period of $h(t)$. Therefore, it is not necessary to have $|n(t)| < |h(t)|$. Instead, it suffices to have

$$h_0 > \sqrt{\frac{\tau_0}{T}} n_0. \quad (3.17)$$

Thus, if we integrate the signal over long period compared to τ_0 , we can average the noise to zero and detect the signal even when $|n(t)| > |h(t)|$. To exemplify, a binary system has a characteristic frequency of $f \sim 100$ Hz, or $\tau_0 \sim 10^{-2}$ s when it is detectable by ground-based detectors. Such signals can be observed in the order of minutes, say $T \sim 100$ s. Therefore, we have $\sqrt{\tau_0/T} \sim 10^{-2}$, meaning that the signal can still be detected if the signal amplitude is factor of 100 smaller than the noise amplitude. For a continuous signal, such as a millisecond pulsar, this factor can be even larger. Suppose we have $\tau_0 \sim 10^{-3}$ s, but we can observe the signal for a period of a year, $T \sim 10^7$ s. Using these numbers, we have $\sqrt{\tau_0/T} \sim 10^{-5}$, meaning we are sensitive to signals with an amplitude of a factor of 100,000 smaller than the noise amplitude.

We can optimise the above by the application of a technique called *matched filtering*. We define

$$\hat{s} = \int_{-\infty}^{\infty} dt s(t) K(t), \quad (3.18)$$

where $K(t)$ is called a filter. Furthermore, we define a signal-to-noise ratio (SNR), $\rho = S/N$, where S is the expectation value of $\hat{s}$ if a GW signal is present and N is the root mean square value of $\hat{s}$ when no GW signal is present. The idea behind matched filtering is to find the filter that maximises the SNR. Since $\langle n(t) \rangle = 0$, we have

$$\begin{aligned} S &= \int_{-\infty}^{\infty} dt \langle s(t) \rangle K(t) \\ &= \int_{-\infty}^{\infty} dt h(t) K(t) \\ &= \int_{-\infty}^{\infty} df \tilde{h}(f) \tilde{K}^*(f). \end{aligned} \quad (3.19)$$

Furthermore, for $h(t) = 0$ we have

$$\begin{aligned} N^2 &= \langle \hat{s}^2 \rangle - \langle \hat{s} \rangle^2 \\ &= \langle \hat{s}^2 \rangle \end{aligned}$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt \, dt' \, K(t) K(t') \langle n(t) n(t') \rangle \\
&= \frac{1}{2} \int_{-\infty}^{\infty} df \, S_n(f) |\tilde{K}(f)|^2,
\end{aligned} \tag{3.20}$$

where we have used the definition of the power spectral density in Eq. (3.11). Subsequently, the SNR is given by

$$\rho = \frac{\int_{-\infty}^{\infty} df \, \tilde{h}(f) \tilde{K}^*(f)}{\sqrt{\int_{-\infty}^{\infty} df \, \frac{1}{2} S_n(f) |\tilde{K}(f)|^2}}. \tag{3.21}$$

This expression can be simplified by introducing an inner product between $a(t)$ and $b(t)$

$$(a|b) = \Re \int_{-\infty}^{\infty} df \, \frac{\tilde{a}^*(f) \tilde{b}(f)}{\frac{1}{2} S_n(f)}. \tag{3.22}$$

With this definition, we can rewrite Eq. (3.21) as

$$\rho = \frac{(u|h)}{\sqrt{(u|u)}}, \tag{3.23}$$

where $\tilde{u}(f)$ is given by

$$\tilde{u}(f) = \frac{1}{2} S_n(f) \tilde{K}(f). \tag{3.24}$$

From this geometrical representation, we can deduce that ρ is maximised when u points in the same direction as h . The optimal filter is therefore given by the so-called *Wiener filter*

$$\tilde{K}(f) \propto \frac{\tilde{h}(f)}{S_n(f)}. \tag{3.25}$$

Furthermore, the SNR for the Wiener filter, also referred to as the *optimal* SNR, is given by

$$\rho = \sqrt{(h|h)}. \tag{3.26}$$

From the above results, we can conclude that the optimal filter depends on the signal searched for, and the properties of the noise. It is therefore critical to gain a good understanding of both aspects.

Assuming that the noise is uncorrelated between detectors, we can also define the optimal SNR for a *network* of detectors by adding the SNR of individual detectors in quadrature. In other words, the network SNR is given by

$$\rho_{\text{net}}^2 = \sum_i \rho_i^2, \quad (3.27)$$

where the index i runs over individual detectors.

In the derivation of the matched filtering, we have assumed that the shape of the true signal and the characteristics of the noise are known. For GW analyses, however, several issues are present. Firstly, the characterisation of the noise is not trivial. For example, the noise might not be stationary and Gaussian. Moreover, there is an inherent ambiguity whether a GW signal is being included in the noise characterisation. Secondly, one rarely has access to the exact same GW signal in the filter. For example, for most realistic GW sources, the waveform is only known in the form of an approximation (cf. the PN formalism in Chap. 2). Furthermore, as the GW signal depends on many parameters, the discretisation of the collection of filters used can cause an SNR lower than the SNR given by the exact Wiener filter. We will further discuss the issues concerning the interpretation of GW signals in Chap. 4.

3.3 Sources of Gravitational Waves

Although every accelerating non-symmetric system emits GWs, only the ones that are able to perturb the metric enough can be detected with current technology. Naturally, we are looking for those systems that are massive, compact and/or violent enough to induce a sufficiently strong gravitational field. It turns out that such sources are only of astrophysical origin, such as black holes (BHs) and neutron stars (NSs) or violent events like supernovae and gamma-ray bursts. In this section, we will look at some of the most promising sources for detection by ground-based detectors. The study of the sources of GWs is an extensive field and citing only a few references will not do justice to the large body of work performed by numerous authors. Therefore, unless explicitly stated, references and/or proofs to relevant statements can be found in Ref. [2] and references therein.

3.3.1 Compact Binary Coalescences

Compact binary coalescence (CBC) is a class of sources in which two compact objects, either a NS or a BH, orbit around each other. CBC sources are further subdivided into binary neutron stars (BNS), black hole neutron star binary (BHNS) and binary black hole (BBH). The emission of GWs carries energy and angular momentum away from the system, causing the two objects to spiral towards each other. The most famous CBC system is perhaps the Hulse-Taylor binary pulsar [9].

The orbital evolution of the Hulse-Taylor binary pulsar provided the first indirect evidence of the emission of GWs. GWs from CBC sources can be divided into three stages, the inspiral, the merger and the ringdown phase.

The *inspiral* phase is when the two objects are spiralling towards each other, while the system loses orbital energy and angular momentum. As a result, the GW amplitude and frequency slowly increase as a function of time, the so-called *chirp*. Sources can remain in the inspiral phase for hundreds of millions of years, but they can only be observed by ground-based detectors towards the end of the inspiral phase. During the inspiral phase, the gravitational fields and velocities are still relatively weak and the GWs emitted can be found through approximation methods such as the PN formalism in Chap. 2.

The *merger* phase is a short-lived phase that follows the inspiral phase and occurs when the two objects are so close to each other that they start to merge into a single object. The gravitational fields are now very strong and the GW emission can only be computed by considering the full EFE numerically. As it turns out, the merger phase can cause a GW luminosity that exceeds the electromagnetic luminosity of the entire Universe.

Once the two objects have merged, they enter the *ringdown* phase. During this phase, the newly formed object tries to reach a quiescent state by radiating away its deformations from the merger phase. The GWs in the ringdown phase can be computed by means of perturbation theory. The resulting GWs consist of superpositions of damped sinusoids, known as quasi-normal modes.

3.3.2 Continuous Wave Sources

Continuous wave (CW) sources are those that emit GWs with roughly constant frequency and amplitude compared to the observation time. The prime candidates to emit such signals are rapidly rotating, non-axisymmetric NSs. Such deformations can arise due to, for example, strain build-up in the crust or in the core, or accretion of matter.

The GW signal from CW sources is relatively simple due to their slow variation of amplitude and frequency. Although the GW amplitude is generally weaker compared to CBC sources, a longer integration time means that CW sources may also achieve detectable SNRs. Finally, the analysis of such sources can be simplified if some of its characteristics can be determined through means other than GWs, e.g. rotational frequency of a NS through the observation of a pulsar.

3.3.3 Burst Sources

Burst sources are associated with astronomical transient phenomena, such as supernovae, gamma-ray bursts or instabilities in NSs. Supernovae occur when massive objects collapse under the influence of gravity. In the process that follows, NSs or

BHs are formed, and GWs are emitted through dynamical processes. On the other hand, GRBs are flashes of gamma rays, for which the progenitors are still uncertain. Possible progenitors are supernova-like events (causing long duration soft spectrum GRBs, also called long soft GRBs) as well as the merger of compact objects (causing short hard GRBs).

These sources are generally difficult to model because of the complicated physics associated with such phenomena. Therefore, the search for GWs from such sources often happens with unmodelled filters, where unmodelled means that there is no astrophysical model associated to the GW. An example of such a filter is a sine-Gaussian signal.

3.3.4 Stochastic Background

The stochastic background comes from the superposition of numerous unresolved GW sources. The stochastic background is divided into two classes: the primordial background and the astrophysical background. The primordial background consists of radiation originating from the early Universe, such as the Big Bang. The astrophysical background comes from the GWGW radiation from astrophysical sources such as CBC systems or cosmic-string cusps and kinks.

The detection of a stochastic background is somewhat different compared to the sources described above. Since random radiation is indistinguishable from instrumental noise, at least for short observation times, it cannot be detected with a single detector. Instead, the output of several detectors are combined to calculate the cross-correlation between detectors. An excess in the cross-correlation can then be an indication of a stochastic background.

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